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Potential formation in a bounded two-electron temperature plasma system – numerical solutions and PIC simulation

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Introduction

In a recent paper [2] we have analyzed the formation of the plasma potential in a bounded plasma system that contains singly charged positive ions and three groups of electrons by a fully kinetic one-dimensional model. The system is bounded by a planar plasma source on one side and by a floating collector that emits electrons on the other side.

In this paper we analyze the numerical solutions of the Poisson equation. These solutions are compared to the results of a compter simulation.

The model and assumptions

An infinitely large planar electrode (collector) has its surface perpendicular to the x axis and is located at $x = 0$. This electrode absorbs all the particles that hit it. On the other hand it may also emit electrons. This electron emission can be thermal or secondary triggered by the impact of incoming electrons and/or ions. The details of the emission mechanism are not essential for the model.

An infinitely large planar plasma source has also its surface perpendicular to the x axis. This source is located at a certain distance $x = L$ from the collector. The distance L is not crucial for the results of the model and may also be considered as infinitely large, $L \to \infty$. This source injects 3 groups of charged particles into the system: singly charged positive ions (index i), the cool electrons (index 1) and the hot electrons (index 2). The emitted electrons from the collector have index 3. The particles i, 1 and 2 are injected from the source with half-maxwellian velocity distribution functions with temperatures T_i, T_1 and T_2. Also the emitted electrons have a half-maxwellian velocity distribution function at the collector with the temperature T_3. We assume that $T_2 > T_1$ and $T_1 \gg T_3$. The ion temperature T_i can in principle be arbitrary but is usually taken smaller or in the best case of the same order of magnitude as T_1.

The collector is biased to a certain (negative!) potential Φ_C. The potential at the source is set to zero and also the electric field at the source must be zero: $\Phi(x = L) = 0$, $d\Phi/dx(x = L) = 0$.

We assume that the plasma is collisionless and the energy of the particles is conserved. An ion that is born at the source with zero velocity has at the distance x from the collector the velocity $v_{mi} = \sqrt{(-2e_0\Phi(x))/m_i}$. On the other hand an electron that is at rest at the collector surface, will have at the distance x from the collector the velocity: $v_{me} = -\sqrt{(2e_0(\Phi(x) - \Phi_C))/m_e}$.

The following dimensionless variables are introduced:

$$z = \frac{x}{\lambda_D},\ \lambda_D = \sqrt{\frac{\varepsilon_0 k T_1}{n_1 e_0^2}},\ \mu = \frac{m_e}{m_i},\ \tau = \frac{T_i}{T_1},\ \Theta = \frac{T_2}{T_1},\ \sigma = \frac{T_3}{T_1},\ \Psi = \frac{e_0\Phi(x)}{kT_1}, \tag{1}$$

$$\Psi_C = \frac{e_0\Phi(x=0)}{kT_1} = \frac{e_0\Phi_C}{kT_1},\ \alpha = \frac{n_i}{n_1},\ \beta = \frac{n_2}{n_1},\ \varepsilon = \frac{n_3}{n_1},\ v_0 = \sqrt{\frac{2kT_1}{m_e}},\ u = \frac{v}{v_0},$$

where n_i, n_1 and n_2 are the respective particle densities at the source and n_3 is the density of the emitted electrons at the collector. With these variables the distribution functions of all 4 particle species are written in the following way:

$$F_i(u,\Psi) = \frac{\alpha}{\sqrt{\pi\tau\mu}}\exp\left(-\frac{\Psi}{\tau}\right)\exp\left(-\frac{u^2}{\mu\tau}\right)H\left(u - \sqrt{-\mu\Psi}\right), \tag{2}$$

$$F_1(u,\Psi) = \frac{1}{\sqrt{\pi}}\exp(\Psi)\exp\left(-u^2\right)H\left(u + \sqrt{\Psi - \Psi_C}\right), \tag{3}$$

$$F_2(u,\Psi) = \frac{\beta}{\sqrt{\pi\Theta}}\exp\left(\frac{\Psi}{\Theta}\right)\exp\left(-\frac{u^2}{\Theta}\right)H\left(u + \sqrt{\Psi - \Psi_C}\right), \tag{4}$$

$$F_3(u,\Psi) = \frac{\varepsilon}{\sqrt{\pi\sigma}}\exp\left(\frac{\Psi - \Psi_C}{\sigma}\right)\exp\left(-\frac{u^2}{\sigma}\right)H\left(-\sqrt{\Psi - \Psi_C} - u\right), \tag{5}$$

where $H(u)$ is the Heaviside unit step function. When the distribution functions are multiplied by different powers of velocity u^n and integrated over velocity moments of the distribution functions can be obtained as functions of the potential Ψ. The zero moments are the densities and the first moments are the fluxes. They are obtained in the following way:

$$N_j(\Psi) = \int_{-\infty}^{\infty} F_j(u,\Psi)\,du,\ \ J_j(\Psi) = \int_{-\infty}^{\infty} uF_j(u,\Psi)\,du, \tag{6}$$

where $j = i$, 1, 2 and 3.

The potential profile $\Psi(z)$ in front of the collector is determined by the Poisson equation. The normalized densities are obtained from (6) and the Poisson equation in one dimension then reads:

$$\begin{aligned}\frac{d^2\Psi}{dz^2} = &-\alpha\exp\left(-\frac{\Psi(z)}{\tau}\right)\mathrm{Erfc}\left(\sqrt{-\frac{\Psi(z)}{\tau}}\right) + \exp(\Psi(z))\left[1 + \mathrm{Erf}\left(\sqrt{\Psi(z) - \Psi_C}\right)\right] + \\ &+\beta\exp\left(\frac{\Psi(z)}{\Theta}\right)\left[1 + \mathrm{Erf}\left(\sqrt{\frac{\Psi(z) - \Psi_C}{\Theta}}\right)\right] + \varepsilon\exp\left(\frac{\Psi(z) - \Psi_C}{\sigma}\right)\mathrm{Erfc}\left(\sqrt{\frac{\Psi(z) - \Psi_C}{\sigma}}\right),\end{aligned} \tag{7}$$

where

$$\mathrm{Erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x \exp(-t^2)dt, \ \ \mathrm{Erfc}(x) = \frac{2}{\sqrt{\pi}} \int_x^\infty \exp(-t^2)dt. \tag{8}$$

At a certain distance $z = z_0$ from the collector the plasma is neutral and the potential at this point has the value $\Psi(z = z_0) = \Psi_P$. So the Poisson equation (7) can be written in the following form:

$$N_i(\Psi_P) - N_1(\Psi_P) - N_2(\Psi_P) - N_3(\Psi_P) = 0. \tag{9}$$

At $z = z_0$ the second derivative of the potential is zero. So the potential has an inflection point at $z = z_0$. The equation (7) is multiplied by $d\Psi/dz$, so that the space differentials dz cancel out and then equation (7) can be integrated once over Ψ from $\Psi = 0$ (at the source) to Ψ_P at the inflection point:

$$\int_0^{\Psi_P} \left(N_i(\Psi) - N_1(\Psi) - N_2(\Psi) - N_3(\Psi)\right) d\Psi = 0. \tag{10}$$

The equation (10) is called the zero electric field condition at the inflection point. If the emission of electrons from the collector is increased, it eventually becomes so high that negative space charge accumulates at the collector surface and the electric field there becomes zero. This is called the critical electron emission. This fact can be used to derive another expression relating Ψ_P and Ψ_C. This expression is called the zero electric field condition at the collector. It is derived in the same way as the zero electric field at the inflection point, only the integration limits are changed. They go from Ψ_P (at $z = z_0$) to Ψ_C (at $z = 0$). The following expression is obtained:

$$\int_{\Psi_P}^{\Psi_C} \left(N_i(\Psi) - N_1(\Psi) - N_2(\Psi) - N_3(\Psi)\right) d\Psi = 0. \tag{11}$$

When the collector is floating, the total electric current to the collector is zero:

$$J_t = \frac{\alpha}{2}\sqrt{\frac{\mu\tau}{\pi}} + \frac{\varepsilon}{2}\sqrt{\frac{\sigma}{\pi}} - \frac{1}{2\sqrt{\pi}}\exp(\Psi_C) - \frac{\beta}{2}\sqrt{\frac{\Theta}{\pi}}\exp\left(\frac{\Psi_C}{\Theta}\right) = 0. \tag{12}$$

The fluxes are obtained from (6). The equations (9) - the neutrality condition, (10) - the zero field condition at the inflection point and (11) - the zero field condition at the collector are the basic equations of the model. Parameters like hot to cool electron density and temperature ratio, ion mass and ion temperature and temperature of emitted electrons are usually determined by the plasma production, so μ, τ, β, Θ and σ can be treated as given parameters. The neutralization parameter α, the critical emission coefficient ε, the plasma potential Ψ_P and the floating potential Ψ_C of the collector can then be found by solving the system of equations (9), (10), (11) and (12). If on the other hand also the emission coefficient ε and the collector bias Ψ_C are given only the equations (9) and (10) solved for α and Ψ_P. The current to the collector is then found from (12).

Results

At the end we present just one example (Fig. 1) of a comparison between the results obtained from computer simulation using the XPDP1 code [3] and numerical solution of equation (7). The parameters are the following: μ = 1/1836, τ = 0.1, β = 0.342, Θ = 20, σ = 0.01,

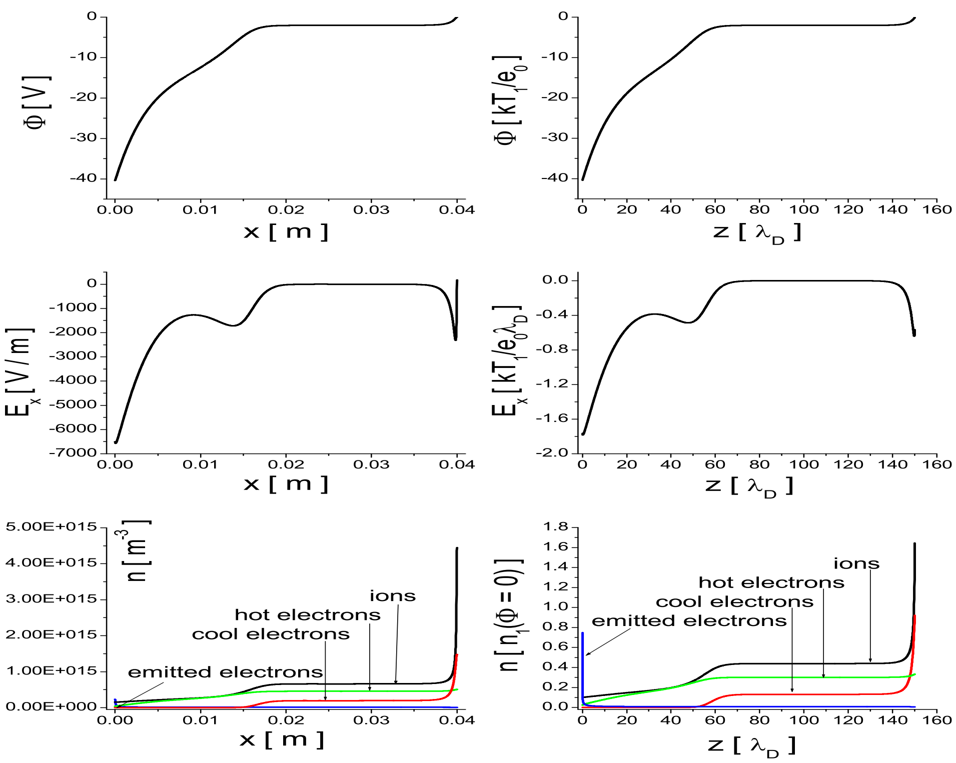

Figure 1: Plasma potential profile (top), electric field profile (middle) and density profile (bottom), obtained from computer simulation (left) and numerical solution (right).

Ψ_C = -40.43, Ψ_P = -2.03, α = 7.18327 and ε = 1.49575. A very good agreement between computer simulation and numerical solution can be observed.

References

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