33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, P-5.016 (2006)

Nonlinear dynamics of interacting intense laser pulses in plasmas

M. Marklund[1,a], B. Eliasson[2,b] and P. K. Shukla[2,c]

[1] *Department of Physics, Umeå University, SE–901 87 Umeå, Sweden*

[2] *Institut für Theoretische Physik IV, Fakultät für Physik und Astronomie, Ruhr-Universität Bochum, D–44780 Bochum, Germany*

Email:
[a] `mattias.marklund@physics.umu.se` [b] `bengt@tp4.rub.de`, [c] `ps@tp4.rub.de`,

Abstract

The nonlinear interaction between two laser beams in a plasma in the weakly nonlinear and relativistic regime is discussed. The laser dynamics is governed by two nonlinear Schrödinger equations, coupled via the slow plasma density response. Numerical studies of the growth rates of the Raman forward and backward scattering instabilities as well of the Brillouin and self-focusing/modulational instabilities are presented. Numerical results on the nonlinear evolution of the instabilities are given.

The interaction between intense laser beams and plasmas leads to a variety of different instabilities, including Brillouin and Raman forward and backward scattering [1–6], as well as modulational instabilities. In multiple dimensions we also have filamentation and side-scattering instabilities. Relativistic effects can then play an important role [1, 6, 7]. Here we consider the nonlinear interaction between two weakly relativistic crossing laser beams in plasmas. We present a set of nonlinear mode coupled equations and nonlinear dispersion relations, and an analysis for Raman backward and forward scattering instabilities as well as for Brillouin and modulation/self-focusing instabilities is given.

In the slowly varying envelope approximation the evolution of the laser pulses in an electron–ion plasma is given by two coupled nonlinear Schrödinger equations [8]

$$-2i\omega_1\left(\frac{\partial}{\partial t}+\vec{v}_{g1}\cdot\nabla\right)\vec{A}_1 - c^2\nabla^2\vec{A}_1 + \omega_{\rm p0}^2 N_s\vec{A}_1 - \omega_{\rm p0}^2\frac{e^2}{m_e^2c^4}(|\vec{A}_1|^2+|\vec{A}_2|^2)\vec{A}_1 = 0, \quad (1)$$

and

$$-2i\omega_2\left(\frac{\partial}{\partial t}+\vec{v}_{g2}\cdot\nabla\right)\vec{A}_2 - c^2\nabla^2\vec{A}_2 + \omega_{\rm p0}^2 N_s\vec{A}_2 - \omega_{\rm p0}^2\frac{e^2}{m_e^2c^4}(|\vec{A}_1|^2+|\vec{A}_2|^2)\vec{A}_2 = 0, \quad (2)$$

where $\vec{v}_{gj} = \vec{k}_j c^2/\omega_j$ is the group velocity, $\omega_j = (\omega_{p0}^2 + c^2k_j^2)^{1/2}$ is the electromagnetic wave frequency, $\omega_{\rm p0} = (4\pi n_0 e^2/m_e)^{1/2}$ is the electron plasma frequency, and we have denoted $N_s = n_{es1}/n_0$. The slow density response is determined via either (a) Raman scattering or (b) Brillouin scattering. In case a we obtain

$$\left(\frac{\partial^2}{\partial t^2} - 3v_{Te}^2\nabla^2 + \omega_{\mathrm{p0}}^2\right) N_s = \frac{e^2}{m_e^2 c^2}\nabla^2(|\vec{A}_1|^2 + |\vec{A}_2|^2), \tag{3}$$

where the electron thermal velocity is denoted by $v_{Te} = (T_e/m_e)^{1/2}$. For case b we find

$$\left(\frac{\partial^2}{\partial t^2} - c_s^2\nabla^2\right) N_s = \frac{e^2}{m_e m_i c^2}\nabla^2(|\vec{A}_1|^2 + |\vec{A}_2|^2), \tag{4}$$

where the sound speed is $c_s = \sqrt{(T_e + 3T_i)/m_i}$ and T_i is the ion temperature.

The nonlinear dispersion relation can be obtained by letting $\vec{A}_j = [\vec{A}_{j0} + \vec{A}_{j+}\exp(i\vec{K}\cdot\vec{r} - i\Omega t) + \vec{A}_{j-}\exp(-i\vec{K}\cdot\vec{r} + i\Omega t))]\exp(-i\Omega_0 t)$, where $|\vec{A}_{j0}| \gg |\vec{A}_{j\pm}|$. The nonlinear dispersion relation then reads

$$\frac{1}{Q} + \left(\frac{1}{D_{1+}} + \frac{1}{D_{1-}}\right)|\vec{A}_{10}|^2 + \left(\frac{1}{D_{2+}} + \frac{1}{D_{2-}}\right)|\vec{A}_{20}|^2 = 0, \tag{5}$$

which relates the complex-valued frequency Ω to the wavenumber $\vec{K}$. Here $D_{j\pm} = \pm 2[\omega_j\Omega - c^2\vec{k}_j\cdot\vec{K}] - c^2K^2$, and

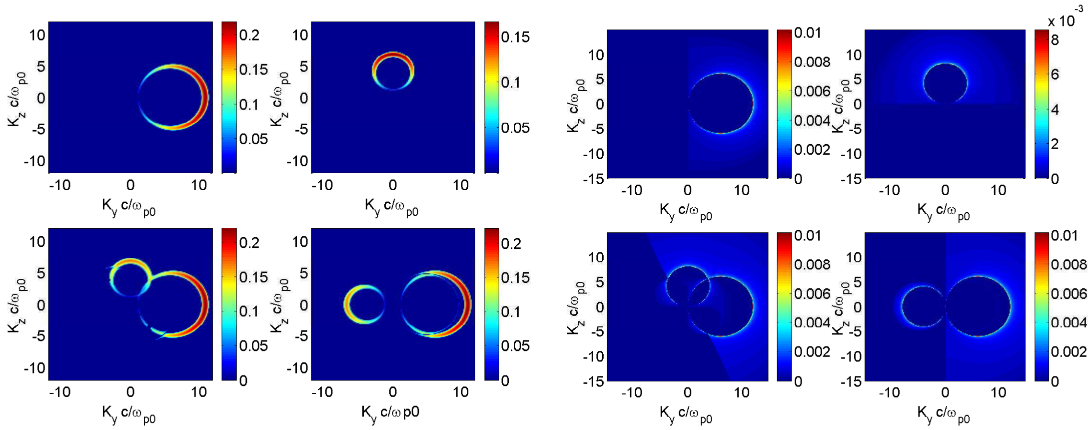
$$Q = \omega_{p0}^2\left(1 - \frac{K^2c^2}{\Omega^2 - 3K^2v_{Te}^2 - \omega_{p0}^2}\right), \tag{6}$$

and

$$Q = \omega_{p0}^2\left(1 - \frac{m_e}{m_i}\frac{K^2c^2}{\Omega^2 - K^2c_s^2}\right), \tag{7}$$

for stimulated Raman (case a) and Brillouin (case b) scattering, respectively. The growth rate obtained from the dispersion relation (5) is depicted in Figs. 1a (Raman scattering) and 1b (Brillouin scattering).

We have also performed numerical simulations of the system of equations (1) and (2) in two spatial dimensions, see Fig. 2. We have used as an initial condition that both beams have a constant amplitude of 0.1 and that they initially have group velocities at a right angle to each other. The background plasma density is slightly perturbed with a low-level random noise. For stimulated Raman scattering, displayed in Fig. 2a, the wave pattern developes local maxima of the laser beam envelope amplitude correlated with local minima of the electron density. However, this pattern is very regular and there is no clear sign of nonlinear structures in the numerical solution. However, for stimulated Brillouin scattering (see Fig. 2b) the waves grow not only in the direction of the laser

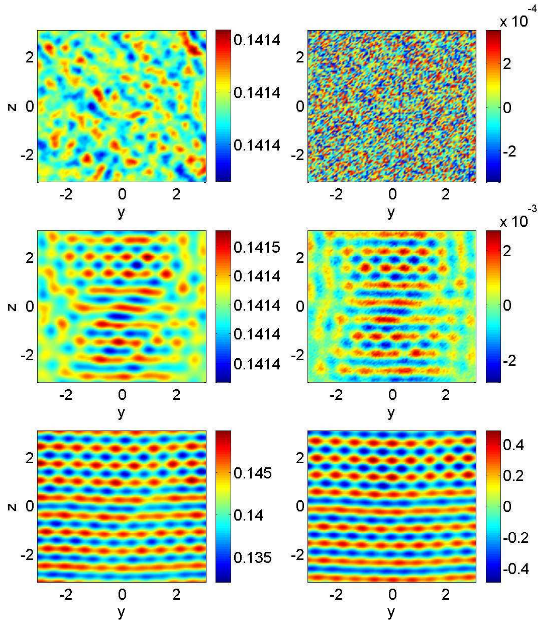


(a) Raman scattering. (b) Brillouin scattering.

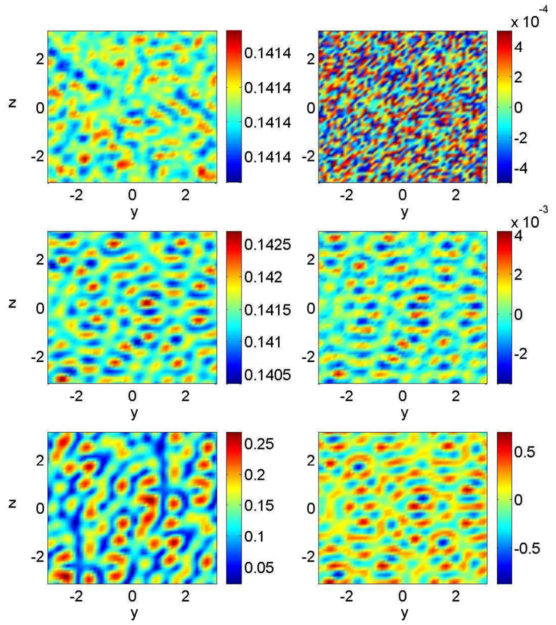
Figure 1: The growth rates for single laser beams (upper panels) and for two laser beams (lower panel), as a function of the wave vector $\vec{K}$, for (a) Raman scattering and (b) Brillouin scattering. In both (a) and (b), the upper left and right panels show the growth rate for the beam $\vec{A}_1$ and $\vec{A}_2$, respectively, where the wave number for $\vec{A}_1$ is $(k_y,\, k_z) = (6,\, 0)\, \omega_{p0}/c$ and the one for $\vec{A}_2$ is $(k_y,\, k_z) = (0,\, 4)\, \omega_{p0}/c$, and in the lower left panel, $\vec{A}_1$ and $\vec{A}_2$ are launched simultaneously at a perpendicular angle to each other, and in the lower right panel, the two beams are counter-propagating. For (a) we use $|\vec{A}_{10}| = |\vec{A}_{20}| = 0.1$ and $v_{Te} = 0.01c$, and for (b) we use the $|\vec{A}_{10}| = |\vec{A}_{20}| = 0.1$, the ion to electron mass ratio $m_i/m_e = 73440$ (Argon), and the ion sound speed $c_s = 3.4 \times 10^{-5}\, c$. (After Ref. [8].)

beam but also, with almost the same growth rate, obliquely to the propagation direction of the laser beam, and a more irregular structure of the instability developes. At the final stage, local "hot spots" are created in which large amplitude laser beam envelopes are correlated with local depletions

In summary, we have investigated the instability and dynamics of two nonlinearly interacting intense laser beams in an unmagnetized plasma. Our analytical and numerical results reveal that stimulated Raman forward and backward scattering instabilities are the dominating nonlinear processes that determine the stability of intense laser beams in plasmas, where relativistic mass increases and the radiation pressure effects play a dominant role. Our nonlinear dispersion relation for two interacting laser beams with different wavenumbers predicts a superposition of the instabilities for the single beams. The numerical simulation of the coupled nonlinear Schrödinger equations for the laser beams and the governing equations for the slow plasma density perturbations in the presence of the radiation pressures, reveal that in the case of stimulated Raman scattering, the nonlinear interaction between the two beams is weaker than for the case of



(a) Raman scattering



(b) Brillouin scattering

Figure 2: The amplitude of two crossed laser beams for (a) Raman scattering and (b) Brillouin scattering, where $|A| = (|A_1|^2 + |A_2|^2)^{1/2}$ (left panels) and the electron density N_s at times $t = 1.0\,\omega_{p0}^{-1}$, $t = 30\,\omega_{p0}^{-1}$ and $t = 60\,\omega_{p0}^{-1}$ (upper to lower panels). The laser beams initially have the amplitude $A_1 = A_2 = 0.1$, and A_1 initially has the wavenumber $(k_{1y},\, k_{1z}) = (0,\, 5)\,\omega_{p0}/c$ while A_2 has the wavenumber $(k_{2y},\, k_{2z}) = (5,\, 0)\,\omega_{p0}/c$. The electron density is initially perturbed with a small-amplitude noise (random numbers) of order 10^{-4}. (After Ref. [8].)

stimulated Brillouin scattering. The latter case lead to local density cavities correlated with maxima in the electromagnetic wave envelope. The present results should be useful for understanding the nonlinear propagation of two nonlinearly interacting laser beams in plasmas, as well as for the acceleration of electrons by high gradient electrostatic fields that are created due to stimulated Raman scattering instabilities in laser-plasma interactions.

Acknowledgment This work was partially supported by the Deutsche Forschungsgemeinschaft through the Sonderforschungsbereich 591.

[1] P. K. Shukla, N. N. Rao, M. Y. Yu, and N. L. Tsintsadze, Phys. Rep. **135**, 1 (1986).
[2] A. Sjölund and L. Stenflo, Appl. Phys. Lett. **10**, 201 (1967).
[3] M. Y. Yu, K. H. Spatschek, and P. K. Shukla, Z. Naturforsch. A **29**, 1736 (1974).
[4] P. K. Shukla, M. Y. Yu, and K. H. Spatschek, Phys. Fluids **18**, 265 (1975).
[5] P. K. Shukla and L. Stenflo, Phys. Rev. A **30**, 2110 (1984).
[6] N. L. Tsintsadze and L. Stenflo, Phys. Lett. A **48**, 399 (1974).
[7] C. E. Max, J. Arons, and A. B. Langdon, Phys. Rev. Lett. **33**, 209 (1974).
[8] P. K. Shukla, B. Eliasson, M. Marklund, L. Stenflo, I. Kourakis, M. Parviainen, and M. E. Dieckmann, Phys. Plasmas **13**, 053104 (2006).