*33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, P-4.167 (2006)*

Plasma equilibrium for a strong coupling of parallel and ExB plasma flows

T.K. Soboleva[1], S.I. Krasheninnikov[2], and J.J. Martinell[1]

[1] *Instituto de Ciencias Nucleares, UNAM, México D. F., Mexico*

[2] *University of California at San Diego, La Jolla, California* 92093-0403*, USA*

I. Introduction. In ideal MHD equilibria the pressure and the poloidal current are flux surface quantities: $p=p(\psi)$ and $I=I(\psi)$, when no plasma flows are present, but more general equilibria that include various classes of flows the plasma pressure ceases to be a flux surface quantity, becoming for instance $p=p(\psi,R)$, R being the coordinate of the torus mayor radius, and thus the constant pressure surfaces are shifted with respect to the magnetic flux surfaces. Experimentally, there is also evidence of the variation of the pressure over flux surfaces, as in the observations made in DIII-D [1], that show that the electric potential and the electron pressure have maxima near the X-point in L-mode plasmas. This drives an $\mathbf{E}\times\mathbf{B}$ circulation about the X-point that takes plasma across flux surfaces and the separatrix. The observations were also supported by numerical simulations with the code UEDGE that self-consistently included $\mathbf{E}\times\mathbf{B}$ and ∇B drifts.

The importance of the flow direction to produce field-aligned pressure variations was pointed out in [2], where it was demonstrated that a strong coupling of $\mathbf{E}\times\mathbf{B}$ and parallel flows can result in a large pressure change along the magnetic flux surfaces, when there is a shear in the $\mathbf{E}\times\mathbf{B}$ flow that breaks the symmetry in the direction perpendicular to **B**. A variable pressure over flux surfaces is of relevance also at the start-up phase of a tokamak, when the magnetic field is mainly toroidal [3]; the $\mathbf{E}\times\mathbf{B}$ drift becomes very important in determining the radial displacement of the plasma, when (B_p/B_T) is very small. These results would suggest that the $\mathbf{E}\times\mathbf{B}$ drift effects can be important for the region near the X-point, where the ratio B_p/B_T is small too. In this paper we focus on an analytic study of the impact of a strong coupling of parallel and perpendicular flows on plasma equilibrium, that can explain the DIII-D results.

II. Plasma Flows. We consider a plasma of electrons and ions with the same temperature. We adopt a cylindrical approximation for a tokamak with a strong "toroidal" magnetic field in z-direction, $B_z = \text{const.}$, and assume that there is no z-dependence of the plasma parameters. Then the velocity of the flow is $\mathbf{v} = \mathbf{b}v_{\|} + \mathbf{v}_{\perp}$ where $\mathbf{b} = \mathbf{B}/B \cong \mathbf{e}_z + \beta(\mathbf{e}_z \times \nabla\psi)$ is the unit vector along the magnetic field **B**; $\psi \equiv \psi(x,y)$ is the magnetic flux function, $v_{\|}$ is the parallel velocity, and $\mathbf{v}_\perp = V_E(\mathbf{b}\times\nabla\varphi)(B_z/B) \cong V_E(\mathbf{b}\times\nabla\varphi)$ is the $\mathbf{E}\times\mathbf{B}$ drift velocity, φ is the electrostatic potential multiplied by electron charge and V_E is a normalization constant. We have assumed that $|B_\perp|/B_z \sim \beta << 1$.

Then, considering the cold ion approximation and no parallel current we have the equations of continuity, parallel plasma momentum balance and the parallel electron balance

$$\nabla\cdot(n\mathbf{v}) = 0, \tag{1}$$

$$\{(n\mathbf{v}\cdot\nabla)\mathbf{v} + \nabla P/M\}\cdot\mathbf{b} = 0, \tag{2}$$

$$\{-\nabla\varphi + \alpha\nabla T + \nabla P/n\}\cdot\mathbf{b} = 0, \tag{3}$$

where n is the plasma density, $P = nT$ the plasma pressure, T the electron temperature, M the ion mass and α is the thermal force coefficient. Recalling that $\partial(...)/\partial z = 0$, the continuity equation (1) can be written as follows

$$n\mathbf{v} = \mathbf{e}_z n v_z + \mathbf{e}_z \times \nabla G, \tag{4}$$

where v_z is the z-component of plasma velocity and $G \equiv G(x,y)$ is the particle flux function. Also, from Eq. (4) we find an estimate $v_\perp \sim \beta v_{||}$. Then, to lowest order in β, we find from Eqs. (4) and (3)

$$w\nabla\psi + V_E\nabla\varphi = \nabla G/n, \tag{5}$$

$$-\nabla\varphi + (1+\alpha)\nabla T + T\nabla \ell n(n/n_0) = K\nabla\psi, \tag{6}$$

with $w = \beta v_{||}$, $K \equiv K(x,y)$, and n_0 is a normalization constant. From Eqs. (2, 5)

$$(\mathbf{e}_z \times \nabla G)\cdot\nabla w + \beta^2(\mathbf{e}_z \times \nabla\psi)\cdot\nabla(nT)/M = 0. \tag{7}$$

Notice that from Eqs. (5, 6) we find

$$n\nabla w \times \nabla\psi = -\nabla \ell n(n/n_0)\times\nabla G, \tag{8}$$

$$\nabla T \times \nabla \ell n(n/n_0) = \nabla K \times \nabla\psi. \tag{9}$$

We may analyze the implications of these equations at this point. Eq. (5) gives the coupling of the parallel and perpendicular $\mathbf{E}\times\mathbf{B}$ velocities: w and V_E, as required by the continuity of the flow. An unbalanced transverse flow (which induces a parallel flow) arises if $\nabla\cdot\mathbf{v}_\perp \neq 0$ which happens when $\nabla\varphi\times\nabla n \neq 0$. In our case, this term can be obtained taking the cross product of Eq. (6), with ∇n and assuming that T = T(n). Thus, $\nabla n\times\nabla\varphi = K\nabla n\times\nabla\psi$, which implies that the non-vanishing of $\nabla\cdot\mathbf{V}_\perp$ must produce a density function that is not constant on the magnetic flux surfaces, and therefore the pressure in not a surface quantity. This exemplifies that the coupling of V_E and w, causes P to vary along a magnetic field line.

Next we change variables from (x, y) to (G, ψ). Then, from Eqs. (5, 6) we find

$$w = V_E \left\{ (1+\alpha)\frac{\partial T}{\partial \psi} + T\frac{\partial \Lambda}{\partial \psi} - K \right\}, \tag{10}$$

$$V_E \left\{ (1+\alpha)\frac{\partial T}{\partial G} + T\frac{\partial \Lambda}{\partial G} \right\} = -\frac{1}{n}, \tag{11}$$

where $\Lambda = \ell n(n/n_0)$. From Eqs. (7-9) we have

$$\frac{\partial w}{\partial \psi} = \frac{\beta^2}{M}\frac{\partial (nT)}{\partial G}, \qquad \frac{\partial w}{\partial G} = -\frac{\partial}{\partial \psi}\left(\frac{1}{n}\right), \tag{12}$$

$$\frac{\partial T}{\partial G}\frac{\partial \Lambda}{\partial \psi} - \frac{\partial T}{\partial \psi}\frac{\partial \Lambda}{\partial G} = \frac{\partial K}{\partial G}. \tag{13}$$

In order to further simplify our equations we will assume that $K = K_\psi \psi$, where $K_\psi = \text{const}$. Then from Eq. (13) it follows that the density n must be a function of the temperature T: $n = N(T)$. Next we notice that from Eqs. (12) we have

$$\frac{\beta^2}{M}\frac{\partial^2 (nT)}{\partial G^2} + \frac{\partial^2}{\partial \psi^2}\left(\frac{1}{n}\right) = 0. \tag{14}$$

Therefore picking $N(T) \equiv n_0 (T_0/T)^{1/2}$, from Eq. (14) we find

$$\frac{\beta^2 n_0^2 T_0}{M}\frac{\partial^2}{\partial G^2}(T/T_0)^{1/2} + \frac{\partial^2}{\partial \psi^2}(T/T_0)^{1/2} = 0. \tag{15}$$

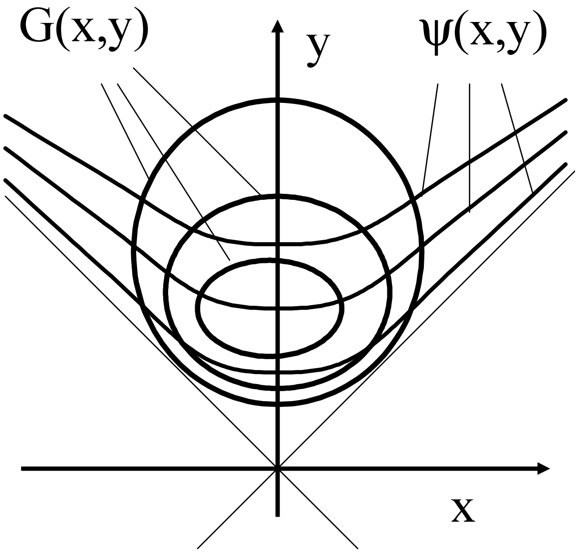
One of the possible solutions of Eq. (15) is $(T/T_0)^{1/2} = C_G G + C_\psi \psi$, where C_G and C_ψ are constants. From here and (10-13) we find $C_G = 1/V_E n_0 T_0 (1+2\alpha)$ and the following relation between constants C_ψ and K_ψ

$$V_E K_\psi - (1+2\alpha) V_E T_0 C_\psi^2 = \frac{\beta^2}{M}\frac{1}{(1+2\alpha)V_E}. \tag{16}$$

Thus, we find that for a strong coupling of parallel and perpendicular dynamics one may have equilibrium solutions with a large variation of plasma pressure on the magnetic flux surfaces:

$$\frac{T}{T_0} = \left(C_\psi \psi - \frac{G}{V_E n_0 T_0 (1+2\alpha)} \right)^2, \qquad \frac{n}{n_0} = \left(C_\psi \psi - \frac{G}{V_E n_0 T_0 (1+2\alpha)} \right)^{-1}, \tag{17}$$

This equilibrium can be used to model the DIII-D results about the formation of high-density, cold plasma in the vicinity of the tokamak X-point [1]. In this case we can, for example, take a magnetic flux function $\psi(x,y) \propto x^2 - y^2$, and $G(x,y)$ in such a way as to have the equi-potentials like those shown in Fig 1, with $G(x,y) \to \text{const}$. far away from the X-point



(x=y=0). Such equilibrium has a strong variation of pressure, P, along the magnetic flux surfaces near the X-point and a flux surface pressure, $P = P(\psi)$, far from it.

A similar analysis can be made when the chosen variables are $\xi = \ell n(n/n_0)$ and ψ or G and ξ instead of x and y. The resulting temperature profile in those cases is [4],

$$T = \theta(n) \equiv \frac{a}{(1+2\alpha-b)n^2} + \frac{C_T}{n^{(1-b)/(1+\alpha-b)}},$$

where $a = G_\psi^2/(V_E^2 K_\psi)$, $b = \beta^2/(MV_E^2 K_\psi)$, C_T is a normalization constant and $K = K_\psi \psi$ and $G \equiv G_\psi \psi + G_\xi(\xi)$ was assumed. Examining this solution we find that it is the extension of the results of [2] to an arbitrary magnetic geometry described by the flux potential ψ.

III. Conclusions. The applicability of the solutions found here is specially important to certain regions of a toroidal plasma that satisfies the condition $B_\perp / B_z << 1$. It is particularly relevant to describe the region around the X-point of the separatrix in a divertor. The solution (17) presents a maximum value near the point on maximum G(x,y) and tends to zero far from this region, since $G(x,y) \to \text{const.}$, returning the pressure to be a flux surface quantity $P(\psi)$.

The establishment of circulation can be traced back to the shearing of the $\mathbf{E}\times\mathbf{B}$ flow, as it was pointed out in [2]. For the case of DIII-D one may check that the magnitude of the relevant quantities confirms the claim. Indeed, an estimate of the $\mathbf{E}\times\mathbf{B}$ shear gives $V_E' \sim V_E / L \sim (e\varphi/T)(cT/eB)/L^2 \sim 16 D_B / L^2$, where D_B ($\sim 10^4$ cm^2/s) is the Bohm diffusion coefficient; so again for L~4 cm, $V_E' \approx (2\times10^{-4} s)^{-1} \sim \tau_x{}^{-1}$, which indicates that the exchange time for the flows coincides with the shearing.

Acknowledgments. Partial support by DGAPA-UNAM grant IN115205 and CONACyT grant 44324-F, Mexico and US DoE Grant DE-FG02-04ER54739 at UCSD is appreciated.

References

[1] M. J. Schaffer, B. D. Bray, J. A. Boedo, *et al.*, Phys. Plasmas **8**, 2118 (2001).

[2] S. I. Krasheninnikov, D. J. Sigmar, and P. N. Yushmanov, Phys. Plasmas **2**, 1972 (1995).

[3] L. E. Zakharov and G. V. Pereverzev, Sov. J. Plasma Phys. **14**, 75 (1988).

[4] S. I. Krasheninnikov, T. K. Soboleva, and J. J. Martinell, Phys. Plasmas **12** (2006) in press.