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Transport equations for high-current relativistic Electron beam in the presence of a background overdense plasma in ICF fast ignition

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I. INTRODUCTION

Since Tabak et al. presented an overview of the fast ignition (FI) scenario and the theoretical models that have been used[1] in inertial confinement fusion (ICF), the idea of fast ignition stirred intense interest around the world with recent developments in high-intensity short pulses laser technology. However, because FI concept is at an earlier stage of development than the main-line hot spot ignition with direct and indirect drive approaches, FI also brings some new major challenging physics[1-4]: (1) transport of the short-pulse, high intensity laser through the under-dense plasma to the critical surface; (2) efficient coupling of the laser light to relativistic electrons; and (3) transport of the relativistic electrons to the high density compressed-fuel “ignition” region. In all FI schemes proposed in recent years, a key element is the conversion of the energy of a Petawatt laser pulse into a beam of strongly relativistic electrons and the transport of the latter into a overdense plasma[2], especially energy transport is the key area of physics uncertainty. As there are greater uncertainties in the physics modeling, in particular the main issues facing the FI concept are in the area of transport and turbulence, the transport of the relativistic electrons to the pre-compressed dense fuel requires more theoretical and experimental research.

II. THE MHD TRANSPORT EQUATIONS

A. **Relativistic electron beams fluid equations**

The behavior of relativistic electrons in the presence of a nonrelativistic background plasma has been great interest in many years because of its connection with nuclear fusion studies[5-7]. To the electron driven FI scheme of ICF, the relativistic electron dynamics in overdense plasma is crucial. The study of fast electron transport in overdense cannot be presently performed with PIC codes, since these require too large computing resources, especially when collisions or ionization processes are included. Therefore, there is a growing attention on fluid and multi-fluid approaches recently[8-11]. Here, it is the aim of this paper that the full set of self-consistently MHD Eqs. are given, which can be utilized to study energy transport for fast ignition scheme Relativistic electron beams fluid equations. The relativistic electron beams fluid Eqs. come from a set of equations developed by Mosher[5]. The continuity equation $\frac{\partial N}{\partial t} + \nabla \cdot (N\vec{U}) = 0$, (1)

where $\vec{U}$ is the relativistic electron-fluid velocity, $\vec{U} = < \frac{\vec{p}}{\gamma m} >$. The momentum-balance equation

$$Nm < \gamma > \frac{\partial \vec{U}}{\partial t} + \nabla \Pi + Ne\left[\vec{E} \cdot (\vec{I} - \frac{\vec{U}\vec{U}}{c^2}) + \vec{U} \times \vec{B} \right] = -4\pi N m_e n_i r_0^2 c(\alpha_i + 2\alpha_e) \frac{< \gamma > \vec{U}}{< p >^3 / m_e^3 c^3}, \quad (2)$$

where $\alpha_i = z_A^2 \ln \frac{\beta^2 \varepsilon}{\mathrm{I}_z} + z^2 \ln \frac{\mathrm{I}_z \lambda_D}{z r_0 mc^2}$, $\alpha_e = z_A \ln \frac{\beta\gamma\sqrt{\gamma - 1} mc^2}{\mathrm{I}_z} + z \ln \frac{\lambda_D \mathrm{I}_z}{2\gamma r_0 mc^2}$, where z is the mean ionization level in the plasma, z_A is the full nuclear charge number, r_0 is the classical electron radius, n_i is the density of scattering ions, I_z is the ionization of the most loosely bound electron, and λ_D is the Debye-shielding length, and that $\gamma = \left(1 + p^2 / mc^2\right)^{\frac{1}{2}}$, $< p >$ is determined from $< \gamma >^2 = 1 + \frac{< p >^2}{m^2 c^2}$, and the thermal pressure of relativistic electron beams, which is assumed to be isotropic $\Pi = \frac{Nmc^2}{3 < \gamma >}\left[< \gamma >^2 (1 - \frac{\vec{U} \cdot \vec{U}}{c^2}) - 1 \right]$, The energy-balance equation

$$\frac{\partial < \gamma >}{\partial t} + \vec{U} \cdot \nabla < \gamma > + \frac{e}{mc^2} \vec{E} \cdot \vec{U} = -4\pi n_i r_0^2 c \alpha_e \frac{< \gamma >}{< p > / mc}, \quad (3)$$

Eqs. (1)-(3) plus the defining equations in the above are the desired relativistic fluid relations. To study the transport mechanism for the intense relativistic electron beam, these equations must be coupled to Maxwell's equations through the electromagnetic field terms and to two-fluid dynamical equations for the background overdense plasma through the collision term.

B. Two-Fluid plasma dynamical equations for the background over-dense plasma and Maxwell's equations

1. Relativistic generalized ohm's law

Eqs. (1)-(3) describe the dynamics and energy loss of the relativistic beam fluid in terms of background plasma fluid and Maxwell's equations. The behavior of relativistic electrons are determined the nonrelativis -tic background plasma two-fluid equations and Maxwell's equations. To derive relativistic generalized ohm's law, we start with the background two-fluid description. The background electron momentum balance equation is

$$m_e n \frac{d\vec{u}_e}{dt} + \nabla p_e = -en\vec{E} - en\vec{u}_e \times \vec{B} - \frac{m_e n}{\tau_{ei}}(\vec{u}_e - \vec{u}_i) + 0.71 n \nabla T_e + 4\pi N m_e n_i r_0^2 c(2\alpha_e) \frac{< \gamma > \vec{U}}{< p >^3 / m_e^3 c^3}, \quad (4)$$

The background ion momentum balance equation is

$$m_i n \frac{d\vec{u}_i}{dt} + \nabla p_i = en\vec{E} - en\vec{u}_i \times \vec{B} + \frac{m_e n}{\tau_{ei}}(\vec{u}_e - \vec{u}_i) - 0.71 n \nabla T_e + 4\pi N m_e n_i r_0^2 c \alpha_i \frac{< \gamma > \vec{U}}{< p >^3 / m_e^3 c^3}. \quad (5)$$

With Eq. (2) plus Eqs. (4)-(5), we can get

$$Nm < \gamma > \frac{\partial \vec{U}}{\partial t} + \nabla \Pi + Ne\left[\vec{E} \cdot (\vec{I} - \frac{\vec{U}\vec{U}}{c^2}) + \vec{U} \times \vec{B} \right] + nm_e \frac{d\vec{u}_e}{dt} + nm_i \frac{d\vec{u}_i}{dt} + \nabla(p_e + p_i) = en(\vec{u}_i - \vec{u}_e) \times \vec{B}, \quad (6)$$

Because the total current is satisfied with $\vec{j}_t = \vec{j}_f + \vec{j}_p = -\gamma eN\vec{U} + en(\vec{u}_i - \vec{u}_e)$, (7)

taking a multiplication cross with magnetic field $\vec{B}$ to Eq. (7), we can get

$\vec{j}_t \times \vec{B} = -\gamma eN\vec{U} \times \vec{B} + en(\vec{u}_i - \vec{u}_e) \times \vec{B}$. (8) After arranging Eq. (8), it takes the form

$\vec{u}_e \times \vec{B} = \vec{u}_i \times \vec{B} - \frac{1}{en}\vec{j}_t \times \vec{B} - \gamma \frac{N}{n}\vec{U} \times \vec{B}$. (9) The inertial term is ignored in Eq. (4), one obtains

$$\nabla p_e = -en\vec{E} - en\vec{u}_e \times \vec{B} - \frac{m_e n}{\tau_{ei}}(\vec{u}_e - \vec{u}_i) + 0.71n\nabla T_e + 4\pi N m_e n_i r_0^2 c(2\alpha_e)\frac{<\gamma>\vec{U}}{<p>^3/m_e^3c^3}, \quad (10)$$

and substituting Eqs. (7) and (9) into Eq. (10) then gives

$$\nabla p_e = -en\vec{E} - en\vec{u}_i \times \vec{B} + \vec{j}_t \times \vec{B} + \gamma Ne\vec{U} \times \vec{B} + \frac{m_e n}{\tau_{ei}}(\gamma\frac{N}{n}\vec{U} + \frac{1}{en}\vec{j}_t) + 0.71n\nabla T_e + 4\pi N m_e n_i r_0^2 c(2\alpha_e)\frac{<\gamma>\vec{U}}{<p>^3/m_e^3c^3}. \quad (11)$$

After arranging Eq. (11), and remembering that $\sigma = \frac{ne^2}{m_e \nu_{ei}}$, where ν_{ei} is the momentum exchange collision frequency of electron-ion, one derives the relativistic generalized Ohm's law

$$\vec{j}_t = \sigma[(\vec{E} + \vec{u}_i \times \vec{B}) + \frac{1}{en}\vec{j}_t \times \vec{B} + \frac{1}{en}\nabla p_e - \gamma\frac{N}{n}\vec{U} \times \vec{B} + \gamma Ne\vec{U} - 4\pi\frac{N}{e} m_e r_0^2 c(2\alpha_e)\frac{<\gamma>\vec{U}}{<p>^3/m_e^3c^3}, \quad (12)$$

2. Background overdense plasma dynamical equations

Because electron mass is very smaller than ion mass, the inertial term is ignored in Eq. (4) in background plasma. So the full derivative d/dt is always relative to ions in two species MHD equations. With Eq.(4) plus Eq. (5) and ignoring the inertial term of electron, one derives the background ion momentum balance Eq.

$$m_i n_i \frac{d\vec{u}_i}{dt} + \nabla p' = \vec{j}_t \times \vec{B} + \gamma eN\vec{U} \times \vec{B} + 4\pi N m_e n_i r_0^2 c(\alpha_i + 2\alpha_e)\frac{<\gamma>\vec{U}}{<p>^3/m_e^3c^3}, \quad (13)$$

where Eq. (9) is used to obtain the form shown. Conservation equation for ion and energy densities are

$$\frac{\partial n_i}{\partial t} + \nabla\cdot(n_i\vec{V}) = 0, \quad (14) \qquad \frac{\partial \varepsilon}{\partial t} + \nabla\cdot[(\varepsilon + p')\vec{u}_i] + \nabla\cdot\vec{Q} = \vec{j}_t\cdot\vec{E} - P_R + 4\pi n_i r_0^2 c\alpha_e\frac{<\gamma>Nmc^2}{(<\gamma>^2-1)^{\frac{1}{2}}}. \quad (15)$$

In the above, $\vec{V}$ is the plasma fluid velocity, P' is the plasma pressure $p' = (1+Z)n_i kT$, $\vec{Q}$ is the heat conduction vector, and ε is the total plasma energy density which includes three parts: kinetic energy + random thermal energy + other energy stored in ionization and excited bound-electron states $\varepsilon = \frac{1}{2}m_i n_i V^2 + \frac{3}{2}(1+Z)n_i kT + \varepsilon_Z$. The right-hand side of Eq. (15) consists of Ohm heating, radiation-dissipation terms and a collision term which represents heating due to relativistic beam electron collisions with background plasma electrons. The collision term is just the right-hand side of Eq. (3) multiplied by Nmc^2.

3. Maxwell's equations

In the problem to study energy transport for fast ignition, the relativistic forward electron beam will transport in background pre-compressed dense plasma, and so the total fluid system isn't quasineutral plasma. Then electrostatic field effects shouldn't be ignored. And that the characteristic time scale of interest is very fast (no more than 10 psec), displacement-current effects need be considered, so that the electric and magnetic fields are determined from the full Maxwell's equations

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}, \quad (16) \qquad \nabla \times \vec{B} = \mu_0 \vec{j}_t + \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t}, \quad (17)$$

$$\nabla \cdot \vec{E} = -\frac{Ne}{\varepsilon_0}, \quad (18) \qquad \nabla \cdot \vec{B} = 0. \quad (19)$$

III SUMMARY

So far it has been presented that the full set of MHD Eqs. (1)-(3), (12), (13)-(15), and (16)-(29) which may be used to study the energy transport mechanism for the intense relativistic Electron beams (current intensity 100~1000MA, electron energy ~ MeV) generated by the petawatt laser in a background overdense plasma for FI scheme. The full set of equations consist of the relativistic beam transport equations which are coupled to Maxwell's equations through the electromagnetic-field terms and to two-fluid plasma dynamical equations for the background overdense plasma through the collision term, so it can describe the dynamic, self consistent collisional and electromagnetic inter- action of relativistic electron beam with hydrogenic overdense plasmas or arbitrary atomic-number plasmas. The full set of MHD Eqs. can be potentially important to study energy transport for fast ignition and high energy density plasma physics. Because of complexity of the MHD Eqs., they can be only self-consistently solved with numerical simulation in principle.

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[1] M. Tabak, J. Hammer, M. E. Glinsky, W. L. Kruer, S. C. Wilks, J. Woodworth, E. M. Campbell, M. D. Perry, and R. J. Mason, Phys. Plasmas **1**, 1626 (1994).

[2] A. Macchi, A. Antonicci, S. Atzeni, D. Batani, F. Califano, F. Cornolti, J.J. Honrubia, T.V. Lisseikina, F. Pegoraro and M. Temporal, Nucl. Fusion **43** 362 (2003).

[3] M. Tabak, D. S. Clark, S. P. Hatchett, M. H. Key, B. F. Lasinski, R. A. Snavely, S. C. Wilks, R. P. J. Town, R. Stephens, E. M. Campbell, R. Kodama, K. Mima, K. A. Tanaka, S. Atzeni, and R. Freeman, Phys. Plasmas **12**, 057305 (2005).

[4] P. A. Norreys, et al., Phys. Plasmas **7**, 3721 (2000).

[5] D. Mosher, Phys. Fluids 18 (1975) 846.

[6] R. A. Grandey, in Strongly Coupled Plasmas, edited by G.Kalman and P.Carini (Plenum, New York, 1978).

[7] D. A. Kershaw, Lawrence Livermore Laboratory Report UCRL 83494 (1979).

[8] R. J. Mason and M. Tabak, Phys. Rev. Lett. **80**, 524 (1998).

[9] J. A. Delettrez, S. Skupsky, C. Stoeckl, and P. B. Radha, 33rd Anomalous Absorption Conference Lake Placid, NY 22–27 June 2003.

[10] S. Hain, P. Mulser, F. Cornolti, and H. Opower, Laser Part. Beams 17, 245 (1999).

[11] S. Hain and P. Mulser, Phys. Rev. Lett. **86**, 1015 (2001).