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Laser pulse amplification by Raman scattering

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Stimulated Raman scatter of energy from a long pump pulse into a short seed pulse has been suggested as a means of producing very intense short pulses in a way which circumvents the need for large and expensive gratings. Malkin et al [1] have shown that there is a nonlinear solution of the appropriate equations representing a self-similar pulse. The amplitude of this is proportional to the distance travelled while the width is inversely proportional to the distance travelled. The energy of the pulse thus increases linearly with distance travelled and energy is fed into it from the pump at a constant rate. Recently Ersfeld and Jaroszynski[2] have shown that similar scaling (superradiance) can be obtained in the linear regime if the pump frequency is appropriately chirped. If there is no chirp then the linear solution gives a pulse whose width increases with time. Our purpose here to show that these various regimes can be brought together within a unified description, so that the transition from linear to nonlinear behaviour can be studied as well as the combined effects of nonlinearity and a chirped pulse.

The equations describing the evolution of the pulse, the pump and the Langmuir wave can be written in the form

$$\frac{\partial a}{\partial t}+\frac{\partial a}{\partial z}=bn \qquad \frac{\partial n}{\partial t}=b^{*}a \qquad \frac{\partial b}{\partial t}-\frac{\partial b}{\partial z}=-n*a \tag{1}$$

with a, b, n are the amplitudes of the pulse, the pump and the Langmuir wave respectively. The scaling is such that frequency times amplitude squared is proportional to the wave energy density, time is in units of the inverse linear growth rate and velocity is scaled to *c,* assuming an underdense plasma in which the group velocity of both transverse waves can be taken as *c*. In order to introduce a frequency chirp into the pump, we follow Ersfeld and Jaroszynski[2] and assume that

$$b(z,t)=B(z,t)\exp\left(\frac{i\beta}{2}(z+t)^2\right) \tag{2}$$

with $B(z,t)$ a slowly varying function of its arguments. Guided by the linear theory we make the changes of variable

$$A = a\exp(-\frac{i\beta}{2}(z-t)^2) \qquad N = n\exp(2i\beta zt)$$

and look for a solution of the form

$$N(z,t) = F(\zeta) \qquad A(z,t) = zG(\zeta) \qquad B(z,t) = H(\zeta) \tag{3}$$

with $\zeta = z(t-z)$. The set of equations (1) then becomes

$$\zeta G' + G = HF \qquad F' - 2i\beta F = H^* G \qquad (3z-t)H' = -zF^* G. \tag{4}$$

The first two of these give equations in the variable ζ, but the third is not in this form. Now, however we can follow the procedure of Malkin et al [1] and note that if the pulse is short then H' is only non-zero when $z \approx t$, so that we can approximate our equations by

$$\zeta G' + G = HF \qquad F' - 2i\beta F = H^* G \qquad 2H' = -F^* G, \tag{5}$$

a set of ordinary differential equations in ζ. Using Laplace transform techniques to analyse the linear regime, it can be shown that this similarity solution is given by a δ-function initial seed.

The only parameter in (6) is the chirp rate (normalised to the square of the linear growth rate in the scaling we use). If it is put equal to zero we get the solution of Malkin et al, except that their similarity variable is $t(z-t)$. This makes little difference when the pulse is localised around $z = t$, but our solution connects smoothly to an exact solution in the linear regime, when the pulse initially broadens.

The behaviour of the pulse is qualitatively as in the solutions of Malkin et al, ie a main pulse whose amplitude increases proportional to time and width varies inversely with time, followed by a train of pulses of decaying amplitude. The effect of a chirp is to produce incomplete pump depletion and a resultant reduction in the pulse amplitude. In the linear regime the pulse behaves in a qualitatively similar way if a chirp is present, but unless the chirp is large enough, neglect of the pump depletion leads to a pulse amplitude which is too large. Only if the chirp parameter goes above about 0.2 does the pump depletion become negligible. This is, of course, a regime in which there is little transfer of energy from the pump to the pulse. However, a strong chirp may well allow experimental demonstration of the superradiant scaling in circumstances where the available combination of pump intensity and plasma length does not allow access to the nonlinear regime. Also, since chirped pumps are generally what is available, the effect of the chirp on the amplification process is important.

As pointed out above, the similarity solution here corresponds to growth from a small δ function seed. In order to see the effect of a finite width initial seed, we return to the initial equations and change the spatial variable to $\xi = z - t$, so that we are in frame moving with the pulse. In the absence of a chirp we obtain

$$\frac{\partial a}{\partial t} = bn \qquad \frac{\partial n}{\partial t} - \frac{\partial n}{\partial \xi} = b * a \qquad \frac{\partial b}{\partial t} - 2\frac{\partial b}{\partial \xi} = -n * a \,. \tag{6}$$

Then, essentially following the procedure of Malkin at al, we assume that the Langmuir wave and the pump do not change significantly in the time it takes the pulse to pass a given point, allowing us to drop the time derivative in the last two equations of (7). The variables can also be assumed to be real, and since there is a conservation relation

$$n^2 + 2b^2 = 1 \tag{7}$$

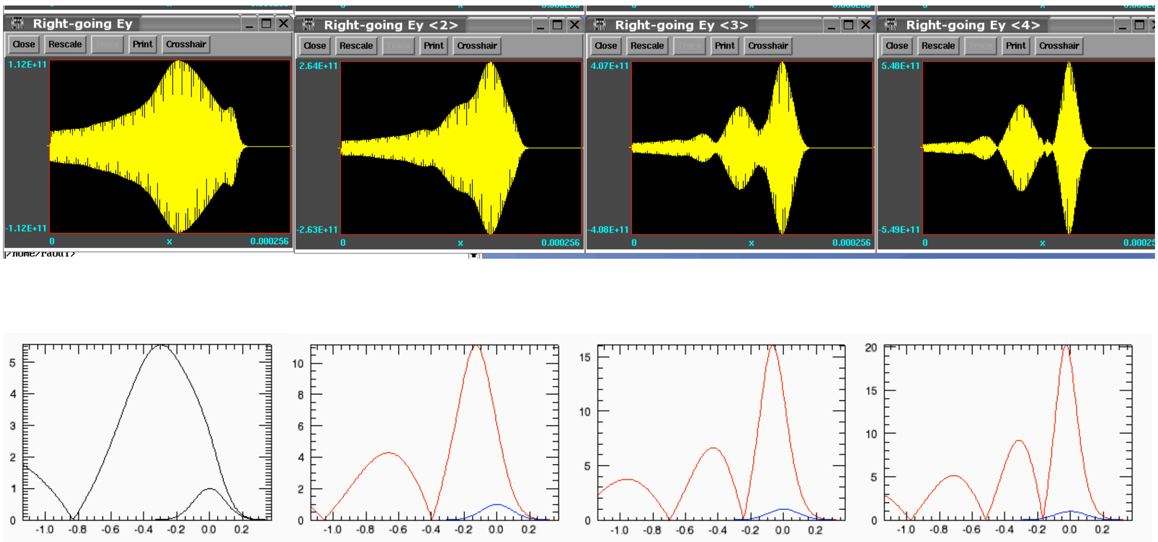

Fig.2 Evolution of a pulse. The top row shows the time evolution from a particle-in-cell code, while the bottom row shows the corresponding results from the analytic approximation. The initial pulse is also shown in the bottom set of graphs.

we can take

$$b = \cos\left(\frac{u}{2}\right) \qquad\qquad n = \sqrt{2}\sin\left(\frac{u}{2}\right). \qquad (8)$$

The resulting equations for a and u are easily solved with a simple numerical algorithm.

In Fig. 2 we show typical behaviour, comparing the analytic results with one dimensional PiC simulations of the same system. Clearly the qualitative behaviour is similar and if the normalised units of the analytic results are translated into the units of the simulation there is also reasonable agreement in the pulse width and amplitude. It is evident that the behaviour seems to be evolving towards the self-similar solution described in the first part of this paper. That this is to be expected can be seen by noting that the pulse amplitude at any point can only depend on the initial value of the pulse amplitude upstream of this point (in a frame moving with the pulse). As the seed pulse evolves the most important feature becomes the narrow leading pulse in a train of pulses of decaying amplitude. As this leading pulse becomes shorter, it depends on an increasingly narrow region at the leading edge of the initial pulse and so the solution with an initial δ-function would be expected to become a good description of it.

To conclude, we have brought together previous similarity solutions predicting superradiant scaling of a short pulse amplified by Raman scattering via a long counter-propagating pump. A single set of equations can describe the evolution of the system in the linear and nonlinear regimes with or without a frequency chirp on the pulse. We have also presented both approximate analytical and simulation results to suggest that any initial pulse will eventually evolve so as to behave like the similarity solution, at least at its leading edge.

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References:

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