Non-linear Alfvén Eigenmode Dynamics of a Burning Plasma in the
Presence of Ion Cyclotron Resonance Heating

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Abstract

Alfvén eigenmodes (AEs) excited by $\alpha$ particles in a burning plasma can degrade the heating efficiency by spatial redistribution of the resonant $\alpha$ particles. Changes of the orbit invariants in phase space by collisions and other waves, such as magnetosonic waves during ion cyclotron resonance heating (ICRH), lead to changes in the phase between the $\alpha$s and AEs, causing a decorrelation of the interactions. ICRH lead to an increased decorrelation of the AE interactions and hence a stronger radial redistribution of the thermonuclear $\alpha$ particles by the AEs. Renewal of the distribution function by thermonuclear reactions and losses of $\alpha$ particles to the wall lead to a continuous drive of the AEs and a radial redistribution of the $\alpha$ particles. The redistribution results in a degradation of the heating efficiency.

1. Introduction

Destabilization of toroidal Alfvén eigenmodes (TAEs) by fusion born $\alpha$ particles has been observed in experiments [1]. These instabilities can degrade the confinement by redistributing the resonant high-energy particles before they are thermalized [2]. The resonances of the wave-particle interactions are defined by surfaces in phase space of drift orbit invariants. Without any decorrelation of the wave-particle interactions, particles will undergo a one dimensional superadiabatic oscillation in phase space without transferring any net energy to or from the mode. Phase decorrelation of the interactions allows the particles to diffuse inside a resonant region, where the boundaries of the resonant region is given by the strength of the decorrelation mechanism, i.e. a strong decorrelation results in a large redistribution of particles. The decorrelation by Coulomb collisions is weak for the high-energy particles, whereas decorrelation by ion cyclotron resonance interactions increases with energy. Gradients along the characteristics of the wave-particle interactions in phase space drives the mode unstable while the distribution function is flattened in the resonant region. Even though the resonant regions can be very small and only contribute to a small redistribution of the particles, several modes which overlap in phase space can have a significant effect on the plasma. The turning points of resonant trapped particles will have a relatively large radial displacement as they interact with a TAE, whereas passing particles are only slightly displaced in real space. To include interactions with AEs [3] in a self-consistent way, the SELFO code [4] has been upgraded. The SELFO code consists of
the Monte-Carlo code FIDO [5], which solves the distribution function during ICRH, and the LION code [6], which solves the electric field given the distribution function.

2. Wave-particle interactions

The guiding centre orbit is described by the orbit invariants 
\[ \mathcal{E} = \frac{mv^2}{2}, \quad P_\phi = mRv_\phi + eZ\Psi \]
and 
\[ \mu = \frac{mv^2}{2B}. \]
The change in energy from wave-particle interaction is given by
\[ \frac{d\mathcal{E}}{dt} = eZ\mathbf{E}_1 \cdot \mathbf{v}_d + \mu \frac{\partial B_{1\parallel}}{\partial t}, \]
where \( \mathbf{v}_d \) is the drift velocity and \( \mathbf{E}_1 \) and \( B_{1\parallel} \) are the perturbed electric and magnetic field respectively. \( \mathbf{E}_1 \) and \( B_{1\parallel} \) can be written on the form
\[ A(t) \Phi(r, \theta) e^{i(n\phi - \omega t - \alpha(t))}, \]
where \( A(t) \) is the mode amplitude, \( \Phi(r, \theta) \) is the structure of the mode, \( n \) is the toroidal mode number and \( \alpha(t) \) is the slowly varying phase of the mode. The characteristic in phase space is defined by
\[ \Delta P_\phi = n\omega \Delta \mathcal{E} \]
and
\[ \Delta \mu = 0 \]
where \( \Delta \phi \) is the precessional drift during a bounce time \( \tau_b \), and \( j \) is an integer. Phase decorrelations occur when the orbit invariants change due to interactions by collisions or ion cyclotron interactions. The particle is moved to a neighbouring characteristic and the orbit time changes slightly leading to phase change over time. Due to the diffusive nature of these interactions the displacement of the orbit from the original characteristic will be small. In the presence of decorrelations of the wave-particle interactions the resonance condition is written as \( |\Theta| \tau_d \leq 2\pi \) resulting in resonant volumes in phase space, where the extent of these volumes increase with the ICRH power [3]. A particle entering a resonant region on the high-energy part of the characteristic will transfer energy to the mode as it is redistributed inside the region, whereas a particle entering on the low-energy part will damp the mode, and the opposite applies for a particle leaving the resonant region. The non-linear evolution of the mode amplitude will depend on the gradients of the distribution function inside the resonant regions, the extent of the regions and the flux of particles into and out of the resonant regions, which both depend on the phase decorrelation strength and renewal rate caused by Coulomb collisions and ICRH. Fusion born \( \alpha \)-particles are mainly passing orbits. The ICRH will redistribute these particles into trapped and non standard orbits, as illustrated in figure 1, where they interact strongly with the AE and are further displaced along the AE characteristic. The radial displacement of the turning points of a trapped particle, as it traverses the resonant region, is given by
\[ \Delta r = \left( \frac{\partial \Psi}{\partial r} \right)^{-1} \Delta P_\phi e. \]
When several modes are overlapping along a characteristic in phase it may result in a large change \( P_\phi \) and hence a significant radial redistribution of the resonant particles.

Figure 1: ICRH and AE characteristics.
3. Simulations

A plasma containing fusion born $\alpha$-particles has been simulated using typical TFTR parameters, $R=2.52$ m, $a=0.9$ m, $B_T=5$ T, $I_p=1.6$ MA, $n_i=5 \cdot 10^{19}$ m$^{-3}$, $n_\alpha/n_i=0.04$%, $T_i=20$ keV and $T_e=5$ keV. A distribution of fusion born $\alpha$-particles is calculated during a slowing down time ($\tau_s=0.21$ s) without any ICRH or AEs. The first simulations are carried out to investigate how the decorrelation by ICRH effects the size of the TAE resonant regions in phase space, as well as the restoration rate of the distribution function. This is carried out for one TAE located at $r_{AE}=0.40$ m with toroidal mode number $n=3$ and ICRH with the second harmonic resonance located at $R=2.91$ m for three different powers; 0, 1 and 10 MW. In absence of ICRH only collisions decorrelate the interactions and the TAE will mainly interact with thermal ions, resulting in a very weak growth rate. The differences in growth rates are shown in figure 2. The mode initially grows exponentially until the ICRH and TAE redistribution reaches a statedy state. The birth rate of new $\alpha$-particles will then dominate the TAE dynamics resulting in linear growth of the amplitude. Stronger decorrelation leads to larger resonant regions and hence more energy can be transferred to the mode. In this scenario it is the continous flow of high-energy $\alpha$-particles into the resonant regions which drives the mode, not the gradients in phase space produced by the ICRH. ICRH alone is not enough to maintain the high energetic $\alpha$-particle distribution, and as the $\alpha$s thermalize to the background temperature, a larger fraction of the RF power is absorbed by the deuterium. In presence of several eigenmodes the redistribution becomes stronger illustrated here by studying the interaction with an additional 2 modes, during 10 MW of ICRH, with $n=2$ and 4 located at $r_{AE}=0.48$ and 0.36 m, respectively. The changes in $P_\phi$ related to three modes are shown in figure 3 for particles in region 7 and 8. The order in which the modes grow up are shown in figure 4. The growth rates for the modes are given by the gradients in phase space and the location of the resonant regions. Particles belonging to regions located higher in energy transfer more energy to the mode. From the resonance condition of the zeroth harmonic, the modes with $n=2,3,4$ resonate with particles with toroidal frequencies $\omega_\phi=83.5$ kHz, 64.0 kHz, 52.3 kHz, respectively. The toroidal precession frequency for trapped thin orbits is proportional to the particle energy, $\omega_\phi \propto E$, which means that the outermost mode is resonating with particles with higher energy than the innermost mode. Even though the mode energy for the $n=2$ seems to saturate, the redistribution of particles continues to increase. Comparing separate simulations of the three modes shows that the innermost $n=4$ mode is not excited on its own, indicating that the particle transport caused by the $n=2,3$ modes is of importance for creating gradients in other resonant regions. The excitation of the $n=2,3$ modes are delayed when the $n=4$ mode is present indicating a very complex interplay where
the modes both build up gradients and extract energy in overlapping regions in phase space. The increased redistribution by 3 modes causes a significant drop in the averaged $\alpha$-particle energy as illustrated in figure 5. The decrease in particle energy is related to the associated particle redistribution of each mode and not its energy.

![Figure 2](image2.png)  
**Figure 2:** Amplitude evolution of a TAE with 1 MW and 10 MW of ICRH power.

![Figure 3](image3.png)  
**Figure 3:** Change in $P_\phi$ for region 7 (—) and 8 (---) type orbits with 3 modes.

![Figure 4](image4.png)  
**Figure 4:** Growth of mode energies for three simultaneous interacting modes, dashed line represent growth of single modes.

![Figure 5](image5.png)  
**Figure 5:** Mean $\alpha$-particle energy, with and without TAEs and ICRH.

4. Conclusions

Interactions between fusion born $\alpha$-particles and several TAEs has been studied using the SELFO code. An increased redistribution of resonant particles is observed as the ICRH power is increased. This is caused by the increased decorrelation of the wave-particle interactions. A significant decrease in $\alpha$-particle energy is observed during mode excitation. The particle energy decreases as the redistribution of high-energy particles increases. The degradation of heating efficiency caused by unstable TAEs is explained by the radial redistribution of resonant particles. At larger minor radius the electron temperature decreases and the $\alpha$s are thermalized faster. In this scenario the redistribution results in 5 % less power to the electrons after one slowing down time.

References