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The nonlinear amplification of magnetic fields by cosmic rays at supernova remnant shocks

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Abstract

A central problem in the theory of cosmic ray acceleration at supernova shock fronts is the generation of turbulent magnetic fields needed to scatter particles across a shock front. In this paper we build on previous studies [1], [2] into the effect of streaming cosmic rays produced by the outer shocks of supernova remnants on the stochastic component of the magnetic field. A three dimensional, MHD code has been constructed which demonstrates the nonlinear growth of the turbulent field.

Introduction

There is clear radio, X-ray and gamma ray observational evidence that electrons, and probably protons, are accelerated to energies in excess of 10^{14} eV in supernova remnants ([1]). Diffusive shock acceleration provides the most natural explanation for the spectral shape. This process requires the presence of magnetic field turbulence that repeatedly scatters particles across the shock front. The timescale for acceleration is constrained by the level of the turbulence, through the particle's diffusion coefficient. Bell ([1]) has recently shown that, in the MHD limit, particle anisotropy upstream of the shock can excite turbulence many orders of magnitude greater than the background interstellar medium value thereby lowering the particle diffusion coefficient and making the process more rapid than previously though. In this paper we confirm Bell's linear analysis by using kinetic theory ([3]) and present some intial results from a code using a three dimensional Godunov scheme.

Linear Instability in Kinetic Theory

The linear dispersion relation for circularly polarised transverse waves propagating parallel to the zeroth order magnetic field is

$$\frac{c^2k^2}{\omega^2} - 1 = \sum_s \chi_s(k,\omega) \tag{1}$$

where the summation is over species f_{0s} with charge q_s and

$$\chi_s = \frac{4\pi q_s^2}{\omega^2}\int dp^3 \frac{v_\perp p_\perp}{\omega \pm \omega_{cs} - kv_\parallel}\left[(\omega - kv_\parallel)\frac{\partial f_{0s}}{\partial p_\perp^2} + kv_\parallel \frac{\partial f_{0s}}{\partial p_\parallel^2}\right] \tag{2}$$

is the susceptibility of each plasma component. We assume the plasma has three components; (i) cold background protons of density n_p, with a small drift velocity u_p along the mean magnetic field, (ii) cold background electrons with small drift velocity u_e, density n_e, also drifting along the mean magnetic field and (iii) anisotropic cosmic rays, for simplicity consists only of protons We also require the plasma to have overall charge neutrality, $\sum_s q_s n_s = 0$, and zero net current, $\sum_s q_s n_s u_s = 0$. If the background proton and electron distributions are Maxwellian with drifts u_p and u_e respectively, the susceptibility of the background plasma is

$$\chi_{bg} = \Sigma \frac{\tilde{\omega}_s \omega_{ps}^2}{\omega^2(\tilde{\omega}_s \pm \omega_{cs})} \left[\frac{(\tilde{\omega}_s \pm \omega_{cs})}{\sqrt{2}kV_{ts}} Z\left(\frac{(\tilde{\omega}_s \pm \omega_{cs})}{\sqrt{2}kV_{ts}} \right) \right] \tag{3}$$

where $\tilde{\omega}_s = \omega - ku_s$ denotes the doppler shifted frequency of each species, ω_{ps} the plasma frequency, ω_{cs} the cyclotron and $Z(\zeta)$ is the plasma dispersion function. For a cold plasma ($\zeta \gg 1$) and waves of short wavelength $\tilde{\omega}_s \ll \omega_c$, the background susceptibility is

$$\omega^2 \chi_{bg} = \frac{c^2}{c_A^2} \left[\tilde{\omega}_i^2 \mp \frac{k^2 V_{ti}^2}{\omega_{ci}} \omega_i \mp \frac{\omega_{ci} k J_{cr}^1}{n_i} \right] \tag{4}$$

in agreement with Achterberg [3]. $J_{cr}^1 = n_{cr} u_{cr} - \frac{\omega}{k} n_{cr}$ is related to the cosmic ray flux density. The J_{cr}^1 term results from the background plasma attempting to compensate for the CR current and corresponds to the return current [1, 2, 4].

The CR distribution is expressed as the first two terms in the Chapman-Enskog expansion $f_{cr}(p,\mu) = f_0(p) + \mu f_1(p)$ giving the susceptibility of the CRs as, with $\lambda = eB_0/kpc$,

$$\omega^2 \chi_{cr} \approx \pm \frac{c^2}{c_A^2} \frac{\omega_{ci} k}{n_i} \left(1 - \frac{\omega^2}{k^2 c^2} \right)^{1/2} \left[\int dp 4\pi p^2 \frac{1}{3} v f_1 \sigma_p \right]_w \tag{5}$$

where

$$\sigma_p(\lambda) = \frac{3}{4} \lambda (1 - \lambda^2) \left[\ln\left(\frac{1+\lambda}{1-\lambda} \right) + i\pi \right] + \frac{3}{2} \lambda^2 \tag{6}$$

We can now combine the background and cosmic ray terms to find the total susceptibility. For the case of an outer shock of a SNR we assume $f_0 = \phi p^{-4}$ in some momentum range and zero outside it (for cosmic ray population), and $f_1 = 3(v_s/c) f_0$. Performing the integration in Eq.(5) for waves $\omega^2 \ll c^2 k^2$, the dispersion relation is

$$\hat{\omega}_i^2 \mp \left(\frac{kV_{ti}}{\Omega_i} \right)^2 \Omega_i \hat{\omega}_i - c_A^2 k^2 \pm \zeta v_s^2 \frac{k}{r_{g1}} (\sigma_1 - 1) = 0 \tag{7}$$

where

$$\lambda_1 \sigma_1 = \frac{3}{8} \lambda_1 (\lambda_1^2 + 1) - \frac{3}{16} (\lambda_1^2 - 1)^2 \ln\left| \frac{1+\lambda_1}{1-\lambda_1} \right| + \frac{3\pi i}{16} \begin{cases} \lambda_1^2 (2 - \lambda_1^2) & \lambda_1 < 1 \\ 1 & \lambda_1 > 1 \end{cases} \tag{8}$$

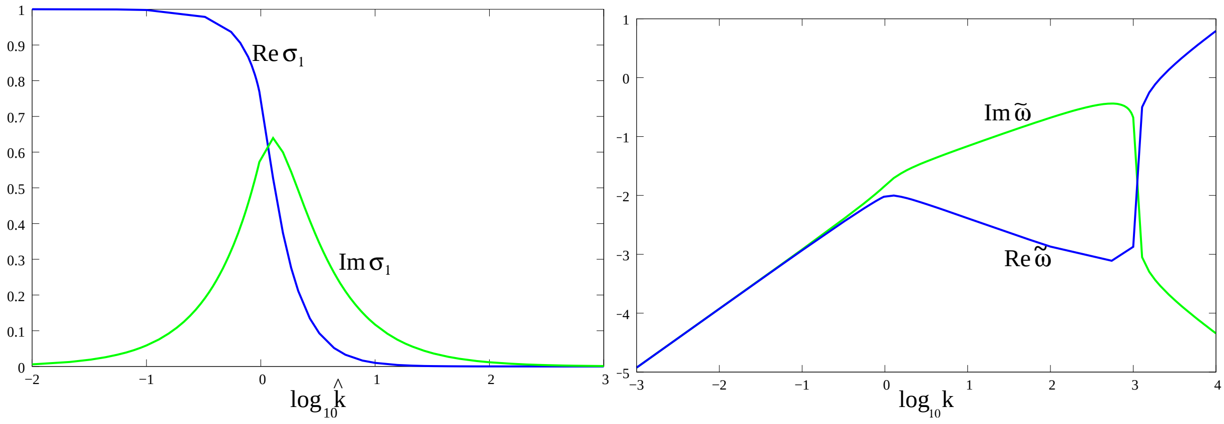

Figure 1: Left: The real and imaginary parts of σ_1 plotted as functions of kr_{g1} Right: Dispersion curves for Eq(7) plotted with k in units of r_{g1}^{-1} and $\tilde{\omega}_i$ in units of v_s/r_{g1}

with $\lambda_1 = (kr_{g1})^{-1}$ and $\zeta = U_{cr}/\rho c v_s \ln(p_2/mc)$. Eq.(7) is identical to that in [1] with an additional term that accounts for thermal effects.

Fig.1 shows the dependence of the complex function σ_1 on wavenumber, and the dispersion relation. The dispersion relation shows a non-resonant mode with a large growth rate. The function σ_1 is small for wavenumbers close to the maximum growth rate. This instability drives the growth of magnetic turbulence upstream of a SNR shock.

Numerical Results

When studying the behaviour of the non resonant growth term, we can neglect σ_1 as it is small for large k, ie. any perturbations to the mean current can be ignored. To examine the growth and behaviour of the non-linear magnetic turbulence we have constructed a numerical MHD model. Computations were performed using an upwind Godunov code which solves the equations of ideal MHD on a 3D regular cartesian grid, ensuring divergence-free magnetic field using lagrange-multiplier method [5]. The current observed by the MHD fluid is the so called return current. This current is represented in the MHD equations through Ampere's law.

$$\nabla \wedge \mathbf{B} = \mathbf{j}_{cr} + \mathbf{j}_{return} - n_{cr} e \mathbf{u} \quad (9)$$

The current driving term can be added to the numerical scheme through source terms. Our simulation has a small seed turbulent field superimposed on the mean background field which is parallel to the return current. The turbulent field is initialised with wavevectors in all directions.

Initially there is an inertial stage of little growth, in fact primarily the damping of waves that have negative growth rates, caused by the same current driven mechanism. At $t \approx 5$ the perpendicular component of the field becomes non-linear and the turbulent field starts to drive convective flow in the plasma. At this point the magnetic field, which is frozen in to the fluid

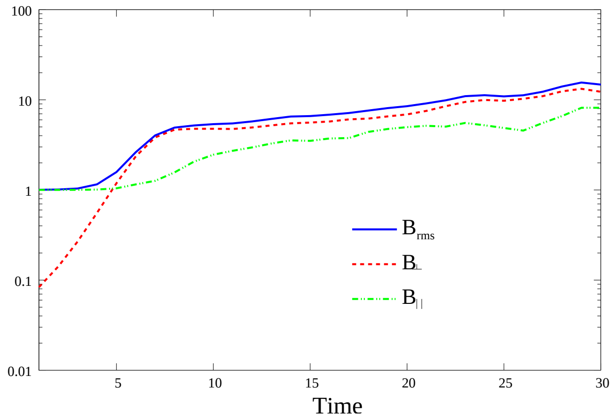

Figure 2: The growth of the different components of the magnetic field in units of the initial mean field

moves in the direction of the beam as well, increasing the overall parallel field. Slow shocks start to form that heat the plasma quite significantly. The perturbations stop growing when the magnetic tension in the field lines overcomes the driving force. But as energy is continually injected into the system this prevents actual "saturation ".

In future work we will determine the effect that this new turbulence generation mechanism has on cosmic ray transport properties. Finding statistical values for the new diffusion coefficients, in our amplified field, will allow us to determine the acceleration timescales in diffusive shock acceleration models. In the case of tightly wound, turbulent magnetic field lines in the vicinity of a shock, anomolous transport effects might be important in trapping particles. We aim to look at the modification of the spectral index of high energy particles at a shock.

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