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Fullwave coupling to a 3D antenna code using Green's function formulation of wave-particle response*

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Introduction

The modeling of radio frequency (RF) waves in the ion cyclotron range of frequencies (ICRF) has reached a maturity in the core physics model in the past decade. So called full wave codes such as TORIC [1], AORSA [2], PENN [3] and ALCYON [4, 5] that couple the plasma dielectric current response with the solution of Maxwell's equations include the effects of diffraction and to a varying degree, finite Larmor radius effects and higher order kinetic effects and operate in frequencies from the Alfvénic to the lower hybrid. More recently, efforts at self-consistency with the kinetic distribution through coupling with Fokker-Planck codes [6] and finite orbit effects [7]. This has all been enabled by access to large parallel computer platforms. So, although much doubtless remains to be learned, the core models of ICRF propagation are fairly mature. In contrast, the modeling of the coupling at the plasma boundary with ICRF sources remains undeveloped. In this paper we present the first steps in coupling a three dimension model of the plasma with a three dimensional antenna model self consistently.

Lower hybrid (LH) coupling codes [8] have proved successful in predicting the coupling to the plasma. This is due to the very small wavelength of the LH wave and so it is sensitive to only local properties like the density. For ICRF, the large wavelengths involved are comparable to the variation in plasma conditions and curvature. Typically, only several wavelengths span the plasma cross-section. Also, existing coupling models impose an approximate representation of a current strap perpendicular to the magnetic geometry rather than evaluate it self-consistently, though some self consistent approaches with simple current models have been applied [9].

The Antenna and Full Wave Codes

TOPICA [10, 11] is an innovative tool for the simulation of the Ion Cyclotron Radio Frequency (ICRF) antenna systems that incorporates commercial-grade graphic interfaces into a fully 3D self-consistent description of the antenna geometry. Rather than a simple constant di-

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electric to represent the plasma, TOPICA may be interfaced with an RF wave code to represent the plasma response. In this way, the experimentally determined plasma geometry and profiles incorporated in the wave code can exert their full volumetric effect on the plasma impedance the antenna sees. The approach to the problem is based on an integral-equation formulation for the self-consistent evaluation of the current distribution on the conductors. The environment has been subdivided in two coupled regions: the plasma region and the vacuum region. The two problems are linked self-consistently by representing the field continuity in terms of equivalent (unknown) sources. This naturally leads to a Green's function formulation and has the added advantage that different plasma models can be used with a given antenna and vice versa without recomputation of the basis functions of either.

TORIC solves Maxwell's equations in the plasma in the frequency domain

$$\nabla\times\nabla\times\mathbf{E} = \frac{\omega^2}{c^2}\left\{\mathbf{E} + \frac{4\pi i}{\omega}\left(\mathbf{J}^P + \mathbf{J}^A\right)\right\} \tag{1}$$

$$\mathbf{J}^P = \overleftrightarrow{\sigma}\,[f_0(\mathbf{x},\mathbf{v}_\perp,v_\|)]\cdot\mathbf{E},$$

where $\overleftrightarrow{\sigma}$ is the plasma conductivity and $\mathbf{J}_A$ is the antenna current density. It uses a discretization of electric field that is Fourier in flux surface and Hermite finite elements in radial dimension [see Eq.(3)]. This results in a block tridiagonal stiffness matrix, where each block is $(6N_m)^2$, with $3N_r$ blocks. The code has dielectric models appropriate for ICRF up to the 2nd harmonic, lower hybrid, and high cyclotron harmonic frequencies and retains appropriate kinetic effects. The cpu load goes $\rightarrow \sim N_m^3 N_\psi N_\phi$ where N_m, N_ψ, and N_ϕ are respectively the number of poloidal, flux surfaces, and toroidal modes.

In a discrete system, S, representing the wave equation in the presence of a plasma, the admittance is related to the discrete Green's function, $G_{m,m'}$.

$$SG_{m,m'} = \delta_{m,m'} \qquad G_{m,m'} = S^{-1}\delta_{m,m'} \tag{2}$$

Note that S^{-1} is only a function of the system and not the boundary conditions. Total work is approximately the same as a single solution with full boundary conditions because we simply retain the inverse to multiply by each source when generating the Green's function. After building up G, we can construct the admittance and the weighted reconstructed solution at only the cost of some matrix-vector products.

Expressing the Surface Impedance in Toroidal Axisymmetric Geometry

Let Eq.(3) below be a solution of Maxwell equations in the plasma. Exclusion of the singular solution at the magnetic axis implies a relation between the tangential components of **E** and **B**

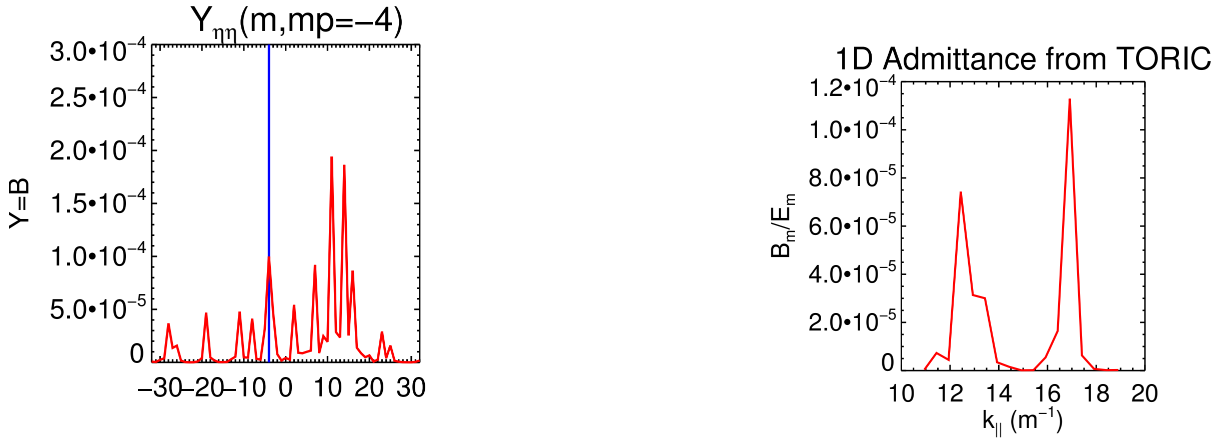

Figure 1: Left figure(A): Broad spectrum response to m=0 ($k_{||}$ = 11 m^{-1}) drive. Simulation was run with 128 poloidal elements and 17 source modes. Right figure(B) Self admittance response, $Y_{m,m}$ vs $k_{||}(m)$

at the plasma surface, $\psi = 1$.

$$\mathbf{E} = \sum_m \mathbf{E}^m(\psi)e^{i(m\theta+n\phi)} \qquad \mathbf{B} = \sum_m \mathbf{B}^m(\psi)e^{i(m\theta+n\phi)} \tag{3}$$

Writing

$$\mathbf{E}_t = \mathbf{E} - (\mathbf{E}\cdot\mathbf{u}_\psi)\mathbf{u}_\psi = E_\eta\mathbf{u}_\eta + E_\zeta\mathbf{u}_\zeta \tag{4}$$
$$\mathbf{B}_t = -\mathbf{u}_\psi \times \mathbf{B} = (B_\eta\mathbf{u}_\eta + B_\zeta\mathbf{u}_\zeta)\times\mathbf{u}_\psi = B_\zeta\mathbf{u}_\eta - B_\eta\mathbf{u}_\zeta$$

we can express this relation in the form

$$\mathbf{E}_t^m(1) = \sum_{m'} \overleftrightarrow{Z}(m,m')\cdot\mathbf{B}_t^m(1) \qquad \mathbf{B}_t^m(1) = \sum_{m'} \overleftrightarrow{Y}(m,m')\cdot\mathbf{E}_t^m(1). \tag{5}$$

The matrices $\overleftrightarrow{Y}$ (admittance) and $\overleftrightarrow{Z}$ (impedance) are the inverse of each other. The vector indices span 1 and 2 with, according to the above definitions

$$\mathbf{E}_{t,1} = E_\eta \qquad \mathbf{E}_{t,2} = E_\zeta \tag{6}$$
$$\mathbf{B}_{t,1} = B_\zeta \qquad \mathbf{B}_{t,2} = -B_\eta.$$

We calculate the response with an excitation specified as

$$\mathbf{E}_{t,j}^{m'}(1) = \delta_{j,j_0}\delta_{m',m_0}. \tag{7}$$

In these indices, $\overleftrightarrow{Y}$ and $\overleftrightarrow{Z}$ are symmetric. Evaluating $\overleftrightarrow{Z}$ is likely to be numerically more robust, but is difficult to implement, because the dependent values of TORIC are the physical components of $\mathbf{E}^m$ and their radial derivatives at the mesh points, rather than the physical components of $\mathbf{B}^m$. Boundary conditions of the form (7), on the other hand are completely straightforward, but might be numerically inaccurate for large values of m_0. However, in this limit, the plane stratified approximation becomes acceptable and the one-dimensional full wave code, FELICE [12], can be used to calculate the response and thus fill in the spectral width that TOPICA requires.

Figure 1A shows the broad spectral response of the admittance to a source at a single poloidal mode number. This differs from the one dimensional plasma model where response can only be measured at the driven mode. Surprisingly, it is not peaked at the driven mode value. Figure 1B is a plot of the self excitation and is the proper quantity to compare against TOPICA coupled with a one dimensional plasma model like FELICE. This will be done in the the large aspect ratio low poloidal field limit. The coupled system will be benchmarked against experiments and simpler plasma models [10].

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