

Figure 3: Ripple well parameter dependences of the canonical angular momentum drift.

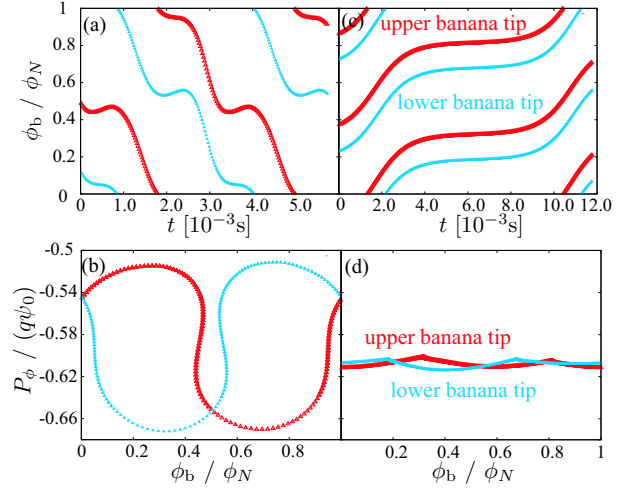


Figure 4: Time evolutions of the toroidal angle of banana tips, (a), (c) and corresponding poicare maps in the  $\phi_b - P_\phi$  phase space, (b), (d).

an abrupt change at every banana tip point which is known as ripple-enhanced banana drift. We evaluated the drift  $\Delta\langle P_\phi \rangle$  by comparing  $\langle P_\phi \rangle$ 's just before and after a banana tip. The calculated banana drift is shown in Fig. 2 as a function of  $\phi_b$ , the toroidal angle of the banana tip where toroidal field coils are placed at  $\phi/\phi_N=0$  and 1,  $\phi_N$  the toroidal-angle difference between adjacent TF coils. Figure 2 shows that the drift can be approximately given by

$$\Delta P_\phi = \Delta P_\phi^* \sin(N\phi_b - \pi/4), \quad (1)$$

where  $N$  is the number of toroidal field coils and  $\Delta P_\phi^*$  is the drift amplitude at the banana tip due to the ripple and is given by an integral over the unperturbed trapped orbit, most of the contribution coming near the banana tip [1]. The amplitude of the canonical angular momentum drift is mainly given by the ripple-well parameter  $\alpha \equiv |\partial\bar{B}/\partial l|/|\partial\tilde{B}/\partial l|$ . The difference between the drifts of canonical angular momentum from EOM and GCO shown in Fig. 2 can be described by a small difference between the ripple-well parameters at banana tips from EOM and GCO. As shown in Fig. 3, if the ripple well parameter at the banana tip is the same, they show almost the same drifts.

### Ripple Resonance

Since the canonical angular momentum drift has the  $\phi_b$ -dependence as shown in Fig. 2, banana particles resonate with toroidal ripples. This ripple resonance condition is expressed as

$$\Delta\phi_b \equiv \phi_b^{i+1} - \phi_b^i = \frac{2k\pi}{N} \quad (k = \pm 1, \pm 2, \pm 3, \dots), \quad (2)$$

ripple anti resonance condition is expressed as

$$\Delta\phi_b = \frac{(2k+1)\pi}{N} \quad (k = \pm 1, \pm 2, \pm 3, \dots), \quad (3)$$

where  $\phi_b^i$  denotes the toroidal angle of the  $i$ th banana tip. If a banana particle satisfies the ripple resonance condition, it bounces at the same toroidal angle at each banana tip. Then it makes a large poincare surface in the  $\phi_b - P_\phi$  phase space (Fig. 4(b)). In contrast, a banana particle in the ripple anti resonance condition cancels the drift between successive banana tips and the poincare surface is narrow (Fig. 4(d)).

The toroidal angle difference between successive banana tips depends on the safety factor, the energy of the particle, and so on. We investigated the energy dependence of the ripple resonance and poincare surfaces.

The results are shown in Fig. 5. Note that there is a substantial difference in resonance energies between solutions to EOM and GCO equation due to their difference in toroidal precessions as shown in Fig. 1. It is also noted that two peaks usually appear on both side of a resonance point (M shaped structure) and a small peak appears at ripple anti-resonance point.

In comparison with GCO equation, the equation of motion needs one more free parameter, the Larmor phase. We executed the same calculation by changing the initial Larmor phase  $\omega = 0, 90$  and  $180$  degree, where Larmor phase,  $\omega_0 = 0$  means a starting point outer side of the torus. Results are shown in

Fig. 6. Although the toroidal angle of a banana tip depends on the initial Larmor phase, the resonance point is about the same in this magnetic field. However their behavior is different.

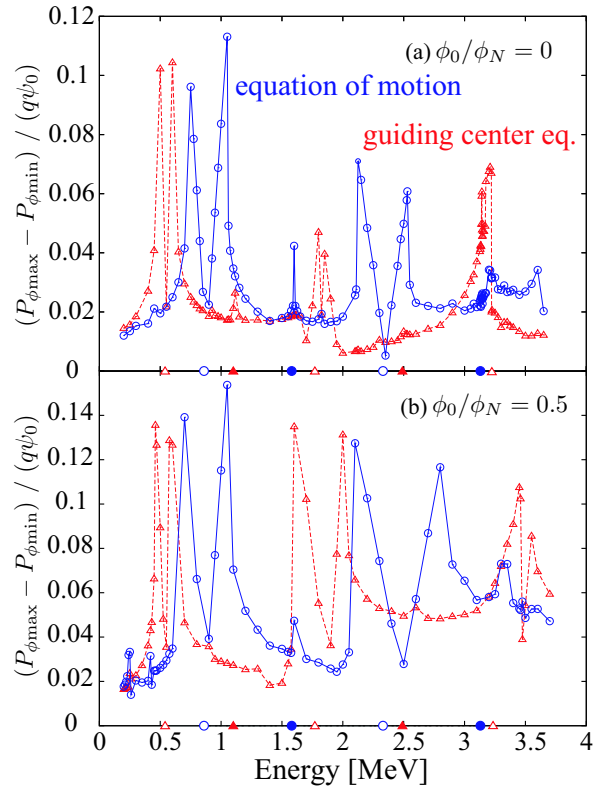


Figure 5: Size of poincare surface calculated by the equation of motion (blue-line) and by the guiding center equation (red-line) against particle energy for  $\phi_0/\phi_N = 0$  and  $0.5$ .

## Ripple Diffusion

We calculated the diffusion coefficients in a parabolic toroidal current, the same magnetic field as Fig. 5. Coulomb collisions were simulated by the Monte-Carlo method [2]. Particles started from  $R = R_0 + 0.75a$ ,  $Z = 0$ , and the initial toroidal angle was randomly given. Diffusion coefficients were evaluated from 1050 particles.

Results are shown in Fig. 7. The diffusion coefficients have peaks near the resonance energies which are shown in Fig. 5. The peaks of solutions to EOM are broader than those of GCO equation since the equation of motion has an additional free parameter to be averaged, the Larmor phase.

## Conclusions

There is a substantial difference between the toroidal angle precession of a banana particle calculated by the equation of motion and that by the guiding center equations. Consequently, ripple-resonance conditions by EOM are also different from those by GCO equation.

The ripple-resonance peaks of diffusion coefficient calculated by EOM are broader than those by GCO equation since the equation of motion has an additional free parameter to be averaged, the Larmor phase.

## References

- [1] R. J. Goldston, R. B. White and A. H. Boozer, Phys. Rev. Lett **47**, 647 (1981)
- [2] K. Tani, T. Takizuka, M. Azumi and H. Kishimoto, J. Phys. Soc. Jpn **50**, 1726 (1981)

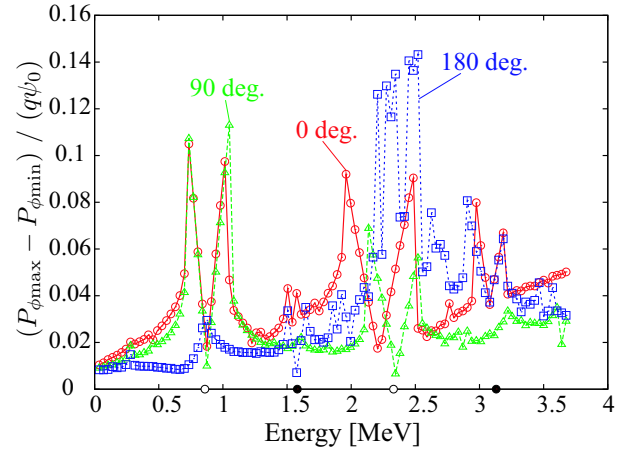


Figure 6: Larmor phase dependences of the size of a Poincaré surface.

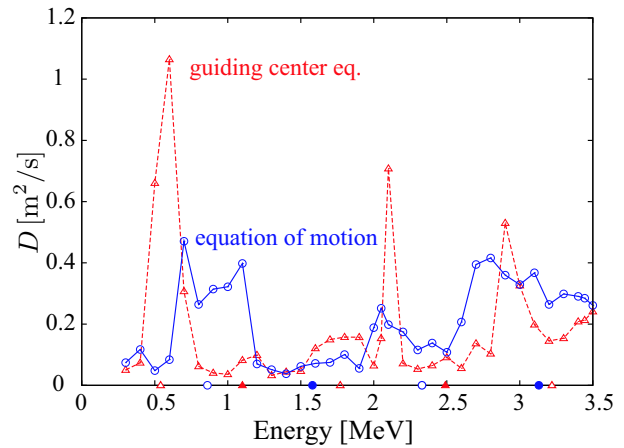


Figure 7: Comparison of energy dependences of diffusion coefficients.