Runaway electrons and the evolution of the plasma current in tokamak disruptions

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In this paper, we model the evolution of the plasma current following the thermal quench of a tokamak disruption, and calculate the post-disruption current, which is entirely carried by runaway electrons. The evolution of the current is governed by two processes, runaway electron production and resistive diffusion of the toroidal electric field $E_{\phi} \simeq E_{\parallel}$. The electric field determines the runaway generation rate, and the runaway current produced modifies the electric field through the induction equation. If the runaway electrons move at the speed of light, the current density becomes $j_{\parallel} = \sigma_{\parallel} E_{\parallel} + n_{\text{run}} ec$, where σ_{\parallel} is the neoclassical parallel conductivity. The equations for runaway generation and induction are [1]

$$\frac{\partial n}{\partial t'} = \left[F(E, t, x) + n \left(E - \frac{n_{\rm e}}{n_{\rm e0}} \right) \right] \Theta \left(E - \frac{n_{\rm e}}{n_{\rm e0}} \right),\tag{1}$$

$$\frac{1}{\alpha x} \frac{\partial}{\partial x} x \frac{\partial E}{\partial x} = \frac{\partial}{\partial t'} (\sigma E + n), \tag{2}$$

where the normalised runaway density and electric field are $n(t,r) = n_{\rm run}ec/j_{\parallel 0}$ and $E(t,r) = E_{\parallel}/E_{\rm c0}$, and where $E_{\rm c} = m_{\rm e}c/e\tau$ is the critical electric field, $\tau = 4\pi \varepsilon_0^2 m_{\rm e}^2 c^3/n_{\rm e}e^4\ln\Lambda$ is the relativistic electron-electron collision time and the subscript zero denotes an initial value on the magnetic axis. Furthermore, $\alpha = (2\pi)^{3/2} j_{\parallel 0} a^2/3I_{\rm A}\ln\Lambda$ is a large parameter in tokamaks with large current, $I_{\rm A} \simeq 17$ kA is the Alfvén current, $\sigma = \sigma_{\parallel} E_{\rm c0}/j_{\parallel 0}$ is the normalised conductivity, and Θ is a Heaviside step function. Minor radius and time are normalised according to x = r/a and $t' = t/\sqrt{2/\pi} 3\tau_0 \ln\Lambda$, respectively.

The runaway electrons are produced both by the primary (Drecier) mechanism and a secondary avalanche caused by short-range collisions. The primary generation rate is represented in Eq. (1) by the function

$$F(E,t,x) \equiv \frac{3\ln\Lambda}{2\sqrt{\pi}} \frac{n_{\rm e0}ec}{j_{\parallel 0}} \frac{\sqrt{n_{\rm e}/n_{\rm e0}}}{u^{15/4}E^{3/8}} \exp\left(-\frac{1}{4u^2E} - \sqrt{\frac{2}{u^2E}}\right),\tag{3}$$

where $u^2 = E_{c0}/E_D$ and E_D is the Dreicer field, and the second term on the right in Eq. (1) is the secondary runaway generation rate. The latter is the dominant in large tokamaks, but primary generation plays an important role in providing a "seed" for the secondary avalanche.

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A simplified, zero-dimensional version of Eqs. (1) and (2) can be solved approximately assuming an infinitely fast thermal quench. This gives the condition that substantial runaway production will occur if H > 0, where

$$H = \alpha - \frac{E_{\rm D1}}{4E_{\parallel 1}} - \sqrt{\frac{2E_{\rm D1}}{E_{\parallel 1}}} + 2\ln\frac{m_{\rm e}c^2}{T_{\rm e1}},\tag{4}$$

and index 1 signifies the value immediately after the thermal quench.

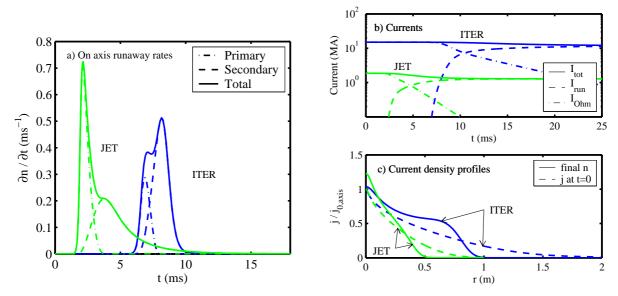


Figure 1: A comparison between JET and ITER. a) On axis runaway production rates. b) Total currents. c) Initial and final current density profiles.

Equations (1) – (2) have been solved numerically with parameters chosen to match a JET discharge (No. 63133): $I_0 = 1.9$ MA, $T_{\rm initial} = (1-0.9x^2)^2 \cdot 3.1$ keV, $T_{\rm final} = (1-0.9x^2) \cdot 10$ eV, $n_{\rm e}(x,t) = (1-0.9x^2)^{2/3} \cdot 2.8 \cdot 10^{19}$ m⁻³, a=1 m. The same equations have also been solved with the ITER parameters $I_0 = 15$ MA, $T_0 = 22.7$ keV, $n_{\rm e0} = 11 \cdot 10^{19}$ m⁻³, a=2 m and with radial profiles according to Ref. [2]. In both cases, the temperature was taken in the simulations to fall exponentially with time as $T_{\rm e} = (T_{\rm initial} - T_{\rm final}) \exp(-t/t_0) + T_{\rm final}$ with $t_0 = 0.5$ ms.

Figure 1 shows that in JET about 2/3 of the plasma current is converted into runaways, in approximate agreement with experimental observations, while the simulations with ITER parameters show a somewhat larger conversion ratio. The density and the initial temperature are higher in ITER, so it takes a longer time to reach sufficiently low temperatures to initiate the primary generation, which is seen in Fig. 1. Furthermore, primary runaway generation is more significant in JET than in ITER, where secondary generation dominates because of the high current.

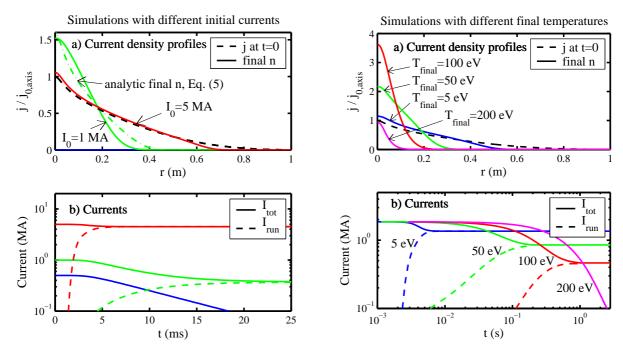


Figure 2: Results from simulations with different initial currents I_0 (left) and different final temperatures T_{final} (right). a) The resulting final current density profile. The left figure also shows an example of a solution to Eq. (5). b) The evolution of the current.

To see how different plasma parameters affect the evolution of the current, several simulations have been performed where one parameter is varied and all other parameters are kept the same as for the reference JET discharge above. Figure 2 presents simulations with different predisruption currents I_0 and post-disruption temperatures $T_{\rm final}$. Together with simulations with different background electron density, these results divide the parameter space into three main areas, one with no runaway production ($I_0 \lesssim 0.5$ MA, $T_{\rm final} \gtrsim 200$ eV, $n_{\rm e} \gtrsim 10^{20}$ m⁻³), a second, middle region with high on-axis runaway current peaking and predominantly secondary generation, and a third where primary generation is effective enough to reproduce the initial current profile ($I_0 \gtrsim 5$ MA, $T_{\rm final} \lesssim 10$ eV, $n_{\rm e} \lesssim 10^{19}$ m⁻³). For realistic thermal quench times ($t_0 \lesssim 0.5$ ms) it is found that the final current is insensitive to t_0 . Thus the approximation of infinitely fast thermal quench used in deriving Eq. (4) should be acceptable.

The simulations show that when most runaways are produced by the secondary mechanism, the runaway current profile becomes more peaked than the pre-disruption current. In fact, the current density often increases on the magnetic axis although the total current decreases. This current peaking is due to electric field diffusion into the centre of the plasma, occuring on a time scale comparable to the runaway avalanche growth time. In addition, electric field diffusion can cause the runaway current to become radially filamented if the runaway seed profile has a small ripple. The seed ripple can for instance be caused by small radial variations in the quench time,

the effect of which can be seen in Fig. 3. A theoretical explanation for these effects of electric

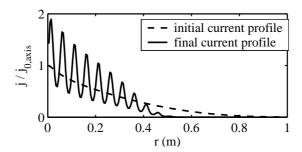


Figure 3: The final current profile becomes filamented when the thermal quench time t_0 has a small ripple, $t_0 = 0.5 \text{ms} \times [1 + 0.02 \sin(40\pi x)]$.

field diffusion can be obtained if one starts by noting that the time scales of primary generation and field diffusion are usually well separated. At some time t_{\star} after the thermal quench, primary generation can be neglected, and the two model equations can be combined to a single equation for the logarithm of the final runaway current profile $N = \ln n$,

$$\alpha^{-1}\nabla^2(N - N_{\star}) + j_0 - e^N = 0, \tag{5}$$

where $\nabla^2 = 1/x \, \partial/\partial x \, x \, \partial/\partial x$. The seed profile $n_\star = e^{N_\star}$ is, for disruptions that are fast compared to the diffusion time scale, approximately given by the runaway density at the time of equal primary and secondary generation rates. An example of a solution to Eq. (5) is shown in Fig. 2 for the 1 MA case. It can easily be shown from Eq. (5) that the final current is always smaller than the initial current. Furthermore, since the highest derivative operates on $N-N_\star$ this difference cannot vary rapidly with radius. This implies that fine-scale variations in the seed profile are inherited by the final current profile, as seen in Fig. 3. Physically, this happens because the induced electric field becomes high where runaway current is low, and vice versa. Hence the field diffuses towards areas with effective runaway generation, strengthening the generation there even more. This process also takes place on longer length scales, where it is responsible for the on-axis current peaking seen in simulations. In other words, primary runaway generation occurring shortly after the thermal quench defines the shape of the initial post-disruption "current channel", which is later filled with current carried by secondary runaways. The final runaway current profile depends on the extent to which the electric field has time to diffuse on the time scale of runaway avalanche growth.

- [1] L.-G. Eriksson et. al., Phys. Rev. Lett. 92, 205004(2004).
- [2] R. Aymar et. al., Plasma Phys. Controlled Fusion 44, 519 (2002).