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Linear and Quasilinear Modeling of DED Induced Plasma Rotation*

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We consider the collisionless kinetic equation in Hamiltonian form

$$\frac{\partial f}{\partial t} = \{H, f\}, \qquad H = H_0 + \mathrm{Re}\left(\tilde{H}\mathrm{e}^{-i\omega t}\right), \tag{1}$$

with the unperturbed Hamiltonian H_0 and the perturbed Hamiltonian $\tilde{H}$

$$H_0 = \frac{m_0\mathbf{v}_0^2}{2} + e\Phi_0, \qquad \tilde{H} = -\frac{e}{c}\mathbf{v}_0\cdot\tilde{\mathbf{A}}, \qquad \mathbf{v}_0 \equiv \frac{1}{m_0}\left(\mathbf{p} - \frac{e}{c}\mathbf{A}_0\right), \tag{2}$$

and the Poisson brackets

$$\{a, b\} = \frac{\partial a}{\partial \mathbf{r}}\cdot\frac{\partial b}{\partial \mathbf{p}} - \frac{\partial b}{\partial \mathbf{r}}\cdot\frac{\partial a}{\partial \mathbf{p}} = \frac{\partial a}{\partial \boldsymbol{\theta}}\cdot\frac{\partial b}{\partial \mathbf{J}} - \frac{\partial b}{\partial \boldsymbol{\theta}}\cdot\frac{\partial a}{\partial \mathbf{J}}. \tag{3}$$

Here, e and m_0 are the particle charge and mass, $\mathbf{A}_0$ and $\tilde{\mathbf{A}} = -ic\tilde{\mathbf{E}}/\omega$ are the unperturbed potential and the amplitude of the perturbed vector potential, $\tilde{\mathbf{E}}$ is the amplitude of electric field perturbation, ω is the perturbation frequency, c is the speed of light, $\mathbf{p}$ is the generalized momentum, and $\boldsymbol{\theta}$ and $\mathbf{J}$ are canonical angles and actions.

For the geometry of a straight periodic cylinder with a rotational transform, the angles $\boldsymbol{\theta} = (\phi, \theta, z)$ are the gyrophase, the azimuth and the z coordinate, whereas the actions $\mathbf{J} = (J_\perp, p_\theta, p_z)$ are the perpendicular adiabatic invariant, the generalized azimuthal momentum and the z-component of the generalized momentum.

When the distribution function is presented in the form

$$f = f_0 + \mathrm{Re}\left(\tilde{f}\mathrm{e}^{-i\omega t}\right), \tag{4}$$

where $\{f_0, H_0\} = 0$ and the perturbed quantities are expanded in Fourier series over canonical angles

$$\tilde{f}(\boldsymbol{\theta}, \mathbf{J}) = \sum_{\mathbf{m}} f_{\mathbf{m}}(\mathbf{J})\,\mathrm{e}^{i\mathbf{m}\cdot\boldsymbol{\theta}}, \qquad \tilde{H}(\boldsymbol{\theta}, \mathbf{J}) = \sum_{\mathbf{m}} H_{\mathbf{m}}(\mathbf{J})\,\mathrm{e}^{i\mathbf{m}\cdot\boldsymbol{\theta}}, \tag{5}$$

one obtains in linear order with respect to the perturbation amplitude

$$f_{\mathbf{m}} = \frac{H_{\mathbf{m}}}{\mathbf{m}\cdot\boldsymbol{\Omega} - \omega - io}\mathbf{m}\cdot\frac{\partial f_0}{\partial \mathbf{J}}, \tag{6}$$

and from this the linear perturbation current density

$$\tilde{\mathbf{j}} = \frac{ie^2 n_0}{m_0\omega}\tilde{\mathbf{E}} + e\int \mathrm{d}^3p\;\mathbf{v}_0\tilde{f}. \tag{7}$$

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In Ref. [1], a finite Larmor radius expansion for this current density had been developed in order to be able to solve Maxwell equations numerically. Results from the developed wave code for realistic profiles of the plasma parameters and for a realistic spectrum of the currents in the DED coils are presented below.

The total time averaged power absorbed in the plasma is obtained as

$$P = \frac{1}{2}\mathrm{Re}\int \mathrm{d}^3 r\, \tilde{\mathbf{E}}^* \cdot \tilde{\mathbf{j}} = -4\pi^4 \omega R \sum_{\mathbf{m}} \int \mathrm{d}^3 J\, \delta\left(\mathbf{m}\cdot\mathbf{\Omega} - \omega\right) \left|H_{\mathbf{m}}\right|^2 \mathbf{m}\cdot\frac{\partial f_0}{\partial \mathbf{J}} = \sum_{\mathbf{m}} P_{\mathbf{m}}. \quad (8)$$

Here, $2\pi R$ is the cylinder length, $\mathbf{\Omega} = \partial H_0(\mathbf{J})/\partial \mathbf{J} = (\omega_c,\, \Omega^\theta,\, \Omega^z)$ are canonical frequencies namely the cyclotron frequency, the azimuthal frequency $\Omega^\theta = h^\theta v_\parallel + \Omega_E^\theta$, and the z-component of the velocity $\Omega^z = h^z v_\parallel + \Omega_E^z$, h^i are the contravariant components of the unit vector $\mathbf{h}$ along the unperturbed magnetic field, and Ω_E^i are the contravariant components of the electric drift velocity due to a quasi-stationary radial electric field.

The Fourier harmonic numbers $\mathbf{m} = (m_\phi, m_\theta, k_z)$ are the cyclotron harmonic index, the azimuthal (poloidal) wave number and the (toroidal) wave number in the z-direction. In the low frequency range which is of interest in this context, only the $m_\phi = 0$ harmonic is of relevance in (8) and the same is true in the quasilinear expressions below.

The resonance condition is then $\mathbf{m}\cdot\mathbf{\Omega} - \omega = k_\parallel v_\parallel + \omega_E - \omega = 0$ with $k_\parallel = m_\theta h^\theta + k_z h^z$, $\omega_E = k_\perp V_E$, $k_\perp = (h_z m_\theta - h_\theta k_z)/r$ and where h_i are the covariant components of $\mathbf{h}$, r is the radius and V_E is the electric drift velocity.

In the next (quadratic) order of the expansion, one obtains the quasilinear equation for f_0 by averaging the kinetic equation over the angles

$$\frac{\partial f_0}{\partial t} = \frac{1}{2}\mathrm{Re}\sum_{\mathbf{m}} \left\{H_{\mathbf{m}}^*, f_{\mathbf{m}}\right\}. \quad (9)$$

When this equation is integrated over the momenta and averaged over the flux surface, the following continuity equation results

$$\frac{\partial n_0}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r} r F_n, \qquad n_0(t,r) = \frac{1}{(2\pi)^2 R}\int \mathrm{d}^3\theta \int \mathrm{d}^3 J\, \delta\left(r - r_c(\boldsymbol{\theta},\mathbf{J})\right) f_0(t,\mathbf{J}), \quad (10)$$

where the density of the radial flux of particles of a given species is

$$\begin{aligned} F_n &= \frac{1}{8\pi^2 r R}\mathrm{Re}\sum_{\mathbf{m}}\int \mathrm{d}^3\theta \int \mathrm{d}^3 J\, \delta\left(r - r_c(\boldsymbol{\theta},\mathbf{J})\right)\left\{r_c(\boldsymbol{\theta},\mathbf{J}), H_{\mathbf{m}}^*(\mathbf{J})\right\} f_{\mathbf{m}} \quad (11)\\ &= -\frac{\pi}{2r}\sum_{\mathbf{m}}\int \mathrm{d}\phi \int \mathrm{d}^3 J\, \delta\left(r - r_c(\phi,\mathbf{J})\right)\mathbf{m}\cdot\frac{\partial r_c(\phi,\mathbf{J})}{\partial \mathbf{J}}\left|H_{\mathbf{m}}\right|^2 \delta\left(\mathbf{m}\cdot\mathbf{\Omega} - \omega\right)\mathbf{m}\cdot\frac{\partial f_0}{\partial \mathbf{J}}. \end{aligned}$$

Here, $r_c(\phi,\mathbf{J}) = r_g(\mathbf{J}) + \rho^r(\phi,\mathbf{J})$, where r_g is the guiding center radius, ρ^r describes the Larmor gyration, and $\mathbf{m}\cdot\partial r_g/\partial\mathbf{J} = ck_\perp/(eB_0)$ with B_0 the unperturbed magnetic field.

In the case of low frequencies $\omega \to 0$ with negligible finite Larmor radius effects $\rho^i \to 0$ and a negligible electric drift $\omega_E \to 0$, one can simplify (11) using

$$H_{\mathbf{m}} \approx -\frac{ev_\parallel}{c}\mathbf{h}\cdot\tilde{\mathbf{A}} = \frac{iev_\parallel}{ck_\perp}\tilde{B}_r, \quad (12)$$

where $\tilde{\mathbf{A}}$ and $\tilde{B}_r$ correspond to the harmonic of the perturbation field with given m_θ and k_z. Replacing in (11) r_c with r_g, taking a local Maxwellian for f_0, $f_0 =$

$(2\pi mT)^{-3/2}\, n_0 \exp\left(-\omega_c J_\perp/T - m_0 v_\parallel^2/(2T)\right)$, and using $\mathbf{m}\cdot\mathbf{\Omega} - \omega \approx k_\parallel v_\parallel$, one recovers the radial particle flux due to the parallel transport of a given species in the ergodic magnetic field

$$F_n \approx -\sqrt{\frac{2}{\pi}} D_M v_T n_0 \left(\frac{1}{n_0}\frac{\partial n_0}{\partial r} - \frac{e}{T}E_{0r} + \frac{1}{2T}\frac{\partial T}{\partial r}\right), \qquad D_M = \frac{\pi}{2}\sum_{\mathbf{m}} \left|\frac{\tilde{B}_r}{B_0}\right|^2 \delta(k_\parallel), \quad (13)$$

where E_{0r} is the quasi-stationary radial electric field, $v_T = \sqrt{T/m_0}$ and D_M is the magnetic field diffusion coefficient. The particle flux (11) which is predominantly an electron flux (the main interaction of the perturbation field is with electrons), leads to a plasma polarization and the acceleration of the $\mathbf{E}\times\mathbf{B}$ rotation in the radial electric field,

$$\frac{\partial E_{0r}}{\partial t} = -4\pi \sum_{species} eF_n. \quad (14)$$

The particle flux (11) can be presented as the result of the radial drift under the action of the perpendicular component of the poloidal and toroidal forces acting on the plasma due to the momentum absorption from the perturbation field. The total poloidal (toroidal) torque due to this interaction has the form (see, e.g. [1]),

$$T_\theta = \sum_{\mathbf{m}} \frac{m_\theta}{\omega} P_{\mathbf{m}}, \qquad T_\varphi = \sum_{\mathbf{m}} \frac{k_z R}{\omega} P_{\mathbf{m}}, \quad (15)$$

where $P_{\mathbf{m}}$ has been defined in (8). Thus, on can see from (13) that the interpretation of the DED effect on the plasma rotation as a result of the equilibrium radial electric field modification through the increased electron transport in the ergodic magnetic field region [2, 3] corresponds to the particular case of resonant interaction of the DED magnetic field with the plasma. At the same time, ergodization of the field is not a necessary condition for the acceleration of the plasma by the perturbation field. This also can take place if a single chain of islands is created by this field.

In Fig. 1 the plasma parameters used in the calculations of the poloidal and toroidal torque are shown in the upper panels. The safety factor profile is chosen such that the main resonance is located at $r = 41$ cm. The plasma radius is at $r \approx 46$ cm. Since density and temperature gradients are present, the diamagnetic velocities of ions and electrons are not zero. The equilibrium profile of the electrostatic field shown in Fig.1c is chosen such that the total poloidal ion velocity (diamagnetic, $\mathbf{E}\times\mathbf{B}$ and projected parallel ion velocity) is zero.

As one can see from the Figs.1d-1f, for typical TEXTOR plasma parameters the resonant interaction leads to a plasma acceleration in the poloidal and the toroidal directions of the plasma current within the whole DED frequency range. In the 12/4 DED operational mode, the poloidal torque acting on the plasma is negative (in the direction of the diamagnetic current) whereas as the toroidal torque is positive (in the direction of the parallel current) over the whole operational range of DED (0-10 kHz). In the 3/1 mode the torque rapidly reverses sign at 16.7 kHz (for 12/4 mode this frequency is 92 kHz). The reversal frequency can be found from the condition that in the rest frame of the electron fluid the frequency of the prinicpal mode ($m/n = 12/4$ or $3/1$) is zero [1].

Remarkably, because of diamagnetic effects and plasma rotation, the plasma experiences also a finite torque at the zero frequency limit (DC mode). At the frequency

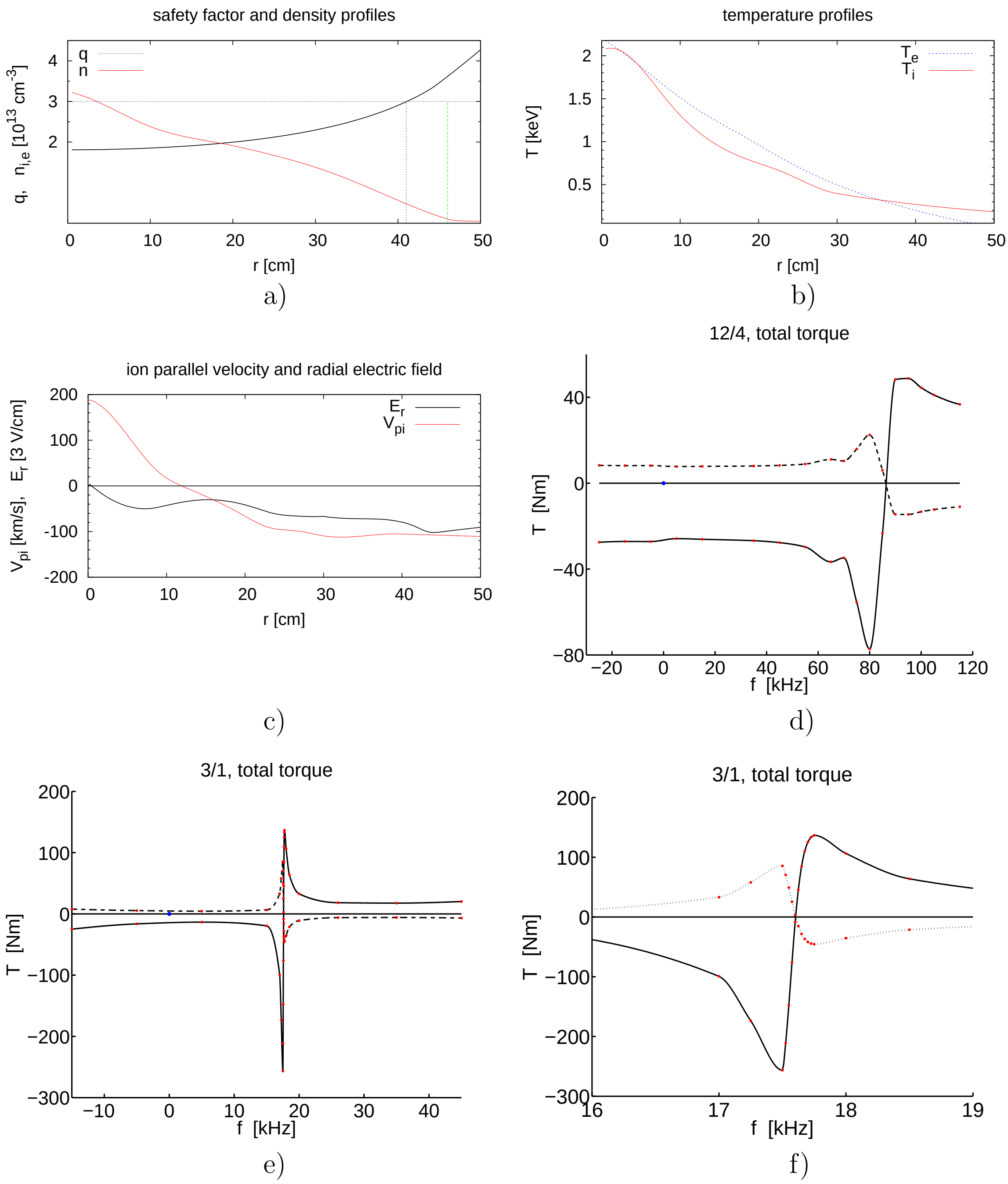

Fig. 1. The safety factor and density profiles are shown in a), temperature profiles in b) and the profile of the parallel ion velocity and radial electric field in c). In d), e) and f) the results for the total, i.e. the sum of the contributions from the individual harmonics to the poloidal (solid) and toroidal (dashed) torque are shown for the nominal 15 kA DED filament current. f) is is a zoom of e).

16.7 kHz where the torque changes sign, the screening of the DED perturbation field by plasma currents is weak and the perturbation will be noticable even far inside the resonance radius.

References

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