Effect of electrostatic plasma oscillations on the kinetic energy of a charged macroparticle in weakly ionized plasma

O. S. Vaulina¹, S. V. Vladimirov², A. Yu. Repin¹

¹Institute for High Energy Density, RAS, Moscow, Russia
²School of Physics, The University of Sydney, New South Wales, Australia

Introduction

A dusty plasma is an ionized gas containing charged macroscopic particles (dust), typically of a micrometer size. The majority of experimental studies of dusty plasmas are done in a weakly ionized plasma of a gas discharge when dissipation due to collisions with the gas atoms/molecules is significant. The dust charge $eZ$ can be significant ~ $10^2$-$10^4e$. The dust kinetic temperature can significantly exceed not only their surface temperature but also the electron temperature (~ 1-5 eV). Possible reasons for these anomalously high dust kinetic temperatures include the space-time fluctuations of the dusty plasma parameters as well as development of various plasma-dust instabilities [1-3].

In this paper, we consider the effect of electrostatic oscillations of a quasi-equilibrium plasma on the kinetic temperature of an isolated dust particle. These oscillations appear as a consequence of the space separation of the plasma charges because of their thermal motions and lead to the time fluctuations of the plasma electric field $E(t)$, with the mean-average $<E(t)^2>$ $\propto T_{pl}$, where $T_{pl}$ is the plasma temperature. The main physics of the influence of the electrostatic plasma oscillations on the dust kinetic energy is that the electric field fluctuations induced by the stochastic motions of the electrons/ions lead, in their turn, to fluctuations of the electric forces, $\sim F_E(t) = eZE(t)$, acting on the dust. This causes extra chaotic movements (in addition to the dust Brownian motion) with the kinetic energy $T_{kd} \propto <Z^2E(t)^2>$ which is non-zero even in the case when the stochastic charge fluctuations (because of the discrete character of charging plasma currents) are ignored, $Z(t)=\text{const.}$

Electrostatic plasma oscillations and dust kinetic energy

Consider electrostatic oscillations of an unmagnetized plasma due to its thermal fluctuations in the two limiting cases: 1) for Langmuir oscillations of electrons on the background of stationary ions, and 2) for electrostatic oscillations of the number density of cold single-charged ions on the adiabatic electron background (with the thermal electron
motions). The last case takes place when the electron temperature significantly exceed the ion temperature, \( T_e \gg T_i \). The field appearing because of the charge separation due to the thermal motion of plasma electrons and ions, is determined by the Poisson equation

\[
\text{div}\mathbf{E} = 4\pi e(n_i - n_e),
\]

where \( n_e(i) \) is the electron (ion) number density and the (space) charge density, \( q = e(n_i - n_e) \), according to the quasi-neutrality condition, is determined by the charge density of the perturbed component. For the cases analyzed here, the condition of the conservation of the electric charge is determined by the continuity equation for the perturbed plasma component,

\[
\frac{\partial n_e(i)}{\partial t} = -\text{div}(n_e(i)v_e(i)),
\]

where \( v_e(i) \) is the electron (ion) velocity. We perform the analysis of the motions of the plasma particles taking into account the electron (ion) friction forces,\( -\nu_e(i) m_e(i) v_e(i) \), where \( \nu_e(i) \) and \( m_e(i) \), are the friction coefficient and the mass of electrons (ions), and the stochastic forces\( \tilde{F}_{e(i)} \)

\[
\frac{dm_e(i)v_e(i)}{dt} = -v_e(i)m_e(i)v_e(i) \pm (e\mathbf{E}) + \tilde{F}_{e(i)}.
\]

In a weakly-ionized plasma, the introduced friction factors characterize mostly the effective friction frequency of the plasma particles with the neutrals of gas (\( \nu_e \approx \nu_{en} \) and \( \nu_i \approx \nu_{in} \)).

The equation of motion of a dust particle (macroparticle) with the mass \( M \) and the charge \( eZ \) in the fluctuating plasma field \( \mathbf{E}(t) \) is given by

\[
\frac{dV}{dt} = -\nu_{fr}V - (eZ/M) \mathbf{E},
\]

where \( V \) is the dust velocity, \( \nu_{fr} \) is the friction coefficient, and the Langevin forces acting on the dust from the neutrals are omitted for simplicity. By assuming that the macroparticle does not affect the plasma fluctuations, its kinetic temperature \( T_d \) can be found from Eqs. (1)-(4):

\[
T_d = Z^2 m_p \omega_p^2 T_p (v_{pn}+v_{fr})/\{[(v_{pn}+v_{fr})v_{fr}+\omega_p^2]MV_{fr}\},
\]

When \( v_{pn} >> v_{fr} \) and \( \omega_p >> v_{fr} \) (where \( \omega_p^2 = 4\pi^2 n_p m_p \)) this equation can be written as

\[
T_d \approx Z^2 T_p m_p v_{pn}/(MV_{fr}),
\]

where \( T_p \) \( m_p \) \( v_{pn} = T_e(i) m_e(i) v_e(i) \) \( n_e(i) \) depending on the type of electrostatic plasma oscillations. In the case of the high-frequency Langmuir oscillations, the field fluctuations are fully determined by the parameters of the electron plasma component (\( v_{pn} = v_{en}, m_p = m_e, n_p = n_e, T_p = T_e \)). In the second case (of the ions on the electron background) the electric field lower-frequency fluctuations are mostly determined by the ion characteristics (\( v_{pn} = v_{tr}, m_p = m_i, n_p = n_i, T_p = T_i \)). In the simulations we can easily establish which plasma component mostly affects the energy gaining by the macroparticle. It is sufficient to consider a particle trapping in the external electric field to determine the value of \( \omega_p \) by analyzing the resonance curve.
Numerical simulations

Simulations were done for the conditions close to those of a typical laboratory experiment in the gas-discharges. A macroparticle with the mass $M$, the radius $R$, and the fixed charge was confined in the center of the cubic simulation box with the length $2L$ by the linear electric field with the gradient $\alpha = 4\pi en$, where $n = N_e/(2L)^3$, and $N_e = N_i$ is the number of electrons/ions in the simulation cell. The macroparticle’s equation of motion was solved taking into account the effect of plasma electrons and ions. For the electrons and ions, the Langevin equations were solved taking into account the electric field of the dust charge as well as the plasma particle interactions.

We have considered argon with the pressure $P = 0.3-10$ Torr. The effective collision frequencies $v_{in}$ ($v_{en}$) were proportional to $P$ and were taken as $v_{in} = 8 \times 10^6$ s$^{-1}$ ($v_{en} = 5.3 \times 10^9$ s$^{-1}$) for $P = 1$ Torr. Simulations were done for $|Z| = 10-1000$, $R = 0.1-3$ µm, $T_i/T_e = 0.01-0.1$, $T_i = 0.03$ eV, $v_f = v_{in}/100$, and $M = (100-10000)m_i$. The simulation box was changed from $\sim 1.5\lambda_{Di}$ to $\sim 15\lambda_{Di}$ (where $\lambda_{Di}$ is the ion Debye length) by varying the number of plasma particles ($n \sim 10^9-10^{12}$ cm$^{-3}$). The simulation was done until the dust temperature achieves a stationary value. The illustration of the time evolution of the heating of the macroparticle in the simulation cell is presented in Fig. 1 for $L/\lambda_{Di} \approx 15$. To determine the plasma eigenfrequency $\omega_p$ in the numerical simulation, we varied the values of $Z\alpha/M$ and $v_{pn}$. As a result, we have obtained that $\omega_p^2 \approx 8\pi e^2 n/(3m_i)$. The temperature $T_{cal}$ determined in the simulation run did not depend on the dust radius $R$. The acquired temperature $T_{cal}$ was dependent on the ratio $\beta = L/\lambda_{Di}$ (see Fig. 2). With the increase of $\beta$, the value of $T_{cal}$ approached $T_d$ (6) with the parameters of the ion plasma component, i.e., $T_d = T_i m_i Z^2 v_{in}/(Mv_{in})$.

Thus the dust particle in a plasma can acquire the additional stochastic kinetic energy $T_d$. The value of $T_d$ is determined by the fluctuations of the electric field caused by the thermal motion of plasma ions. If to consider the conditions of typical laboratory experiments in a gas-discharge (in argon) and write the coefficient for the dust friction in the free-molecular approach and the dust charge in the Orbit-Motion Limited (OML) approach, then the acquired energy can be estimated as $T_{app} \approx 0.5 (T_e [eV])^2 T_i$. Therefore, for $T_e \sim 2-5$ eV and $T_i \sim 0.03$ eV, the dust kinetic energy can achieve $\sim 0.14-0.34$ eV.

Conclusions

To conclude, we have presented here the analytic relations for the stochastic energy that is acquired by an isolated solid macroparticle in a weakly-ionized plasma because of the
thermal electrostatic plasma fluctuations. The fluctuations can be related to the Langmuir plasma mode as well as to the electrostatic (cold) ion mode. The derived expressions allowed us to estimate the kinetic temperature of the dust in quasi-equilibrium plasma for the conditions where there is no plasma-dust instabilities and no propagated plasma waves. Numerical simulations have demonstrated that for the conditions of typical laboratory experiments complex plasma (when, in particular, $T_i/T_e << 1$), the most significant contribution to the energy acquired by the macroparticle comes from the plasma fluctuations associated with the ion component. The kinetic temperature of the macroparticle acquired by this channel, can significantly exceed the room temperature of the background gas/plasma ions.

![Figure 1](image1.png)  
**Figure 1.** The $MV^2/T^0$ vs. time for:  
1 - $M=10^4 m_i$, $Z=10^3$; 2 - $M=10^3 m_i$, $Z=10^3$; 3 – $M=10^2 m_i$, $Z=10$; $T^0 = 300 T_i$.

![Figure 2](image2.png)  
**Figure 2.** The dependence of the ratio $T^{cal}/T_d$ on $\beta=L/\lambda_D$.

Acknowledgments

This work was partially supported by the Russian Foundation for Fundamental Research (No. 04-02-16362), CRDF (No. RU-P2-2593-MO-04), the Program of the Presidium of RAS.

References

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