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A Fermi-like model for wave-particle interaction in plasmas

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The wave-particle interaction process in plasmas is of fundamental importance in many space and laboratory settings because, as shown by Landau [1], it is responsible of the dissipation of waves even in absence of collisions. The physical content of the linear interaction is conceptually simple: a particle traveling at velocity v near the wave phase velocity v_p, gains energy if its velocity is just below the wave phase velocity, and lose it just above. As $(\partial f/\partial v)_{v=v_p} < 0$ (f is the distribution function of particles) the wave exhibits exponential damping. In the nonlinear regime the damping should be prevented [2] due to the trapping of particles which oscillate in the potential well of the wave. In a heuristic way particles either gain and lose energy, thus generating a saturation of the damping on times of the order of the trapping time.

The nonlinear Landau damping has been investigated by numerical simulations of the Vlasov-Poisson system [3, 4, 5, 6]. Analytical results [7] show the existence of a critical initial amplitude of the wave that marks the transition between a scenario in which the wave is definitively damped to zero, and a scenario in which the Landau damping is stopped. The latter seems to be characterized by an oscillating behavior of the wave amplitude around an approximately constant value.

To examine the wave-particle interaction we developed a model [8] of the type of that used by Fermi to study the acceleration of cosmic rays. We simulate the interaction as the net result of a large number of collisions between an ensemble of particles and two infinitely massive, one-dimensional barriers with variable length, set at a fixed distance $L = 2$. A test particle, moving with velocity v is extracted from a Maxwellian distribution function linearized around v_p the wave phase velocity: $f(v) \approx f(v_p) + (v - v_p)(\partial f/\partial v)_{v=v_p}$. The whole system moves with speed v_p, so, in this frame of reference, each elastic collision of a particle with a barrier will reverse the direction of motion of the particle, i.e. $v' = -v$ (v' is the velocity after the collision), and the amplitude of the barriers, $A(t)$, will vary due an energy exchange $\Delta E = 4vv_p$ between the particle and the barrier itself. If we identify the energy of the wave with the squared amplitude $E_W \approx |A(t)|^2$, after each collision this will vary by $A(t') = [A(t)^2 + \sigma(t)\Delta E]^{1/2}$, where $\sigma(t) = vv_p/|vv_p|$.

A Monte Carlo simulation of the above model can be easily performed. Velocities are normalized to the thermal velocity of plasma, i.e. $u = v/v_{th}$ and $u_p = v_p/v_{th}$. Each particle is identified by the random initial position $x_j^{(0)}$ ($j = 1, 2, \ldots, N$, being N the number of particles), uniformly distributed in the range $[-L/2, L/2]$, and by a constant speed u_j, extracted from the normalized distribution function:

$$f_0(u) = \frac{1}{\sqrt{2\pi}} e^{-u_p^2/2}(1 - uu_p) \tag{1}$$

where $-\sqrt{A_0} \le u \le \sqrt{A_0}$, with $A_0 = A(0)$. The y-position of the particles is taken equal to their squared velocity, $y_j = u_j^2$. The dynamics of the j-th particle between two collisions is trivially described by the equation of motion of a point mass moving at constant speed u_j. When a particle arrives to the barrier two situation can arise: the altitude of the particle is either below or above the actual height of the barrier. In the first case the particle will reverse its direction of motion and the amplitude of the barrier will vary of the amount of energy gained or lost by the particle. In the second case the amplitude of the barrier remain unchanged, and the particle continues its flight. It can be retrapped later, because periodic conditions are imposed in the numerical domain.

The sequence of collisions modifies both the amplitude of the barrier and the distribution function of the particles in a characteristic way. Studying these modifications it is possible to follow the system to a state of complete equilibrium. The time evolution of the wave energy is reported in Fig. 1. In the picture we can distinguish three stages: i)a first stage in which we recover a linear damping regime; ii)an intermediate regime, where the wave oscillates regularly around a saturation value A_{sat}; iii)for longer times the oscillations of $A(t)$ become lower and irregular.

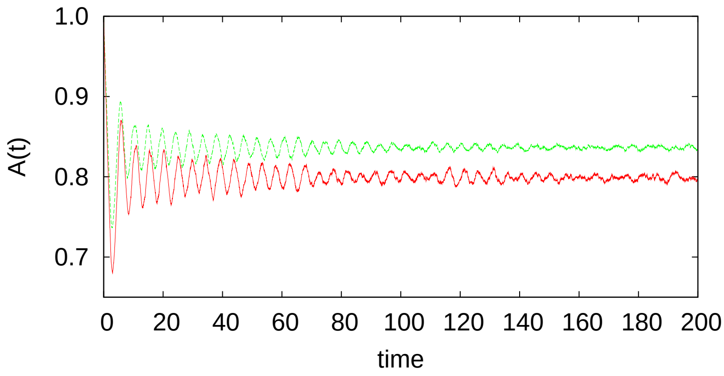

Figure 1: The time behavior of the amplitude $A(t)$, calculated with $N = 10^5$ particles, for $u_p = 1.0$ (green), $u_p = 1.5$ (red) and for $A_0 = 1$.

In the first regime, for low times, the trapping of particles is not still at work. In this case, the amplitude decays exponentially. An expression of the damping rate can be obtained by conjecturing that the time variation of energy is proportional to an energy exchanged during a single collision divided by a typical transit time between barriers:

$$\frac{dE_W}{dt} = \gamma_0 \int_{-\sqrt{A_0}}^{\sqrt{A_0}} 2u_p u|u| f_0(u) du \tag{2}$$

(γ_0 is a proportional constant). From (1) we immediately get $dE_W/dt = -\gamma E_W$, where the damping rate results to be

$$\gamma(u_p) = \frac{\gamma_0}{\sqrt{2\pi}} u_p^2 e^{-u_p^2/2} \tag{3}$$

The above expression is proportional to the usual Landau damping rate of the linear theory [1, 2]. In Fig. 2 we report the values of γ, as a function of u_p, calculated through a linear fit of the curves $A(t)$ vs. t, in the range $t \leq 2$. After this time the linear damping is rapidly stopped and high-amplitude oscillations are observed. This is the nonlinear strong trapping regime.The value of A_{sat} around which the amplitude oscillates depends on the parameters of the model. A threshold value $A_0^{(thr)}$ (function of u_p and A_0), below which the damping is stopped, is observed. For each u_p, this is due to a bifurcation between two states, namely $A_{sat} = 0$ for $A_0 \leq A_0^{(thr)}$ and $A_{sat} \neq 0$ for $A_0 > A_0^{(thr)}$. A scaling relation between the period of large oscillations τ_{tr} and the saturation level A_{sat} is also found. This has the form $\tau_{tr} \sim A_{sat}^{-\mu}$, with $\mu = 0.53 \pm 0.01$. The result is nicely consistent with that found in Ref. [5].

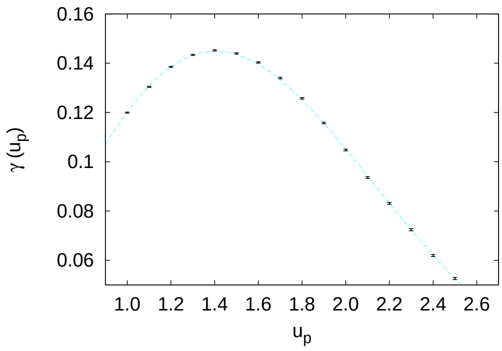

Figure 2: Black symbols: different values of the damping rate γ, calculated by numerical simulations with $N = 10^5$ and $A_0 = 1$. The curve represents the function (3) with $\gamma_0 \simeq 0.50$.

Beyond the oscillatory regime a new behavior is observed, namely oscillations of $A(t)$ become irregular and with a low amplitude, around the same value A_{sat}. This trend is the result of a complete phase mixing as predicted by O'Neil [2].

As a consequence of the complex dynamics, the distribution function of particles changes in time. In the phase space (x, u), as sketched in Fig. 3 (left panel), each trapped particle follows, at its own speed, a closed path (the rectangular whose corners are the four points $(\pm 1, \pm u_j)$). This generates a filamentation in the distribution of the particles, that becomes narrower, up to a time where a complete phase mixing is established. The filamentation in phase space corresponds to the presence of antisymmetric spikes in the distribution function $f(u)$ (Fig. 3, right panel). The spikes become finer and finer for long times, till a complete flattening of the distribution function. The filamentation is the dynamical process that is responsible for the presence of oscillations of the amplitude of the wave. In fact, in a given time interval, there is an imbalance between collisions in which the wave gains energy an those in which wave loses energy. When the stripes are large enough the synchronization of collisions on a given barrier is able to support the oscillation of the amplitude. When stripes become narrower the synchronization is felt by

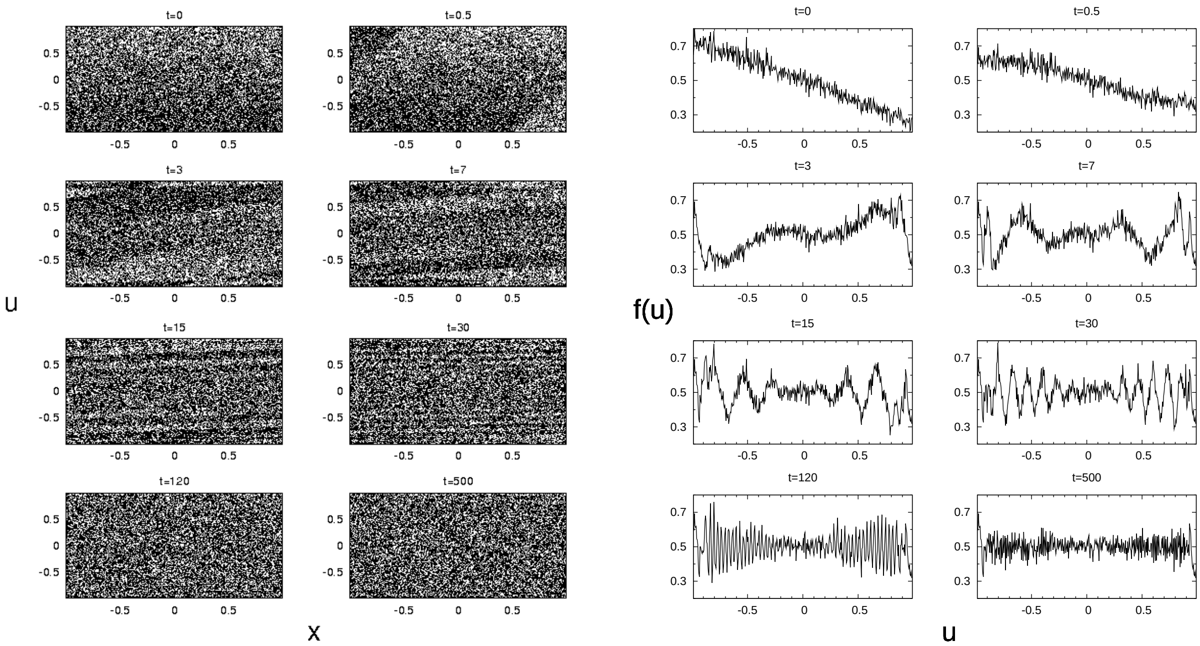

Figure 3: The phase-space (x,u) and the distribution function $f(u)$ at different times, for $A_0 = 1$, and $u_p = 1.0$.

a lower number of particles and oscillations become less regular, till the system reaches the equilibrium configuration.

References

[1] L. D. Landau, J. Phys. (Moscow) **10**, 25 (1946)

[2] T. O'Neil, Phys. Fluids **8**, 2255 (1965)

[3] G. Manfredi, Phys. Rev. Lett. **79**, 2815 (1997)

[4] L. Galeotti, and F. Califano, Phys. Rev. Lett. **95**, 015002 (2005)

[5] A.V. Ivanov, I.H. Cairns, and P.A. Robinson, Phys. Plasmas **11**, 4649 (2004)

[6] F. Valentini, *et al.* Phys. Rev. E **71**, 017402 (2005)

[7] C. Lancellotti and J. J. Dorning, Phys. Rev. Lett. **81**, 5137 (1998)

[8] R. De Marco, V.Carbone, P. Veltri, Phys. Rev. Lett. **96**, 125003 (2006).