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Drift-Tearing and Electron Temperature Gradient Driven Turbulence*

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Contrary to relevant experimental observations in high-temperature toroidal plasmas [1], both the collisionless and weakly collisional theory of drift-tearing modes predict [2, 3, 4] that in the presence of a significant radial electron temperature gradient these modes should remain stable. In collisionless regimes [3] this result is due to electron Landau damping associated with a transverse electron temperature gradient whereas in the (weakly) collisional regime the effect of Landau damping is replaced by that of finite inverse thermal conductivity [4]. In particular, there exists a critical value of the stability parameter Δ', the jump in the first derivative of the perturbed magnetic field across the reconnection layer, such that the mode is stable for $\Delta' < \Delta'_{\rm crit}$. In Fig. 1 the quantity $\Delta'_{\rm crit}a$, as predicted by a weakly collisional analysis, is shown as a function of $\eta_e = d\log(T_e)/d\log(n)$. Here $a = 20$cm is a reference scale-length, and the values of the plasma parameters used in the evaluation of $A \propto (m_{\rm i}/m_{\rm e})(cT_{\rm e}/eB)\nu_{\rm ei}^{-1}L_s^{-2}$ pertain to typical ones found in the Alcator C-MOD device. The two curves in Fig. 1 correspond to two different values of A, showing $\Delta'_{\rm crit}$ to increase with A. Here L_s is the shear length, more precisely defined below. Note that $\Delta'_{\rm crit}$ is a growing function of η_e for a given A, indicating that the electron temperature gradient has a strong stabilizing influence.

In modern toroidal experiments the estimated values of Δ' for the observed modes are well below the corresponding $\Delta'_{\rm crit}$. Having established that in the high temperature regimes the perpendicular temperature gradient and the associated Landau damping or the effect of finite longitudinal thermal conductivity has a stabilizing influence, we propose that a local depression in thermal conductivity, caused for example by the interaction of the drift-tearing mode with a background spectrum of microscopic modes, is the process responsible for the excitation of the mode.

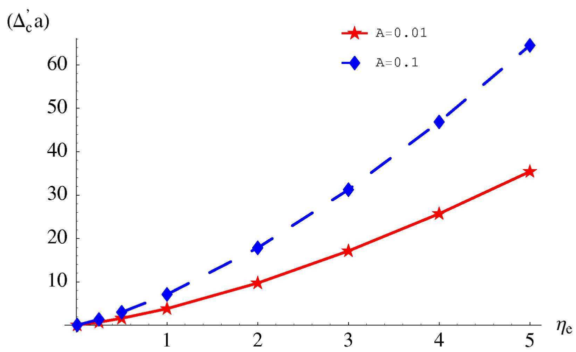

Figure 1: Critical Δ' versus η_e

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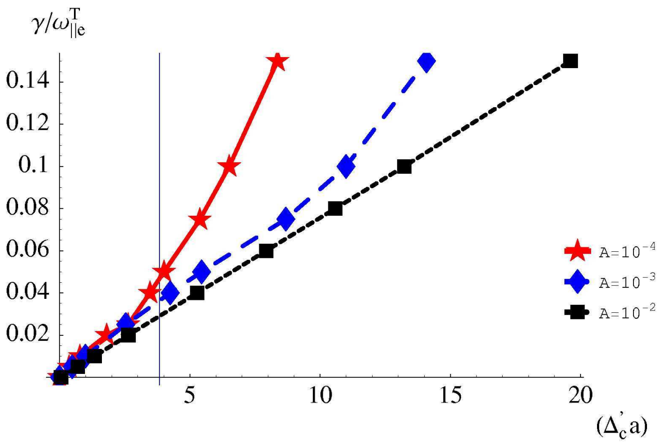

Figure 2: Effect of a longitudinal thermal conductivity depression.

We consider a magnetic field represented by $\underline{B} \approx B_0 \underline{e}_z + (B_y' x)\underline{e}_y$. The local depression in the parallel thermal conductivity is taken to be

$$D_{\|e} = D_{\|e}\left(\left(x/\delta_{\mathrm{L}}^{\mathrm{T}}\right)^2/\left(1+\left(x/\delta_{\mathrm{L}}^{\mathrm{T}}\right)^2\right)\right),$$

where $\delta_{\mathrm{L}}^{\mathrm{T}} \sim L_{\mathrm{S}}(\nu_{\mathrm{e}}/\Omega_{\mathrm{ce}})^{1/2}$ is the characteristic width of the reconnection layer, ν_e is the collision frequency and $L_{\mathrm{S}}^{-1} = B_y'/B_0$. As can be seen in Fig. 2 such a depression has a destabilizing effect on the mode [5], reducing the value of Δ_{crit} to approximately zero. In this figure the growth rate normalized to the mode characteristic frequency $\omega^{\mathrm{T}}_{\|\mathrm{e}} = \omega_{*\mathrm{e}}(1+1.71\eta_{\mathrm{e}})$, with $\omega_{*\mathrm{e}} = (ck_y T'_{\mathrm{e}\|})/(eB)$, is represented as a function of Δ', and the vertical line represents the critical Δ' without parallel thermal conductivity depression. The other characteristics of the mode, such as the width of the reconnection layer, remain close to those obtained without the depression.

We also note that a background spectrum of microscopic modes may cause local flattening of the electron temperature profile. To assess the effects of this, we model the flattening as a local well in η_e around the magnetic surface where the drift-tearing mode is developing. Not surprisingly, this leads to the destabilization of the drift-tearing mode. In particular, for $\eta_e = \eta_{e0}\left[1-\exp\left(-x^2/\sigma^2\right)\right]$, the critical value of Δ' is reduced to zero if σ is taken to be of the order of $\delta_{\mathrm{L}}^{\mathrm{T}}$.

Background of "micro-reconnecting" modes

To reconcile the experimental observations with the theory, we propose that the presence of a spectrum of pre-existing electromagnetic microinstabilities driven by the electron temperature gradient and producing microscopic reconnection allows for the excitation of the reconnecting mode.

To be specific we consider electromagnetic modes in the fluid ($k_\| v_{te}/\omega \ll 1$), long-wavelength ($k_\perp \rho_{te} \ll 1$) limit, where ρ_{te} is the electron gyroradius and $v_{te} = \sqrt{T_e/m_e}$ is the thermal velocity. Furthermore, we consider for simplicity the density gradient scale length L_n to be long compared to the temperature gradient scale length L_{Te} such that ω_{*e}/ω may be neglected compared to ω_{*Te}/ω where $\omega_{*Te} = k_y \rho_{te} v_{te}/L_{Te}$ and $\omega_{*e} = k_y \rho_{te} v_{te}/L_n$. We refer to a slab geometry with $\vec{B}(x) = B_0[\hat{z} + (x/L_s)\hat{y}]$, where L_s is the magnetic shear scale-length and the x coordinate models the "radial" coordinate of toroidal geometries. We also take the ion response to be adiabatic so that $\delta F_i \simeq -(e/T_i)F_M\Phi^1$ and neglect the parallel component of the perturbed magnetic field. Under these assumptions the quasi-neutrality condition together with the mass and momentum conservation equations give an algebraic relation between $\hat{\Phi}^1$ and $\hat{A}^1_\parallel$, which when used in the parallel component of Ampére's law yields the following single second-order ordinary differential equation for $A^1_\parallel$, where $\hat{A}_\parallel \simeq A^1_\parallel \exp(-i\omega t + ik_y y)$.

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2} - k_y^2\right] A^1_\parallel + \frac{1}{d_e^2}\left[\frac{\omega^2\omega_{*Te}}{\omega^3 + \omega_{*Te}k_\parallel^2 c_{se}^2}\right] A^1_\parallel = 0 \tag{1}$$

Here $d_e = c/\omega_{pe}$ is the electron skin depth and $c_{se} = (T_e/m_i)^{1/2}$ is the "electron sound" velocity.

We are interested in modes that are strongly localized around $\vec{k}\cdot\vec{B} = 0$ surfaces. Expanding the magnetic field around such a surface x_0 we have $k_\parallel = \vec{k}\cdot\vec{b} \simeq k_y(x - x_0)/L_s$. To analyze Eq. (1) we first note that in the limit where $x \gg 1$ the asymptotic solutions behave like $A^1_\parallel \propto \exp(\pm k_y x)$. We consider $k_y\delta \sim 1$, where δ is the characteristic scale of the mode. This leads to $\delta \sim \varepsilon^{3/4} d_e^{3/2}(L_{Te}/L_s)^{1/2}(1/\rho_e)^{1/2}$ under the condition $\varepsilon/\beta_e < L_s^2/L_{Te}^2$. Then with the following normalizations

$$\bar{x} \equiv k_y(x - x_0), \qquad \bar{\omega} \equiv \left(\omega_{*Te}\frac{c_{se}^2}{L_s^2}\right)^{-1/3}\omega, \qquad \varepsilon \equiv \frac{1}{k_y^2 d_e^2}\left(\omega_{*Te}\frac{L_s}{c_{se}}\right)^{2/3} \tag{2}$$

Eq. (1) may be rewritten as

$$\frac{\mathrm{d}^2 A^1_\parallel}{\mathrm{d}\bar{x}^2} + \left[-1 + \frac{\varepsilon\bar{\omega}^2}{\bar{\omega}^3 + \bar{x}^2}\right] A^1_\parallel = 0 \tag{3}$$

where $\bar{\omega}$ is the "proper value" of the problem, and the boundary conditions are that $A^1_\parallel$ vanishes at infinity and $\mathrm{d}A^1_\parallel/\mathrm{d}\bar{x} = 0$ at $\bar{x} = 0$. When $\varepsilon \ll 1$ the solution exhibits a transition layer, whose width is related to $\Delta\bar{x}_{tp} = 2\varepsilon\bar{\omega}\sqrt{1-\bar{\omega}}$. Forming a quadratic form from Eq. (3), we obtain a cubic equation in $\bar{\omega}$, and using trial functions we find that for $\varepsilon \leq 0.8$ complex conjugate solutions exist with $\bar{\omega} \simeq 1.1 \pm 0.7j$. A direct and detailed numerical solution of Eq. (3) confirms these simple estimates.

Now we argue that a quasi-mode consisting of the superposition of adjacent elementary modes can form and have a "finite" extent of the order of $|\mathrm{dB_y/dy}|/|\mathrm{d}^2 B_y/\mathrm{d}x^2|$. This quasi-mode is localized along z and referring to the electron temperature it will be represented by

$$\widehat{\widehat{T}}_e \simeq \int_{-\infty}^{\infty} T_e^1(k_y(x - x_0))W(x_0)e^{ik_y\left(y - \frac{x_0}{L_s}z\right)}\mathrm{d}x_0 \simeq \hat{T}_e^1\left(\frac{z}{L_s}\right)W(x)e^{ik_y\left(y - \frac{x}{L_s}z\right)} \tag{4}$$

where $W(x)$ is a "weight" function localized over a scale distance smaller than $a = |B_y'/B_y''|$ with $W(x = x_0) = 1$. Note that $T_e^1(x - x_0)$ is the perturbed temperature due to the individual mode computed from $A_{\parallel}^1$ and the algebraic relation to Φ^1, and $\hat{T}_e^1(z/L_s)$ is its Fourier transform.

We note that each elementary mode creates a string of magnetic islands whose width is of the order of

$$\delta_I \sim \left|\frac{B_x^1}{(\mathrm{d}B_y/\mathrm{d}x)k_y}\right|^{1/2} \sim \left|\frac{B_x^1}{a(\mathrm{d}B_y/\mathrm{d}x)k_y}a\delta\right|^{1/2} \tag{5}$$

where a represents the plasma radius and $k_y\delta \sim 1$. For the validity of the linearized approximation we require that $\delta_I < \delta$ and this corresponds to $|B_{x0}^1/(a\mathrm{d}B_y/\mathrm{d}x)| < \delta/a$ that is quite restrictive. Therefore the formation of a quasi-mode will involve that of a sequence of island strings that are not coplanar as the magnetic field "rotates" as a function of x. Each string of islands is centered on the same surface at which $A_{\parallel}^1$ for each elementary mode is peaked.

Finally, we observe that nice time dependent parallel temperature gradients are maintained by the considered quasi-modes. We may consider this to correspond to a state of low thermal conductivity as required for the onset of drift-tearing modes.

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