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Oscillations of three-dimensional plasma clusters

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Introduction

Throughout the recent decades there appeared several physical systems actualizing J.J. Thomson's plum pudding model of the atom. Among such systems of particular interest for the plasma science are Coulomb clusters confined by various electromagnetic traps [1] and three-dimensional dust clusters [2, 3]. To a certain approximation, both Coulomb clusters and dust clusters may be treated as a set of interacting point particles confined by an external potential well.

In the present work, we develop the theory of oscillations of plasma clusters in a spherically symmetric potential well. We consider a set of N particles of the same mass, M, interacting via an arbitrary pairwise potential, $U(r)$. Making use of the finite group theory (e.g., [6]), we provide the complete classification of the cluster spectra for $N = 4\dots 13$. Under certain conditions, the polarization of an oscillation is independent of the interparticle potential; in these cases the explicit expressions for the frequencies are found.

Below we briefly outline the calculations. As an example, we consider the oscillations of the cluster consisting of six particles. More complete discussion may be found in the preprint [7].

Outline of calculations

The equation of motion for the set of N particles is written as

$$\ddot{\mathbf{r}}_n = -\frac{\partial W}{\partial \mathbf{r}_n}, \qquad n = 1\dots N, \tag{1}$$

where W is proportional to the potential energy,

$$W(\Gamma) = \frac{\omega_0^2}{2}\sum_{n=1}^{N} r_n^2 + \frac{1}{2}\sum_{\substack{n,m=1\\ n\neq m}}^{N} u(|\mathbf{r}_n - \mathbf{r}_m|^2). \tag{2}$$

Here ω_0 is the oscillation frequency of a single particle in the external confining potential and $u(r^2) = U(r)/M$ (to reduce the subsequent expressions we consider $u(r^2)$ as a function of distance squared). In equilibrium the particles are situated at certain fixed positions, $\mathbf{r}_n^0$, that are found from Eqs. (1).

Introducing the small displacements, $\mathbf{r}_n = \mathbf{r}_n^0 + \mathbf{s}_n$, and expanding the potential energy (2) we arrive at the quadratic form $W_2 = \frac{1}{2}\sum_{i,j=1}^{3N} \kappa_{ij} s_i s_j$, where the indices i, j label both different particles and the components of displacements. The problem is to evaluate eigenvalues and eigenvectors of the symmetric force matrix, κ_{ij}.

There are two sets of particle displacements corresponding to the rigid-body motions of the cluster for which the eigenvalues are easily evaluated. First, when the entire cluster is shifted from its equilibrium position so that all the displacements, $\mathbf{s}_n$, are equal, one can easily verify that the corresponding frequency is $\omega_{tr} = \omega_0$. Second, since the potential energy (2) is invariant under rotations, three eigenvalues of κ_{ij} are identically zero. In a particular case of the Coulomb interaction potential there also exists the symmetric breathing oscillation with the displacements proportional to the equilibrium vectors, $\mathbf{s}_n(t) = \lambda(t)\mathbf{r}_n^0$; the corresponding frequency is $\omega_{br}^2 = 3\omega_0^2$.

The displacements, $\mathbf{s}_n$, form a $3N$-dimensional vector space, L_0, which is represented as the direct sum of two sub-spaces, $L_0 = L^{rb} + L^{osc}$. One of the subspaces, L^{rb}, corresponds to the rigid-body motion of the cluster, the second one, L^{osc}, represents the oscillations. If the equilibrium configuration obey a certain symmetry group, then the oscillation space, L^{osc}, is represented as a sum of sub-spaces invariant under the action of this group, $L^{osc} = \sum_{i,\alpha} n_i^\alpha L_i^\alpha$ (e.g. [6]). Here L_i^α stands for the i-dimensional invariant space of an irreducible representation, the superscript, α, distinguishes non-equivalent irreducible representations. For every symmetry group there is a finite set of standard invariant subspaces. The integers n_i^α show how many times the subspace L_i^α occurs in the oscillation space L^{osc}.

There exists a regular algebraic procedure that allows to determine the number and the dimensions of the invariant sub-spaces as well as the corresponding basis vectors. Upon transforming to the new basis, the force matrix κ_{ij} takes the block-diagonal form. If there are n_i equivalent irreducible representations of dimension i, then the corresponding block consists of i equal $n_i \times n_i$ symmetric matrices. The corresponding set of modes in this case is of degeneracy i. If $n_i = 1$ then the polarization of the mode is determined by the symmetry group only and is independent of the interparticle potential. In this case the explicit expression for the oscillation frequency may be found.

The general tendency is that the higher is the symmetry the larger is the number of non-equivalent invariant sub-spaces and the smaller are the numbers n_i^α. The utility of the group technique may be illustrated by the following example. The cluster composed of twelve particles is of the form of the regular icosahedron; the corresponding symmetry group consists of 120 elements and has 10 irreducible representations. The application of the sketched procedure to this cluster shows that only one five-dimensional sub-space enter twice and for all other non-equivalent subspaces $n_i = 1$ [7]. In other words, to obtain the oscillation spectrum we have to investigate only one 2×2 matrix instead of analyzing the eigenvalues of a 36×36 matrix. Below we discuss the oscillations of a simpler cluster composed of six particles.

Oscillations of the octahedral cluster

In equilibrium, six particles settle at the sphere of radius a at the vertices of the regular octahedron. The equilibrium radius is found from Eq. (1): $\omega_0^2 + 8u'(2a^2) + 4u'(4a^2) = 0$. The symmetry group of the octahedron, O_h, consists of 48 elements. The decomposition of the oscillation space was found to be $L^{osc} = L_1 + L_2 + L_3^1 + L_3^2 + L_3^3$, that is, there are one non-degenerate oscillation, one oscillation of degeneracy 2, and three oscillations of degeneracy 3.

The only non-degenerate oscillation is the breather, all particles oscillate in the radial direction with equal amplitudes. The frequency of the breather is

$$\omega^2 = 32a^2\left[u''(2a^2) + u''(4a^2)\right]. \tag{3}$$

For the mode of degeneracy 2, an arbitrary pair of opposite particles is immobile, others move in antiphase as shown in Fig. 1a with the frequency

$$\omega^2 = 8a^2\left[u''(2a^2) + 4u''(4a^2)\right]. \tag{4}$$

There are three types of oscillations of degeneracy 3. The frequency of the first mode is

$$\omega^2 = 16a^2u''(2a^2). \tag{5}$$

The polarization is depicted in Fig. 1b. Any two opposite particles are immobile and other four particles move in the same plane with equal amplitudes.

The second oscillation is shown in Fig. 1c, where a pair of opposite particles is also immobile, while others move along the same direction with equal amplitudes. The frequency of this mode is given by

$$\omega^2 = 4u'(2a^2) - 4u'(4a^2) + 8a^2u''(2a^2). \tag{6}$$

The last mode is shown Fig. 1d. Here all particles oscillate along the same direction with the frequency

$$\omega^2 = 4\left[u'(2a^2) - u'(4a^2) + 6a^2u''(2a^2)\right]. \tag{7}$$

The amplitude of an arbitrary opposite pair doubles the amplitudes of the remaining four particles.

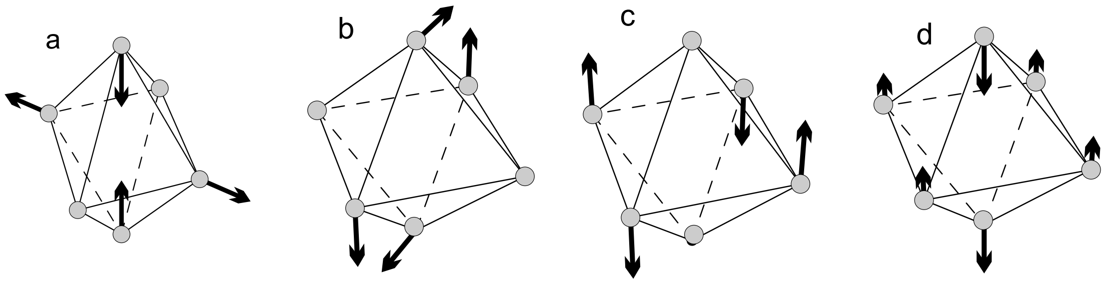

Figure 1: Oscillations of the octahedral cluster.

Conclusion

We have developed the theory of oscillations of plasma clusters with an arbitrary interaction between particles. The complexity of the spectrum depends essentially on the symmetry group of the cluster. High symmetry of clusters in the form of Platonic solids ($N = 4, 6, 12$) allows to obtain explicit expressions for nearly all eigenfrequencies. With less symmetric configurations, the group technique implemented here also turns out to be a very effective tool.

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