Multi-channel analyser and perpendicular ion energy distribution in magnetised plasma

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Introduction

The electrical methods for the diagnostic of magnetised plasmas are commonly used for their high spatial and temporal resolution. A summary of such methods is presented in the ref. [1]. Other paper [2] focuses only on ion diagnostic. This paper proposes a multi-channel analyser (MCA) for the measurement of the perpendicular ion energy distribution within magnetised plasma. The correlated results from different collectors of the analyser, with different diameters, may supply information about the ion energy distribution function in perpendicular direction to the magnetic field.

Analyser design

The constructive detail of the MCA is shown in the fig.1. It consists of two multi-channel plates (MCP) made of graphite (1,3), separated by graphite spacer (2). The MCP (3) is used to fix the metallic collectors (5) through ceramic insulators (4), while the MCP (1) is exposed to the plasma.

Fig.1. Constructive detail of the multi-channel analyser (MCA):

1,3 – graphite multi-channel plates (MCP);
2 – graphite spacer;
4 – ceramic insulators;
5 – metallic collectors

The MCP (1) is pierced by a number of parallel holes or cylindrical channels of different diameters. One collector is associated to one channel. The collectors are metallic wires of the same diameter as the corresponding channels in MCP (1). Their collecting surfaces are situated in the same plane, parallel to the MCP. The MCA has to be orientated normally to the magnetic field lines so that the axis of the channels became parallel with direction of the magnetic field $B$. 

Theoretical description

The purpose of the analytical development is to determine the ratio between the ion current that passes through one channel and reaches the corresponding collector and the total current entering in that channel. Let us consider an ion, with the mass \( m_i \) and the electric charge \( q_i \), entering in the channel at a certain radial position \( \rho \) with respect to the channel axis (fig.2). Its normal velocity on the magnetic field lines \( v_{\perp} \) makes an angle \( \phi \) with the radius \( \rho \).

![Fig.2. Geometrical description of limit trajectory of the ions inside the channel, in a perpendicular plane to the magnetic field lines, for given \( \rho \) and \( \phi \).](image)

The condition imposed for the ion to pass through the channel is

\[
a_c \leq R - r_G,
\]

where \( a_c = m_i v_{\perp} / q_i B \) is the ion Larmor radius, \( R \) is the channel radius and \( r_G \) is the radial position of the guiding centre (G) of the ion trajectory. The inequality (1) is obtained considering that the channel length is longer than the helical pitch of the ion trajectory. The dotted circle in the fig.2 shows the limit trajectory of the ion.

The general expression of the collected ion current is:

\[
I = \int j dS = \int q_i n_0 f(v) v_{||} d\nu_{||} v_{\perp} d\nu_{\perp} d\phi 2\pi \rho d\rho,
\]

where, \( f(v) \) is the velocity distribution function of the ions normalised to 1, \( v_{||} \) is the velocity of the ion parallel to the magnetic field and \( n_0 \) is the density of the ions in the plasma. To obtain the ion current intensity (2) all the ions entering the channel have to be considered with \( 0 \leq \rho \leq R \), \( 0 \leq \phi \leq 2\pi \) and, for given \( \rho \) and \( \phi \), with \( 0 \leq v_{\perp} \leq v_{\perp\text{max}} \). The value of \( v_{\perp\text{max}} \),

\[
v_{\perp\text{max}} = \frac{q_i B}{m_i} \frac{R^2 - \rho^2}{2(R - \rho \sin\phi)},
\]

is obtained from the relation (1), by replacing the expressions of both \( a_c \) and \( r_G \), respectively, where \( r_G \) can be easily calculated from geometrical reasons, in accordance with fig.2.

For a maxwellian distribution function of the ions, with the temperature \( T_{i\perp} \) in the normal plane to the magnetic field, the relation (2) becomes:
\[ I = q_n_0 \sqrt{\pi R} v_0 \int_0^{2\pi} \int_0^{v_{\text{max}}} \frac{m}{2\pi kT_{\perp}} \exp \left( -\frac{m v^2}{2kT_{\perp}} \right) v_\perp dv_\perp d\phi \left( 2\pi \rho d\rho \right), \quad (4) \]

with \( \sqrt{\pi} v_0 = \int_0^\infty f(v_\parallel) v_\parallel dv_\parallel \). Knowing that the ion current entering the channel from the plasma is \( I_0 = q_n_0 \sqrt{\pi} \pi R^2 \), the ratio between the ion collected current and the entering one is:

\[ f = \frac{I}{I_0} = \frac{1}{\pi} \int_0^{2\pi} \left[ 1 - e^{-\frac{q^2 v^2 R^2}{8kT_{\parallel}m_0} (1-x^2)^2} \right] dx d\phi. \quad (5) \]

The expression (5) was obtained after the integration over \( v_\perp \) and using the quantity \( x = \rho/R \). Another way to calculate the ratio \( f \) is presented in the ref. [3], obtaining a different analytical expression but the same results. It can be observed that, for a given magnetic field strength \( B \) and for a specific type of ions, the ratio \( f \) depends on the ion temperature \( T_{\parallel} \) and on the channel radius. This result can be used to determine the ion temperature in the normal plane on the magnetic field. Combining the results from different channels a complete picture of the ion energy distribution in perpendicular direction might be obtained.

**Results**

The results presented hereafter were obtained for hydrogen plasma, by numerically integrating the expression (5). In the fig.3 was plotted the variation of the ratio \( f \) with respect of the channel radius \( R \), having the ion temperature \( T_{\parallel} \) as parameter.

![Fig.3](image_url) **Fig.3.** The variation of the ratio \( f = I/I_0 \) with respect of the channel radius \( R \), for different values of \( T_{\parallel} \)

![Fig.4](image_url) **Fig.4.** The variation of the ratio \( f = I/I_0 \) with respect of the ion temperature \( T_{\parallel} \), for \( R = 0.4 \) mm

From the experimental point of view, the dependence of the ratio \( f \) with \( R \) can be obtained using an analyser with several channels of different diameters. If the experimental
points fit perfectly with a theoretical curve which is similar with those plotted in fig.3, it means that the distribution function of the ions in perpendicular direction on the magnetic field is maxwellian. The theoretical curve gives also the perpendicular ion temperature $T_{\perp}$. Hence, to obtain these results it is necessary that only the ions coming from the plasma could enter into the channels, the electrons being rejected. This condition requires that the diameter of the channel to be smaller than the sheath thickness formed in front of the MCA. The latter one is of the order of a few Debye lengths. As an example, for hydrogen plasma with the density of $10^{12}$ cm$^{-3}$ and $T_e = T_i = 10$ eV, the Debye length is about 20 $\mu$m. With proper negative bias of the plate, the sheath thickness can be larger even as 0.2 mm. It means that diameters of the holes less as 0.2 mm have to be used. Moreover, in a magnetic field of 1 T, the corresponding Larmor radius for the above ion temperature is about 0.35 mm. Thus, for a good energetic resolution, the analyser is suited for a lower plasma density or a higher magnetic field strength.

The fig.4 shows the dependence of the ratio $f$ with the ion temperature $T_{\perp}$ for a fixed channel radius of 0.4 mm. It can be observed that the resolution of the collected currents decreases when the ion temperature increases. This imposes another constriction to the range of use of the analyser.

Conclusions

This paper proposes a multi-channel analyser (MCA) that can be used in magnetised plasmas to obtain information about the energetic distribution of the ions in a perpendicular direction on the magnetic field. By comparing the experimental results with a theoretical curve, we can conclude on the distribution function of the ions and a perpendicular temperature can be obtained for them. For a good resolution of the results, the use of the analyser is limited by plasma parameters and by the dimensions of the channels.

References