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Dynamics Of Nonlinear Vortexes In High-Current Plasma Lens

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The spatial structure and nonlinear dynamics of vortices in the high-current plasma lens have been investigated on computer model base. The charged plasma lens, intended for focusing of high-current ion beams, has crossed configuration of magnetic and electric fields. In small inhomogeneous electron density in the real experimental lens the preference is realized in behavior of vortices. The acceleration of evolution of vortices in electron plasma was observed in laboratory experiments. It is important to know properties of vortices at nonlinear stage of their evolution. It has been shown in this paper, that at reaching of quasi-stationary state the electrons in a field of a vortex rotate around its axis with the greater velocity in comparison with velocity of azimuth drift of electrons in fields of the lens. The development of instability in initially homogeneous plasma lens causes that the vortices are born pairs. It had been investigate slow vortices.

Introduction

The theoretical and numerical; investigation of vortical structures in plasma has been an active field of research for last several decades. The acceleration of evolution of vortices in electron plasma was observed in laboratory experiments [1]. The same dynamics of vortices should take place in near wall turbulence of nuclear fusion installations, where the crossed configuration of electrical and magnetic fields is realized.

The charged plasma lens (PL) is typical plasma optic devices, intended for focusing of high-current ion beams. It has the same crossed configuration of ExH-fields [2]. The possibility arising vortex structures in PL has been shown theoretically before [3, 4]. It is important to know properties of vortices at nonlinear stage of their evolution. It has been shown theoretically early [5] that at reaching of quasi-stationary state the electrons in a field of a vortex rotate around its axis with the greater velocity in comparison with velocity of azimuth drift of electrons in fields of the lens. Slow and quick vortices are contacting combinations of two vortices rotated in the opposite directions. The numerical investigation of arising and evolution of slow vortices in PL has been made in this paper. The instability development in initially homogeneous plasma causes that the vortices are born pairs (bunch-hole). It has been shown, that in small inhomogeneous electron density in the real experimental lens the preference is realized in behavior of vortices: the bunch goes to the region of greater electron density, and hole goes to the region of smaller.

Base equations

For the description of the electrons and ions dynamic in plasma lens will use hydrodynamic approximation:

$$\begin{aligned}
&\frac{\partial v}{\partial t} + (v\nabla)v = \frac{e}{m}\nabla\varphi + \omega_{ce}[k_z \times v] = 0 \\
&\frac{\partial n_e}{\partial t} + div\,(n_e v) = 0 \\
&\frac{\partial V_i}{\partial t} + (V_i\nabla)V_i = -\frac{Q_i}{M_i}\nabla\varphi \\
&\frac{\partial n_i}{\partial t} + div\,(n_i V_i) = 0 \\
&\Delta\varphi = -4\pi(Q_i n_i - e n_e),
\end{aligned} \tag{1}$$

here φ is electrostatic potential, V_i, n_i, Q_i, M_i and v_e, n_e, q_e, m_e –velocity, density, charge and mass of ions and electrons, $\omega_{he} = eH/mc$ - an electron cyclotron frequency. Neglecting non-stationary and nonlinear on φ terms, we derive the following equation for electron velocity:

$$v_\perp = -\frac{e}{m\,\omega_{He}}\left[e_z \vec{E}_{r0}\right] + \frac{e}{m\,\omega_{He}}\left[e_z \nabla\varphi\right] \tag{2}$$

From the equation of electron motion and Poisson equation it is possible to derive approximately expression for the vorticity: $\boldsymbol{\Omega} \equiv \boldsymbol{e}_z rot\boldsymbol{v}$, which is characteristic of the vortical motion of electrons, from (2) follows:

$$\Omega \approx -\frac{2eE_{r0}}{rm\,\omega_{He}} + \frac{\omega_{pe}^2}{\omega_{He}}\frac{\delta n_e}{n_{e0}}, \tag{3}$$

It describes quasi-stationary dynamics of electrons in fields of the lens and vortical perturbation. Here $\omega_{pe}^2 = \frac{4\pi e^2 n_{e0}}{m_e}$ - plasma electron frequency, E_{0r}- radial focusing electric field. From equation (1) we can get according [6] equation, describing transversal electron dynamics in nonlinear approximation and longitudinal in linear approximation in assuming: $\boldsymbol{\omega}_{He} = \boldsymbol{e}_z \omega_{He}$ and $\partial_z \omega_{He} = 0$:

$$\begin{aligned}
&\frac{d\left(\frac{\Omega - \omega_{He}}{n_e}\right)}{dt} = \frac{\Omega_0 - \omega_{He}}{n_{e0}}\frac{\partial v_z}{\partial z}; \qquad \frac{d}{dt} = \frac{\partial}{\partial t} + (v_\perp \nabla_\perp) \\
&\frac{\partial v_z}{\partial t} + \omega_\theta \frac{\partial v_z}{\partial\theta} = \frac{e}{m_e}\frac{\partial\varphi}{\partial z}
\end{aligned} \tag{4}$$

Let's consider the vortex with velocity v_{vort} that less in comparison with the drift electron velocity, $v_{vort} << v_{dr}$ (v_{dr}=-eE_{ro}/$m_e\omega_{He}$). For the description of such slow vortex structure one can also use the equation:

$$d_t\omega_{He}/n_e \approx 0, \quad d_t=\partial_t+(v_\perp\nabla_\perp)-v_{vort}\nabla_\theta, \quad n_e=n_{eo}+(q_i/e)\delta n_i+\Delta_\perp\varphi/4\pi e\ , \qquad (5)$$

where for perturbation ion density we can write in linear approximation:

$\delta n_i = n_{i0}\frac{q_i\varphi}{M_i}\frac{k^2}{(\omega - k_z V_b)^2}$. From eq. (2) we can find: $v_\theta=-(e/m_e\omega_{He})E_{ro}=\frac{\omega_{pe}^2}{2\omega_{He}}\frac{\Delta n}{n_{e0}}r$.

Thus, for description of arising slow vortex we can derive equation:

$$\frac{\partial\Delta_\perp\varphi}{\partial t} \approx -4\pi Q_i(v_\theta\nabla_\theta)\delta n_i \qquad (6)$$

Results of the Computer Simulation

Now, we will look for the numerical model for arising slow vortex when magnetic field is represented in the form $H(x)=H_0I_0(x/2R)$, here R – radius of PL and I_0.-Bessel function. Note, such kind of distribution of $H(x)$ in the PL is close to the optimal distribution minimizing spherical aberrations [2]. Fig. 1 present results of computer modeling for arising and saturation of slow vortex.

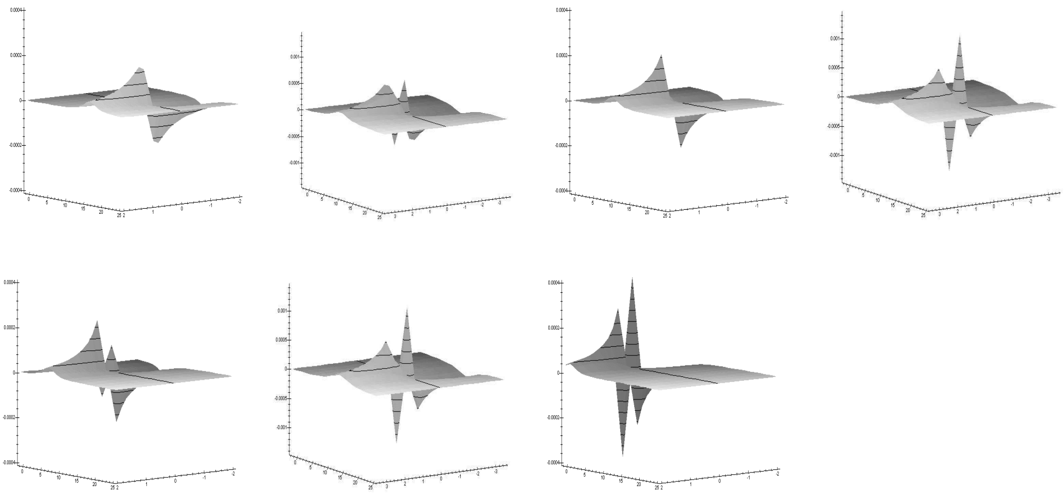

Fig. 1 Evolution vortex structure for different moments of time

For slow vortices the reason of the instability is the interaction of the drifting electron stream with ions, therefore amplitude of the saturation of the slow vortex is determined from the condition of the ion trapping or from the condition of the electron trapping and is determined by smaller of them. The spatial structure of the electron trajectories in its field for small amplitudes of the vortex looks like, shown in Fig. 2a).

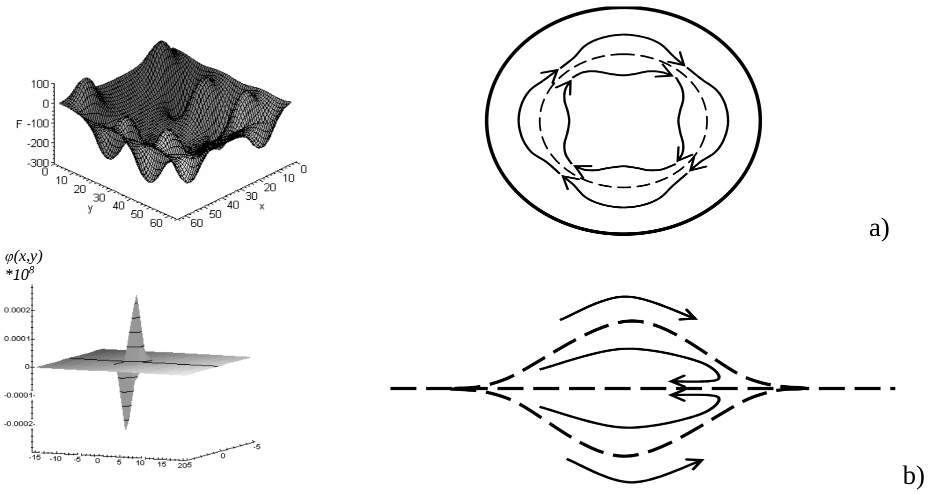

Fig. 2. The electron trajectories in chain of the slow vortices a)small amplitude with $V_{ph} \ll V_{\theta o}$,
b) large amplitude .

The slow vortex is a dipole perturbation of the electron density, disjointed on radius. At $\delta n_e > \Delta n$ the structure of the slow vortex is similar to structure of the Rossby vortex (see Fig.2b). The vortex size determines by vortex amplitude, radial non-uniform -ω_{He} and power of ions overcompensation - $\Delta n/n_{oe}$. Then more non-uniform ω_{He} and less $\Delta n/n_{oe}$, than vortex size is smaller with the same amplitude ϕ_o.

Conclusion

Thus, it is shown that ion-electron instability in the high-current PL caused by the radial gradient of the magnetic field axial component can lead on the non-linear stage to creation of long-live electron vortexes. The development of instability in initially homogeneous plasma lens causes that the vortices are born pairs: if the vortex - bunch of electrons is generated, the vortex - hole of electrons occurs near it. The small amplitude slow vortices with $v_{vort} \ll v_{dr}$ have structure like chain of the bunch-hole. Slow turbulence is not separated into single vortices. At large amplitudes, the slow vortex structure is similar to Rossby vortex.

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