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On the Dynamics of Alfvén Eigenmode Excitation

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Abstract

Alfvén eigenmodes (AEs) excited by fast ions can cause losses of fast ions in thermonuclear experiments. To describe the dynamics of the AE excitation, it is necessary to include the decorrelation of the AE interactions and the renewal of the distribution function in the unstable regions on time scales that are short compared to the decorrelation time. For simulation of AE excitation for finite decorrelation times, a Monte Carlo operator describing the phase decorrelation between ions and AEs has been developed for implementation in the SELFO code.

Introduction

In fusion experiments, Alfvén eigenmodes (AEs) excited by fast ions can give rise to losses or redistribution of the fast ions resulting in a degradation of the heating efficiency. For AEs to be excited, there must exist weakly damped eigenmodes that can be driven unstable. This can occur if the distribution function of the resonant ions is increasing in energy along the AE characteristics in phase space. The resonant regions in phase space are in general small and increase with the decorrelation of the wave-particle interaction. The AE interactions are decorrelated by collisions and interactions with other waves, e. g. during ion cyclotron heating (ICRH), so that the orbit time differs from what it would have been in absence of these interactions. The interactions with the AE flattens the distribution function along the characteristics, and the drive is reduced. The drive will increase as the distribution function in the resonant region restores.

In experiments with ion cyclotron resonance heating, a pitch-fork splitting of the mode frequencies of the AEs has been observed. This has been interpreted as a nonlinear oscillation of the mode amplitudes caused by the renewal of the distribution function by oscillations and ICRH, and by the relaxation of the drive by the nonlinear AE interactions [1, 2, 3].

To describe the dynamics of the AE excitation, it is necessary to include the decorrelation of the AE interactions and the renewal of the distribution function in the unstable regions. On short time scales, compared to the decorrelation time, it is also important to follow the evolution of the particle phase with respect to the AE phase.

With SELFO [4], it is possible to self-consistently model the non-linear interaction between fast ions and AEs, including the phase decorrelation. The SELFO code couples the global wave code LION with the orbit averaged Monte Carlo code FIDO. The code LION solves for the wave field given the dielectric tensor and the distribution function, whereas FIDO solves for the distribution function in the three orbit invariants given the wave field. FIDO includes collisional effects and wave-particle interactions with ion cyclotron and MHD waves [5], and is now expanded from 3D to 4D to allow the evolution of the particle phase relative the mode phase to be studied.

Theory

The motion of particle guiding centres follows orbits described by the invariants of the equation of motion in absence of wave-particle interactions and collisions. Here we use energy, E, magnetic moment, μ, and canonical angular momentum, P_ϕ,

$$E = \frac{mv^2}{2}, \qquad \mu = \frac{mv_\perp^2}{2B}, \qquad P_\phi = mRv_\phi + eZ\psi, \tag{1}$$

where $2\pi\psi$ is the poloidal magnetic flux.

The electric wave field will accelerate or decelerate particles that are close to a resonance. The change in particle energy due to wave-particle interaction is given by integration of the equation of motion, including the wave field, along the drift orbit

$$\frac{\partial E}{\partial t} = \frac{eZ}{m_i}\tilde{\mathbf{E}}\cdot\mathbf{v}_D + \mu\frac{\partial \tilde{B}_\parallel}{\partial t}, \tag{2}$$

where $\mathbf{v}_D$ is the drift velocity, and $\tilde{\mathbf{E}}$ and $\tilde{B}_\parallel$ are the perturbed electric and magnetic fields due to the wave. For shear Alfvén waves, the last term in equation (2) equals zero and

$$\mathbf{E}\cdot\mathbf{v}_D = \sum_{n,m}\left(A(t)\Phi(r,\theta)e^{i(n\phi - m\theta - \omega t - \alpha(t))}\right), \tag{3}$$

where $A(t)$ is the mode amplitude, $\Phi(r,\theta)$ describes the radial and polodial structure of the mode, n and m are toroidal and polodial mode numbers, ω is the mode frequency, and $\alpha(t)$ the slowly varying phase of the mode. The resonance surfaces are defined by

$$\Theta \equiv \frac{n\Delta\phi}{\tau_b} - \omega \pm j\frac{2\pi}{\tau_b} = 0, \tag{4}$$

where $\Delta\phi$ is the precessional drift during one bounce time τ_b, and j is an integer representing higher harmonics.

The guiding centre orbits follow characteristics defined by

$$\Delta P_\phi = \frac{n}{\omega}\Delta E, \qquad \Delta\mu = 0. \tag{5}$$

With no other wave-particle interactions or collisions present, a particle close to the resonance will be trapped by the wave field and experience a non-linear superadiabatic oscillation. After a decorrelation time $\tau_d \gg \tau_b$, the collisions and ion cyclotron interactions have randomized the phase of the particles. When studying dynamical effects that occur on timescales shorter than a decorrelation time, it is therefore necessary to follow the particle phase along the drift orbit.

Implementation in FIDO

The orbit averaged Monte Carlo code FIDO uses the invariants $(E\,, \Lambda \equiv \mu B_0/E\,, P_\phi\,)$, and during one time step Δt, the invariants change by AE interactions according to

$$\Delta E^i \;=\; \sum_{n,m}\left(\frac{eZ}{m_i}\int_0^{\Delta t}\tilde{\mathbf{E}}\cdot\mathbf{v}_D\,dt\right) = \mathbb{I}(0,\tau_b)\sum_{l=1}^{L} e^{i(l-1)\Theta\tau_b} + \mathbb{I}(L\tau_b,\Delta t)\,, \tag{6}$$

$$\Delta\Lambda^i \;=\; -\frac{-\Lambda\Delta E^i}{E^i+\Delta E^i}\,, \tag{7}$$

$$\Delta P^i_\phi \;=\; \frac{n}{\omega}\Delta E^i\,, \tag{8}$$

where $L\tau_b < \Delta t < (L+1)\tau_b$, and

$$\mathbb{I}(\tau_1,\tau_2) = \sum_{n,m}\left(\frac{eZ}{m_i}\int_{\tau_1}^{\tau_2} A(t)\,\Phi(r,\theta)\,e^{i(n\phi-m\theta-\omega t-\alpha(t))}\,dt\right)\,. \tag{9}$$

A particle is considered to be in resonance with the wave if $|\Theta|\,\tau_b \le 2\pi$. To include the wave-particle interaction during time scales shorter than the decorrelation time, we upgrade FIDO to also follow the phase of the particle.

The change in in phase between the particle and the wave during one poloidal bounce time is defined as $\Delta\dot{\vartheta}$, and the evolution of the phase is given by

$$\vartheta_{N+1} = \vartheta_N + \Delta\dot{\vartheta}\Delta t + \xi\sqrt{\langle(\Delta\dot{\vartheta})^2\rangle}\,, \tag{10}$$

where ξ is a random number, $\xi \in (-1,1)$. The term $\Delta\dot{\vartheta}$ is determined by the actual changes of the invariants (E,Λ,P_ϕ) due to stochastic wave-particle interactions and collisions, whereas the term $\sqrt{\langle(\Delta\dot{\vartheta})^2\rangle} \sim (\Delta t)^{3/2}$ takes into account the stochastic contribution to the change in particle phase from wave-particle interactions and collisions during the time step.

The energy of the mode evolves as

$$\Delta E_{TAE} = -\sum_{i=1}^{N}\Delta E^i\,, \tag{11}$$

where the mode amplitude is given by $A_{mode} = \sqrt{E_{TAE}}$. For computational reasons, the mode is modelled by two separate modes, phase shifted by $\pi/2$. The phase $\alpha(t)$ of the mode is then given by the relation between the two different mode amplitudes.

References

[1] A. Fasoli *et al.*, Plasma Phys. Control. Fusion **39**, B287 (1997)

[2] A. Fasoli *et al.*, Phys. Lev. Lett. **81**, 5564 (1998)

[3] K. Wong, Plasma Phys. Control. Fusion **41**, R1 (1999)

[4] J. Hedin *et al.*, Proc. Joint Varenna-Lausanne Workshop on Theory of Fusion Plasmas (1998)

[5] T. Bergkvist, T. Hellsten, T. Johnson and M. Laxåback, Nucl. Fusion **45**, 485 (2005)