Nonneutral Plasma Confinement in a Spindle Cusp

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INTRODUCTION

A magnetic quadrupole (cusp) is the simplest configuration belonging to Min-B fields. When cold anti-hydrogen atoms are synthesized in the central region of the cusp, they will be extracted as an ultra-slow spin-polarized beam¹). In order to mix positrons and anti-protons for the synthesis, it is first requisite to examine the confinement of nonneutral plasmas in this strongly non-uniform magnetic field.

This paper describes the presence of the rigid-rotor equilibrium of a nonneutral cold plasma with a finite length in the spindle cusp region. Preliminary experiment on electron confinement in the spindle cusp has been performed using a superconducting magnet. Obtained results are briefly summarized.

RIGID ROTOR EQUILIBRIUM IN SPINDLE CUSP

The cusp field is described in the cylindrical coordinates \((r, \theta, z)\) in terms of vector potential \(A_\theta = \{B_0/(2L)\} r z\), where \(L\) is measure of length and \(B_0\) is the magnetic field strength at \(r=0\) and \(z=L\). The magnetic field components and the flux are respectively given by

\[
\begin{align*}
(B_r, B_\theta, B_z) &= (-B_0 \frac{r}{2L}, 0, B_0 \frac{r}{L}), \quad \text{and} \quad \Phi = 2\pi A_\theta = \frac{\pi B_0}{L} r^2 z
\end{align*}
\]

We shall consider a cold nonneutral plasma of single-species, which is confined in the spindle cusp region \(z>0\) in a thermal equilibrium state. The plasma rotates about the axis with a constant frequency \(\omega\). We discuss the case that the Debye length is small compared with the dimensions of the plasma and image charges induced on surrounding walls can also be neglected. Then, the effective potential for the plasma particles of the mass \(m\) and the charge \(q\) in the rotating frame becomes²):

\[
\phi_R(r, z) = \phi_V(r, z) + \frac{\omega}{2\pi} \Phi(r, z) - \frac{1}{2q} (m\omega^2 r^2),
\]

where \(\phi_V\) is the externally applied electric potential. The plasma space charge potential \(\phi_p\) is related to \(\phi_R\) in the equilibrium as

\[
\phi_R(r, z) + \phi_p(r, z) \sim \text{const.}
\]

This equation leads the plasma frequency depending on \(z\)

\[
\omega_p^2 = 2\omega \{ \Omega_0 (z/L) - \omega \},
\]
where $\Omega_0 = qB_0/m$ is the cyclotron frequency at $r=0$ and $z=L$. The Brillouin density limit is determined at the edge of the plasma boundary nearest to the plane of symmetry, $z_{b^-}$, as

$$n_{BL} = (\varepsilon_0 B_0^2 / 2m) (z_{b^-} / L)^2.$$

To settle a plasma with a finite-length in the equilibrium, $\phi_R$ needs to form a well. Here is adopted the following external potential

$$\phi_v(r,z) = \phi_0 \{ [(z - L)^2 - r^2] + L\beta \ (z - \kappa L) \} / L^2,$$

with dimensionless parameters $\beta$ and $\kappa$. The first term is a harmonic potential and the second an added term for adjusting the potential distribution. Using dimensionless quantities $\rho = r/L, \varsigma = z/L, \gamma = \omega / \Omega_0, \varphi_v = \phi_v / \phi_0, \varphi_p = \phi_p / \phi_0, \varphi_R = \phi_R / \phi_0$,
eq(2) is rewritten as

$$\varphi_R(\rho,\varsigma) = \{(\varsigma - \gamma) \Gamma - 1\} \rho^2 + 2(\varsigma - 1)^2 + \beta \ (\varsigma - \kappa),$$

where

$$\Gamma = \frac{\omega B_0 L^2}{2\phi_0} = qn_0 L^2 \epsilon_0 \phi_0$$
and $n_0$ is the density at $\varsigma = 1$.

This parameter $\Gamma$ plays an essential role in determining the characteristics of plasmas.

**Shape of confined plasma**

Since the equipotential surfaces of $\varphi_R(\rho,\varsigma)$ are not spheroidal but ovoid, the plasma boundary is also shaped like an ovoid surface. In order to determine the boundary, the dimensionless form of eq.(3) : $\varphi_R(\rho,\varsigma) + \varphi_p(\rho,\varsigma) = 0$ is numerically solved using a trial

![Fig.1 Equipotential surfaces for $\Gamma=8.0$, $\beta=0.11$ and $z_{b^-}/L=0.6$.](image)

(a) External field $\varphi_V(\rho,\varsigma)$, (b) Plasma potential $\varphi_p(\rho,\varsigma)$,
(c) Sum $\varphi_R(\rho,\varsigma) + \varphi_p(\rho,\varsigma)$, (d) Potential in the experimental frame $\varphi_V + \varphi_p$. 

function that expresses a boundary with several arguments. Here, \( \gamma = 0 \) is assumed because the magnetic field is high enough in usual cases. Figures 1(a,b,c,d) show the obtained results for the case that \( \Gamma = 8.0, \beta = 0.11 \) and \( z_b/L = 0.6 \). The plasma boundary is drawn by a bold solid line and the space in the boundary is filled up with dots. The external potential in (a) is a hyperbolid. The sum of \( \phi_R \) and \( \phi_p \) in (c) becomes constant inside the plasma. Equipotential surfaces in the experimental frame in (d) coincide with the field lines, so that no electric field is present along the lines as we expected. Generally, the plasma radius decreases as \( \Gamma \) becomes larger because of the increase in the rotation frequency. This examination proves that the rigid-rotor equilibrium can be formed in the spindle cusp region.

**EXPERIMENT**

The experiment was performed using a superconducting quadrupole magnet, of which the coils are cooled down to 5 K with a cryogenic refrigerator. The maximum magnetic field on the axis is 3.5 T 15 cm away from the plane of symmetry and its maximum field gradient is 38 T/m. Inside the warm bore of the magnet is installed an aluminium vacuum tube that houses electrodes of the trap, a tiny electron gun and a Faraday cup, as depicted in Fig. 2. A set of twelve electrodes of 8 cm inner diameter produce the electric potential of the trap as expressed by eq.(5). A burst of pulsed electron beams are injected into the trap. Synchronously to the pulsed beams, the potential on the injection side is shallowed to allow the entering of the beam electrons into the trap. Electrons are thus stacked and then confined in the trap. The trapped electrons are dumped to the Faraday cup, which is radially segmented to eight parts for measurement of the radial line-density distribution of the trapped electrons. Also, the total trapped electron number, \( N \), is obtained by taking the whole sum of signals from the segments. The system is evacuated down to the vacuum pressure of \( 2 \times 10^{-7} \) Pa.

![Fig. 2 Cross-sectional view of the setup inside the vacuum tube.](image)

**Evolution of confined electron number**

Figure 3 shows the time dependence of \( N \) for the case that the magnetic field at \( r = 4 \) cm on the plane of symmetry was \( B_c = 0.77 \) T and the well depth of the electric potential was 42 V. The initially stacked electron number of \( 1.1 \times 10^8 \) was kept nearly constant until the time \( t_c \approx 800 \) s and then began to decay. Such a flatness of \( N \) with time means that the plasma was...
confined in this duration without touching the electrodes. However, the plasma periphery was slowly expanding across the magnetic field. Once the periphery contacted the electrodes, particle losses arose. This slow expansion was inferred from the change in the radial line-density distribution.

**Radial compression with rotating wall and cyclotron radiation cooling**

It was experimentally assured that rotating electric field, i.e. rotating wall, of the azimuthal mode number \( m=1 \) effectively compressed the plasma in the radial direction. Figure 4 shows the increase of the ratio of the electrons collected on the central segment, \( N_c \), to the total electron number, \( N_t \) at the frequency of \( f=9.15 \) MHz. 80% of the plasma was compressed in the radius of 1.4 mm. The frequency range, in which the compression effectively worked, was wide as \( 6\sim9.3 \) MHz in this case.

The cooling rate of the plasma due to cyclotron radiation was also measured. This e-folding time was about 1.5 s at \( <B> \sim 1 \) T. This rate is nearly the same as the theoretical one.

![Figure 3](image3.png)  
**Fig. 3** Time dependence of \( N \) after injection of electrons.

![Figure 4](image4.png)  
**Fig. 4** Increase of the ratio \( N_c / N_t \) during the compression. Here, \( f=9.15 \) MHz, \( B_e=0.34 \) T and \( N_t=6\times10^7 \).

**SUMMARY**

It is proved that thermal equilibrium of a cold nonneutral plasma can be formed in the spindle cusp region by applying the axially shifted quasi-harmonic potential. The plasma shape is ovoid with a bulge on the lower magnetic field side, and its deviation from spheroid depends on the parameter \( \Gamma \). Experiments using the superconducting magnet showed that such a plasma is practically formed and stably confined for 800 s. Radial compression by rotating electric fields is applicable to this plasma and the cyclotron radiation cooling works well.

**References**