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Electrostatic forces on particles in plasma in presence of an electric field

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'Dusty' or 'complex' plasmas are plasma containing ions, electrons, highly charged micron sized particles (dust grains) and neutral gas. Dust plays an exceptionally important role in technological plasma applications, associated with the utilization of plasma deposition and etching technologies in microelectronics, as well as with the production of thin films and nano particles. We are concerned here with the force exerted on an isolated particle by the ambient plasma. In an uniform electric field we have derived the potential distribution around the dielectric particle analytically and compared it with the metal particle case which was calculated by Dougherty et al [1]. However in this paper they didn't consider the particle sheath distortion due to a flowing plasma and trapped ions. Later Hamaguchi and Farouki[2] extended the theory considering density gradient in the plasma with an externally applied constant electric field. They showed that a polarization force, proportional to the density gradient should be added in order to account for the deformation of the sheath. However Ivlev et al[3] calculated the stationary self consistent potential of a dielectric spherical particle in a flow of a collisionless plasma with the assumption that ion drift velocity (v_{dr})>> ion thermal velocity (v_{th}) in the sheath region. In this case the potential of the dielectric particle becomes strongly anisotropic because of the significant inhomogeneity of the surface charge. In our case we consider, $eV_f \gg E_0\lambda_D$, i.e the voltage drop within the debye sheath is much smaller than particle floating potential. The spherical symmetry of the problem is employed. Estimation of the total force on a particulate involves not only the calculation of the sheath polarization force, but also the contribution of the pressure forces due to electrons and ions on the particulate due to ambient plasma. The pressure force is due mainly to positive ions which are electrostatically attracted by the negatively charged particulate. A slight imbalance between the pressure force on one side of the particulate and that on the other due to nonuniformity of plasma may result in a finite net force. The imbalance derives from the variation of potential on the surface of the dielectric sphere in the OML approximation or from the momentum of the ions in the radial motion. The radial theory represent the physical situation of ions accelerated radially towards the negative particle in the hypothesis $T_i = 0$ and when $T_i \ll T_e$ and the orbital momentum of the ions is lost by collisions far from the particle. In this second acception it is possible to define a coulomb radius even for radial motion. The coulomb radius is different for Radial and OML theory. In this article all the distances are normalized w.r.t debye length ($\lambda_D = 12.81\mu m$ for $n = 10^{16}/m^3$ and $T_e = 3eV$) and $E_0 = 30kV/m$.

Our model starts with the solution of the Debye-Hückel equation in spherical coordinate taking into account azimuthal symmetry as:[4]

$$\nabla^2\phi = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial\phi}{\partial r}) + \frac{1}{r^2 sin\theta}\frac{\partial}{\partial\theta}(sin\theta\frac{\partial\phi}{\partial\theta}) = k_D^2\phi \tag{1}$$

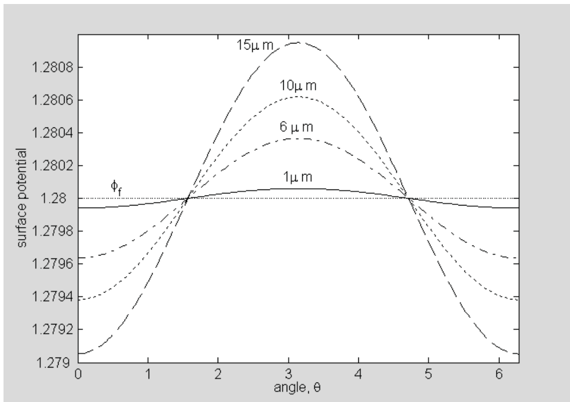
where $k_D^2 = \lambda_{De}^{-2} + \lambda_{Di}^{-2}$. Using the method of separation of variables and applying the proper boundary conditions for dielectric particle the solution can be written as:

$$\phi_{in}(r,\theta) = -\frac{\alpha k^2a^2 + 3\alpha(ka+1)}{\alpha k^2a^2 + (ka+1)(2\alpha+1)}E_0 rcos\theta \tag{2}$$

$$\phi_{out}(r,\theta) = -E_0 rcos\theta + \frac{(1-\alpha)k^2a^3}{\alpha k^2a^2 + (ka+1)(2\alpha+1)}e^{-k(r-a)}(\frac{1}{kr} + \frac{1}{k^2r^2})E_0 cos\theta \tag{3}$$

In the limit of no plasma condition ($k \longrightarrow 0$) we get back the vacuum result. On the surface of the dielectric sphere $\phi_{in}(a,\theta) = \phi_{out}(a,\theta) = \phi_p cos\theta$, shows the consistency of the boundary condition for the dielectric particles where $\phi_p = -[\frac{3(ka+1)+k^2a^2}{\alpha k^2a^2+(2\alpha+1)(ka+1)}]a\alpha E_0$ and $\alpha = \frac{\varepsilon_0}{\varepsilon}$

However in dusty (complex) plasma due to higher mobility of electrons compared to the ions, the surface of the dielectric particle becomes negatively charged. In the linear regime the potential ϕ_f is assumed to be uniform for these charge accumulation with a modulation term so that on the surface of the charged dielectric sphere we can write: $\phi(a,\theta) = \phi_f + \phi_p cos\theta$ which is shown in the figure (1) for different particles in units of (kT_e/e) and $\phi_f = 1.28$ for $\beta = \frac{T_i}{T_e} = 0.01$.



fig(1): Variation of surface pot with angle

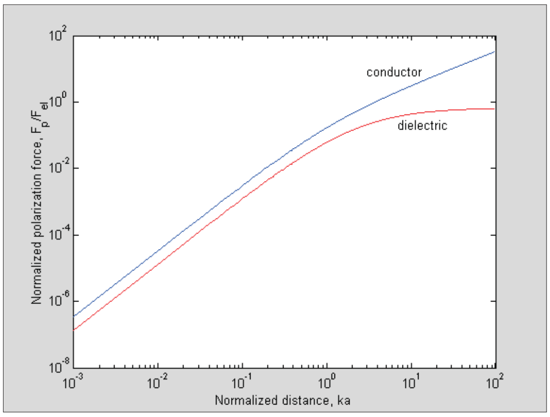
Taking this boundary condition if we now solve the Debye-Hückel equation then we get the complete expression for the potential outside the dust particle which can be written as:

$$\phi_{out}(r,\theta) = -E_0 rcos\theta + \phi_f\frac{a}{r}e^{-k(r-a)} + \frac{(1-\alpha)k^2a^3}{\alpha k^2a^2 + (ka+1)(2\alpha+1)}e^{-k(r-a)}(\frac{1}{kr} + \frac{1}{k^2r^2})E_0 cos\theta \tag{4}$$

The potential in equation(4) is clearly the superposition of three separate potential fields. The first term is the potential due to the constant applied field E_0. The second term is the spherically symmetric Debye-Hückel potential. The last term is the polarization response of the plasma, and it depends on $cos\theta$ and represents the characteristics of the dipole distribution of the dust grain charge The total electrostatic force on the dielectric sphere along the z direction i.e in the direction of the external field is: $F = 2\pi a^2 \varepsilon_0 \int_{\theta=0}^{\pi} [\frac{1}{2}(E_r^2 - E_\theta^2)\cos\theta - E_r E_\theta \sin\theta] \sin\theta d\theta$

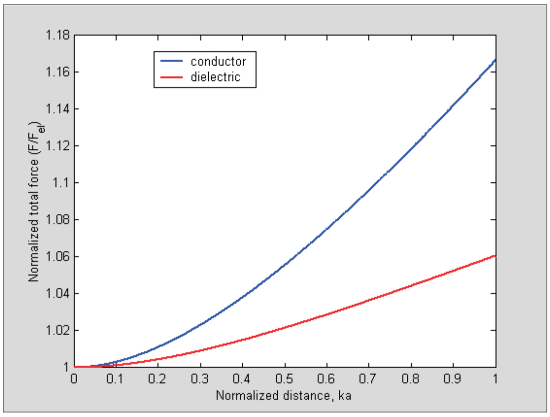
After performing the integration the total electrostatic force on the dielectric particle can be written as: $F = QE_{eff}$ where $Q = 4\pi\varepsilon_0 a(ka+1)\phi_f$ and $E_{eff} = E_0[1+(\frac{1}{3})\{\frac{(1-\alpha)(ka)^2}{\alpha k^2 a^2 + (2\alpha+1)(ka+1)}\}]$

This result is similar to that of metal particle and implies that the space charge does not shield the particle from the bulk electric fields and enhances the electrostatic force. In absence of plasma we get the total force expression in vacuum: $F_{el} = QE_0$. In fig(2) and fig(3) we have plotted the polarization force and total electrostatic force both for metal and dielectric particles.



fig(2): norm polarization force vs norm dist

From the potential distribution we can identify the third term as the potential for an electric dipole. The dipole moment $\vec{p}$ is the first moment of the polarized charge distribution which can be found out using Poisson equation. Considering the symmetry of the system we can write the z component of the dipole moment as: $p_z = 2\pi \int_{\theta=0}^{\pi} \int_a^{\infty} r^3 \rho_{dp}(r,\theta) cos\theta sin\theta d\theta dr$ where $\rho_{dp}(r,\theta) = -\varepsilon_0 k^2 \phi_{dp}(r,\theta)$ where ϕ_{dp} is the third term in eqn (4).

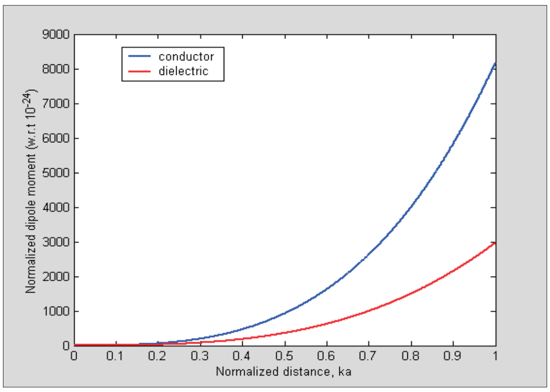


fig(3): norm total force vs norm distance

After performing this integration we get:

$$p_z = 4\pi\varepsilon_0(\frac{1-\alpha}{2\alpha+1})a^3 E_0[1+(\frac{1}{3})\{\frac{(1-\alpha)k^2a^2}{\alpha k^2 a^2 + (2\alpha+1)(ka+1)}\}] = 4\pi\varepsilon_0(\frac{1-\alpha}{2\alpha+1})a^3 E_{eff} \quad (5)$$

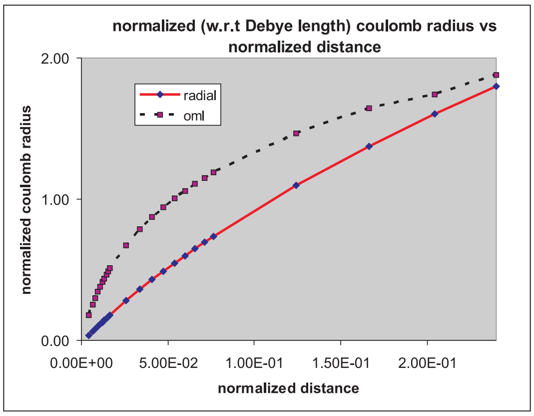
Here also we can see the appearance of the enhanced field E_{eff} due to the polarization of the plasma in response to the bulk field. These things are similar to that of the metal particle. In absence of plasma we get back the dipole moment of a dielectric sphere in vacuum in uniform electric field. In the fig(4)we have shown the normalized dipole moment for the metal and dielectric particle.



fig(4): norm dipole moment vs norm dist

In order to calculate the ion force due to asymmetrical bombardment from the radial motion theory the potential difference between the particle surface and the plasma at Coulomb radius (r_c) can be written as: $V(\theta) = V_f + r_c E cos\theta$ so that ions heat the surface with radial velocity after performing a collision at r_c where it loses it's orbital angular momentum is:

$$v_r = \sqrt{\frac{2e(V_f + r_c E cos\theta)}{M}} cos\theta \qquad (6)$$



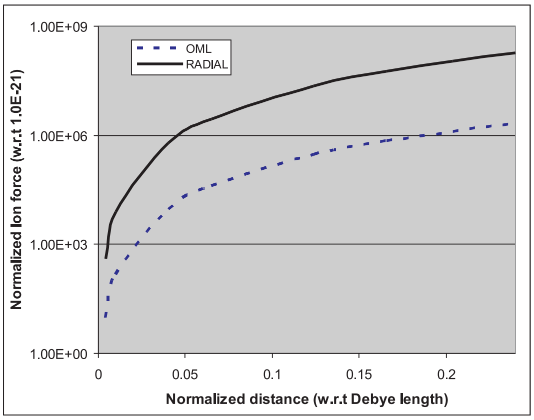
fig(5):coulomb radius with particle radius

The total momentum flux to the particle in the z-direction is:

$$P = \frac{I_e}{e} M \int_0^{\pi} \sqrt{\frac{2eV_f(1 + \frac{r_c E cos\theta}{V_f})}{M}} cos\theta sin\theta d\theta$$

After performing this integration we get the total ion force as:

$$F_i = -\frac{I_e}{e} \sqrt{\frac{V_f M}{2}} \frac{r_c E}{3V_f}$$



fig(6):variation of ion-force with distance

Instead using OML theory and taking electrons and ions to be in Boltzmann distribution we can write the complete force expression due to asymmetric ion bombardment as:

$$F_{EP,IP} = 2\pi k_B T_e a^2 \int_0^{\pi} n_0 e^{\pm \frac{e\phi out(a,\theta)}{k_B T_e}} \cos\theta \sin\theta$$

After integration we find the final expression for electron and ion pressure:

$$F_{EP,IP} = \mp \frac{4}{3} \pi a^3 e n_0 exp(\pm \frac{e\phi_f}{k_B T_e})[1 - \frac{(1-\alpha)(ka+1)}{\alpha k^2 a^2 + (2\alpha+1)(ka+1)}] E_0 \qquad (7)$$

In conclusion we can say that the polarization force can't be neglected in particular for large particles in the sheath edge. The ion force calculated from the radial theory can't be neglected as well and we have to take into account all these cases.

References:

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(4) J.D. Jackson, Classical Electrodynamics (Wiley, New York, 1963)