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Analysis of horseshoe cyclotron maser instability

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When a beam of electrons with a thermal spread moves along converging magnetic field lines, the distribution function evolves into a horseshoe shape in ($p_{\|}$, $p_{\perp}$) space. In such a distribution there is a population inversion of particles in perpendicular momentum which results in the plasma being subject to a cyclotron maser instability. There is strong evidence that this type of instability is the source of auroral kilometric radiation[1]. An evident horseshoe-shape distribution function is seen in a data from a satellite passing through the acceleration region.

The theory of the instability[2] indicates that the phenomenon can be scaled to laboratory dimensions, with centimetre rather than kilometre wavelengths, and an experiment to do this has been constructed at the University of Strathclyde[3] (see the paper of K.Ronald and D.C. Speirs et al at this conference). Here we present an analysis of the instability in cylindrical geometry, with a circular beam partially filling a circular conducting waveguide. The plasma frequency is small compared to the cyclotron frequency in both the auroral zone and the experiment and we model this regime.

We consider an electron beam moving into a convergent magnetic field. The magnetic moment is an adiabatic invariant, i.e. $\frac{v_{\perp}^{2}}{B}$ is approximately constant so long as the field varies over a scale >> Larmor radius. Energy conservation implies that $v_{\perp}^{2}+v_{\|}^{2}=const$.

For a drifting Maxwellian initial distribution the laws of conservation of magnetic moment and particle energy result in the transformed distribution

$$f(v_{\|},v_{\perp})=A\,e^{-\frac{m}{2T}\left(\left(\sqrt{v_{\|}^{2}+(1-B_{0}/B)v_{\perp}^{2}}-v_{0}\right)^{2}+(B_{0}/B)\,v_{\perp}^{2}\right)} \qquad (1)$$

with a shape of a horseshoe in velocity space, as shown in Fig.1,

This distribution has a population inversion in the perpendicular direction and is subject to instability in the cyclotron range of frequencies. The electron cyclotron resonance

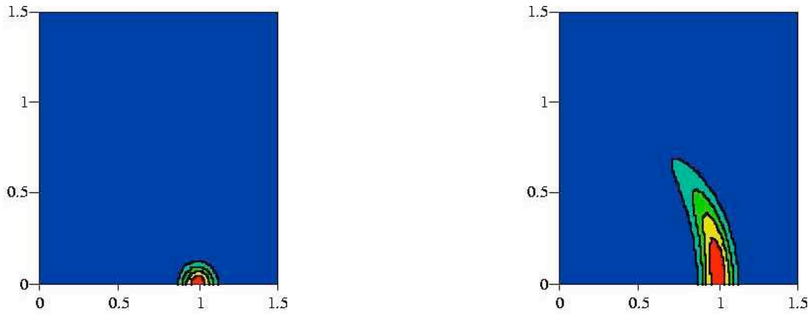

Fig1. Evolution of the horseshoe distribution. Velocities are normalised to the initial mean parallel velocity and the magnetic field increases by a factor 30.

condition at the fundamental frequency considered here is $\omega = \frac{\omega_{ce}}{\gamma} + k_{\|} v_{\|}$, where ω_{ce} is the electron cyclotron frequency, and γ is the usual relativistic factor. For $k_{\|} = 0$ this specifies a circle and if this circle lies around the inside of the horseshoe, then we may expect instability. We evaluate semi-analytically the integrals in the dielectric tensor

$$\hat{K}_{\perp} = \begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} = \left(1 - \frac{\omega_{pe}^{2}}{\omega^2}\right)\hat{I} - \frac{\omega_{pe}^{2}}{\omega^2} \int \left\{ \frac{\hat{T}}{k_z v_{\|} - \omega + \omega_{ce}\sqrt{1 - (v_{\|}^{2} + v_{\perp}^{2})/c^2}} \cdot \left(-\frac{n\,\omega_{ce}}{v_{\perp}} \frac{\partial f}{\partial v_{\perp}} + k_z \frac{\partial f}{\partial v_{\|}} \right) \frac{1}{n_0} \right\} d\vec{v},$$

$$\hat{T} = \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \qquad k_{i3} = k_{3j} = 0, \qquad k_{33} = \left(1 - \frac{\omega_{pe}^{2}}{\omega^2}\right) \tag{2}$$

for the horseshoe distribution function *f* described by (1).

To model the radiation, we consider an electron beam, located in the central part $0 \le r \le R_1$ inside the transverse cross section of a circular metallic tube with its radius equal to R_0 (Fig.2). Both central and annular beams are considered because the former is closer to natural phenomenon and the later is used in the experiment mentioned above.

Introducing a representation of the electromagnetic field in the plasma region as a superposition of TE and TM modes as is needed in an anisotropic medium, the following dispersion relations were obtained[4]:

$$\left(k_{11} - k_z^{2}\right)\left[k_{11} - k_z^{2} - \frac{\beta_{TE}^{2}}{R_1^{2}}\right] + k_{12}^{2} = 0$$

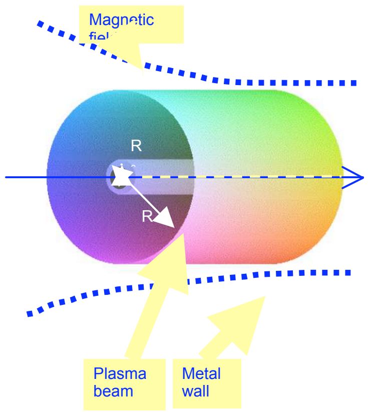

Fig.2. Geometry of the problem.

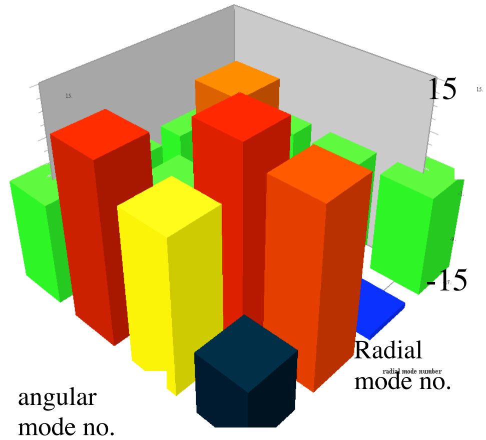

Fig.3. Spatial growth rate, m^{-1}, for various modes

$$\left[k_{11}\left(1-\frac{k_{33}R_1^{\,2}}{\beta_{TM}^{\,2}}\right)+\frac{k_z^{\,2}k_{33}R_1^{\,2}}{\beta_{TM}^{\,2}}\right]\left(k_{11}-k_z^{\,2}-\frac{\beta_{TM}^{\,2}}{R_1^{\,2}}\right)+k_{12}^{\,2}\left(1-\frac{k_{33}R_1^{\,2}}{\beta_{TM}^{\,2}}\right)=0\,. \qquad (3)$$

They were analyzed numerically together with two more equations obtained from boundary conditions, which are omitted here because of limitations on space.

Fixing the radiation frequency in (3), imaginary and real parts of k_z give us growth rate and propagation characteristics for the different modes. Typical results for the growth rate are shown in Fig.3 and confirm the expected high growth rate of a number of modes. The highest growth rate is for those modes propagating nearly perpendicular, near their cut off . The radiated field has a very small TM mode component in the field outside the beam, having almost TE structure. However, inside the plasma modes are strongly coupled (Fig.4).

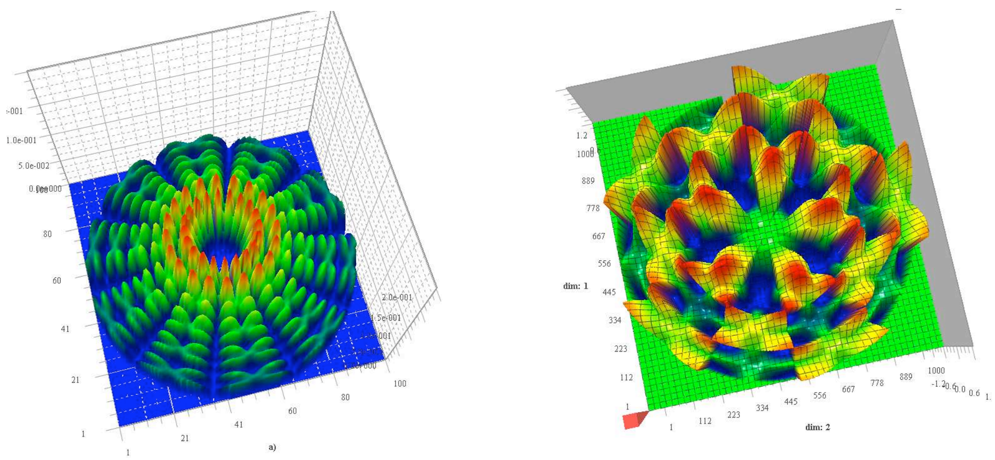

Fig.4. Coupled field distribution inside plasma region for annular and central beams respectively.

We model the non-linear distribution function behaviour by assuming quasilinear flattening mainly around the resonant circle for the frequency of radiated field (Fig.6). It is natural to assume such behaviour because modelling, the experiment and satellite observations all show narrow bandwidth radiation.

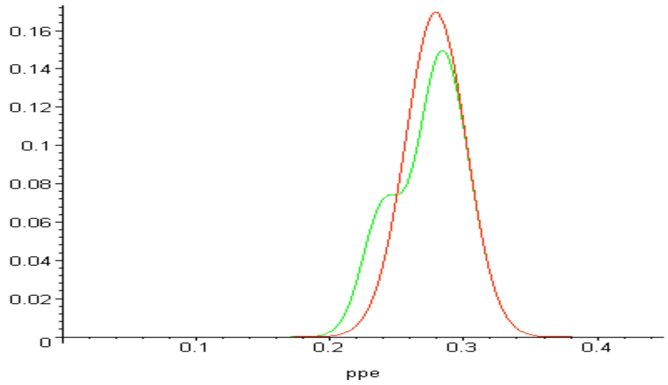

Fig.5. Horseshoe cross-section at constant $v_{||}$. Red line showing the distribution function at the beginning of intensive radiation and green line showing flattened distribution function after the main peak of radiation

This simple model gives us about 2% energy efficiency estimate, which is in line with results of the experiment and PIC code simulations.

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