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The Water Bag model for gyrokinetic simulations

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Turbulence transport has emerged as one of the most important topics in magnetized fusion plasma research. Electrostatic low frequency turbulence is indeed assumed to be responsible of the inadequate thermal confinement. For improving the performance of the fusion devices it is of prime importance to get a better understanding of the mechanisms of instability. Plasma turbulence can be studied by using either a fluid or a kinetic description. The kinetic description is the most difficult way to solve the problem but it allows us to describe the resonant interactions between waves and particles. This process cannot indeed be fully described with fluid equations which then lead to overestimates turbulent fluxes. Moreover we deal with a weakly collisional plasma whereas the fluid model needs a strongly collisional regime to be justified.

A drift-kinetic model in cylindrical geometry has been used to study slab Ion Temperature Gradients (ITG) instabilities as depicted in a former work [1]. A periodic cylindrical collisionless plasma is considered as a limit case of a stretched torus. The uniform and constant magnetic field is along the axis of the column (z coordinate). Electron inertia is ignored (adiabatical response to the low frequency fluctuations), and concerning the ions, finite Larmor radius effects are neglected so that only the guiding-center trajectories are taken into account. There is no equilibrium radial electric field. The plasma approximation $n_i = n_e$ is sufficient for low frequency electrostatic perturbations. With these assumptions the evolution of the ion guiding-center distribution function $f(\mathbf{r}_\perp, z, v_\parallel, t)$ is described by the following drift-kinetic Vlasov equation :

$$\frac{\partial f}{\partial t} + \mathbf{v}_\perp \cdot \nabla_\perp f + v_\parallel \frac{\partial f}{\partial z} + \frac{q}{m} E_\parallel \frac{\partial f}{\partial v_\parallel} = 0 \qquad (1)$$

where $\mathbf{v}_\perp = \mathbf{E} \times \mathbf{B}/B^2$ is the $E \times B$ drift velocity.

Compared to the Ref. [1] where the equation (1) is solved numerically with a continuous maxwellian distribution function, a new Water Bag model has been used for both analytical and numerical studies with the aim of decreasing the numerical difficulty required by the parallel direction. Thirty years ago this Water Bag model had been used only for non magnetized plasmas ([2],[3]) because this model is well suited mainly for problems involving a phase space with one velocity component. For magnetized plasmas it gives an interesting alternative way to study turbulence thanks to the gyrokinetic approximation which allows to reducing the 3D velocity space into a 1D space ($v_\parallel$).

With the Water Bag model a discrete distribution function f taking the form of a multi-step-like function is used in place of the continuous distribution function along the velocity direction (fig. 1).

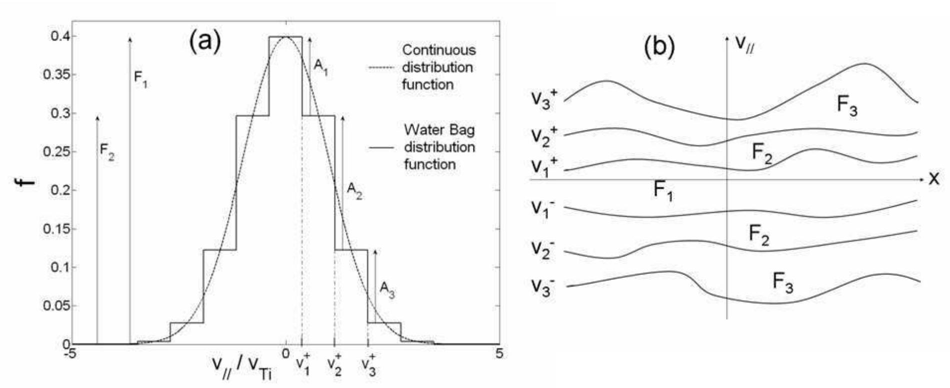

Figure 1: Continuous (dashed line) and Water Bag (solid line) distribution functions (a). Contours of the bags in the phase-space for a three-bag system (b). Between two contours $f = F_j$ remains constant.

At initial time the distribution function can be written of the form :

$$f_{eq}(r, v_\parallel) = \sum_{j=1}^{M} A_j \left[H(v_\parallel + a_j) - H(v_\parallel - a_j) \right] \tag{2}$$

where $A_j = F_j - F_{j+1}$ is the height of the step j and $v_j^{\pm} = \pm a_j$ its corresponding velocity abscissa, as depicted in fig. 1. M is the number of bags. H is the Heaviside function. When time is increasing the heights A_j remain constant thanks to phase space conservation properties ($df/dt = 0$ for a collisionless plasma), meanwhile velocities a_j change and are now labelled $v_j^{\pm}$. A particle on one of the contours remains on this contour, therefore the problem is entirely described by the functions $v_j^{\pm}(\mathbf{r}, t)$ (fig. 1) which obey to the equation

$$\frac{\partial v_j^{\pm}}{\partial t} + \mathbf{v}_\perp \cdot \nabla_\perp v_j^{\pm} + v_j^{\pm} \frac{\partial v_j^{\pm}}{\partial z} = \frac{q}{m_i} E_\parallel \tag{3}$$

Let us now introduce for each bag j the density $n_j = (v_j^+ - v_j^-) A_j$, average velocity $u_j = (v_j^+ + v_j^-)/2$ and pressure $P_j = m_i n_j^3 / (12 A_j^2)$. For each bag j equations (3) allow to recover the continuity and Euler equations namely

$$\frac{\partial n_j}{\partial t} + \nabla_\perp . (n_j \mathbf{v}_\perp) + \frac{\partial}{\partial z}(n_j u_j) = 0 \tag{4}$$

$$\frac{\partial u_j}{\partial t} + \mathbf{v}_\perp \cdot \nabla_\perp u_j + u_j \frac{\partial u_j}{\partial z} = -\frac{1}{m_i n_j} \frac{\partial P_j}{\partial z} + \frac{q}{m_i} E_\parallel \tag{5}$$

where m_i and q are ion mass and charge. Consequently the one bag case is equivalent to the fluid description, and the case of M bags is equivalent to multi-fluids coupled by the quasi-neutrality. The limit M going to infinity allows us to find the continuous distribution function case.

A linear study of this model has been achieved with the aim of predicting the ITG instability characteristics as a function of plasma parameters. In this approach density, plasma potential and electric field are expanded to first order around the equilibrium state in order to characterize the behavior of first-order fluctuating quantities, under the form $G = G_{eq} + G_1$, where G_{eq} is the quantity at the equilibrium and G_1 the perturbation which writes on a Fourier basis in θ and z directions as

$$G_1(r) = G_{1_0}(r) e^{i(m\theta + k_\parallel z - \omega t)} \tag{6}$$

The Fourier wave number k_θ is m/r. The distribution function writes $f(\mathbf{r}, v_\parallel) = f_{eq}(r, v_\parallel) + f_1$, where $f_{eq}(r, v_\parallel)$ is the relation (2) and where f_1 is

$$f_1 = f_{1_0}(r, v_\parallel) e^{i(m\theta + k_\parallel z - \omega t)} \tag{7}$$

Finally, (2) together with the quasi-neutrality, the Boltzmann relation, and expressions (6,7) give the dispersion relation

$$1 + Z_i^\star \sum_{j=1}^{M} \alpha_j \frac{\omega \Omega_j^\star - 1}{\omega^2 - a_j^2} = 0 = \varepsilon(\omega) \tag{8}$$

where $Z_i^\star = Z_i T_e / T_i$, $\alpha_j = 2 a_j A_j / n_o$ is the relative density of the bag number j and $\Omega_j^\star = k_\theta \frac{KT_i}{qB} \partial_r \ln n_{0_j}$ is the ion diamagnetic frequency of the bag j. In (8) velocities and frequencies are normalized quantities respectively to the ion thermal velocity v_{Ti} and $k_\parallel v_{Ti}$. Let us introduce two macroscopic plasma parameters : $\Omega_n^\star = k_\theta \frac{KT_i}{qB} \partial_r \ln n_0$ and $\Omega_T^\star = k_\theta \frac{KT_i}{qB} \partial_r \ln T_i$. The behavior of the linear growth rate γ is given by the zeros of the linear dispersion relation (8) where $\omega = \omega_r + i\gamma$. Of course this linear description does not give any information about the saturation behavior. The instability threshold corresponds to the case Im(ω)= $\gamma \to 0$. Finding the instability threshold is equivalent to solve the system $\varepsilon(\omega) = 0$ and $d\varepsilon(\omega)/d\omega = 0$. Also we have been careful with choosing an equilibrium Water Bag distribution function eqivalent to the continuous distribution function in the sense of the moments. For each value ω we solve the system and obtain the dependence of $\Omega_T^\star$ on $\Omega_n^\star$ at the threshold. Then it is possible to compare these curves with the analytical result given in [1] in the case of a maxwellian continuous distribution function (fig. 2). It is interesting to note that an instability threshold appears only for a number of bags $M \geq 2$. The one bag case, equivalent to a fluid description, does not have indeed any instability. The figure 2 (left) shows the three bag case with some quantitative difference compared to the continuous distribution curve. However the qualitative behaviour is recovered with the two stable and unstable areas clearly appearing. As one can see from the figure 2 (right) the accuracy of the water-bag description is rapidly improving with an increasing number of bags.

We also observed that only slow particles, i.e. which have a velocity lower than the thermal speed, are related to the bottom left and top right parts of the graph, whereas fast particles are related to the bottom right and top left parts.

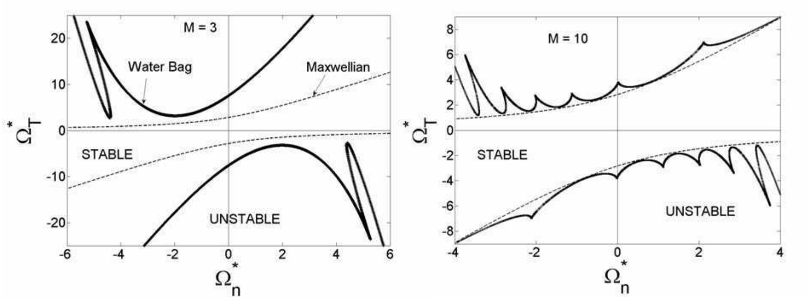

Figure 2: Dependence of $\Omega_T^\star$ on $\Omega_n^\star$ at the threshold for $M = 3$ and $M = 10$. The dashed and solid lines correspond respectively to continuous and Water Bag distribution functions.

Approximately ten bags seem to be sufficient to correctly describe the physics of the instabilities, which enables us to undertake numerical simulations of the contours equations (3) using this water-bag model. The discontinuous Galerkin (DG) method [4] has been used to investigate these equations. This is a finite element methods space discretization by discontinuous approximations, that incorporates the ideas of numerical fluxes and slope limiters used in high-order finite difference and finite volume schemes. The DG method can be combined with Runge-Kutta or Lax-Wendroff time discretization scheme to give stable, high-order accurate, highly parallelizable schemes that can easily handle h-p adaptivity, complicated geometries and boundaries conditions. The first step was to write a 2D phase-space code (z, $v_\parallel$) without taking account the perpendicular direction. The validation was performed in particular with Landau damping. Now we are implementing a 4D ($\mathbf{r}_\perp$, z, $v_\parallel$) code where the velocities in azimuthal direction are linearized. The last step will be to consider all the equations (3) and observe the saturation behavior of the instabilities. Currently we are also working on taking into account the finite Larmor radius effect by gyro-averaging and polarization drift due to ion inertia. Both corrections do not generate any additional difficulties to implement into the Water Bag description. Next we intend applying this model to the toroidal geometry of a tokamak.

References

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