2D modeling of blob dynamics in tokamak edge plasmas

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I. Introduction. Intermittent convective-like transport associated with such meso-scale structures as blobs is often dominant in the cross-field transport in the scrape off layer (SOL) of tokamaks, stellarators and linear devices [1]. In recent years it also became evident that the ELMs propagate into the SOL region in very much the same way as blobs do [2].

Significant amount of theoretical and computational work on blob physics have been done to date. Often reduced two 2D blob models [3] consider the outer part of tokamak edge structures as blobs is often dominant in the cross-field transport in the scrape off layer (SOL) of tokamaks, stellarators and linear devices [1].

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As a result of such normalizations, Eq. (1) one obtains the following equations for the normalized electrostatic potential \( \phi = \frac{\epsilon \phi}{T_e} \) and plasma density \( n \)

\[
\rho_s^2 \nabla \cdot (n \nabla \phi / \partial t) + (\rho_s C_s / R) (\partial n / \partial y) = \hat{\mu} (\phi), \quad \frac{dn}{dt} = D \nabla^2 n \tag{1}
\]

where the operator \( \hat{\mu} (\phi) \) describes different closures for different dissipation effects and gives the relation between the electrostatic potential and parallel current; \( R \) is the tokamak major radius, \( C_s = (T_e / M)^{1/2} \), \( M \) is the ion mass; \( \rho_s = C_s / \Omega_i \) and \( \Omega_i \) is the ion gyrofrequency; \( \frac{d(...)}{dt} = \frac{\partial (...)}{\partial t} + \nabla \times B \cdot \nabla (...) \), \( \nabla \times B = c (B \times \nabla \phi) / B^2 \); \( c \) is the speed of light, and \( D \) is the diffusion coefficient.

So far the major attention is paid to the sheath-limited model [3], which deals with the blob sitting on magnetic field lines in the SOL, which are going through conductive material surfaces. Recently another model was suggested, which describes the dynamics of blobs with high plasma beta, which can cause so significant bending of magnetic field lines that blobs can penetrate deeply into the SOL region while the magnetic field lines, blob sitting on, still do not intersect the material surfaces [4]. As a result we have two different closures:

\[
\hat{\mu} (\phi) = \hat{\mu}_{\text{SOL}} (\phi) = (2C_s / L_b) n_0, \quad \hat{\mu} (\phi) = \hat{\mu}_{\text{HB}} (\phi) = - (2V_A / L_b) n_{\text{amb}} \rho_s^2 \nabla^2 \phi, \tag{2}
\]

where \( L_b \) is length of the region with increased plasma density/pressure, \( n_{\text{amb}} \) is the ambient plasma density, and \( V_A \) is the Alfvén speed calculated with the density of ambient plasma.

II. Normalization. For numerical simulation it is convenient to use dimensionless variables: \( \tilde{n} = n / n_0, \quad \tilde{r} = r / \delta, \quad \tilde{t} = t / t_s, \quad \tilde{\phi} = \phi / \phi_*, \) where \( n_0 \) is the initial highest plasma density in the simulation domain, and \( \delta, \ t_s, \) and \( \phi_* \) are the characteristic scale length, time, and normalized electrostatic potential.

In our normalization procedure we choose such coefficients in front of all the three terms in Eq. (1) will be equal to unity. The results we have for the blobs with the “SOL” and “high b” closures read:

\[
(\delta_*)_{\text{SOL}} = \rho_s \left( \frac{L_b}{2R} \right)^{1/5}, \quad (V_*)_{\text{SOL}} = (\delta_*)_{\text{SOL}} / (t_*)_{\text{SOL}} = C_s \left( \frac{2 \rho_s}{R} \right)^{1/5} \left( \frac{L_b}{R} \right)^{1/5}, \tag{3}
\]

\[
(\delta_*)_{\text{HB}} = (L_b / 2R) (\beta / \alpha^2), \quad (V_*)_{\text{HB}} = (\delta_*)_{\text{HB}} / (t_*)_{\text{HB}} = C_s \beta^{1/2} (L_b / \alpha R), \tag{4}
\]

where \( \alpha = n_{\text{amb}} / n_0 \) and \( \beta = n_{\text{amb}} T_e / (B^2 / 4\pi) \). As a result of such normalizations, Eq. (1) with the closures (2) can be written, correspondingly, as follows:

\[
\nabla \cdot (n \nabla \phi / \partial t) + \partial n / \partial y = n \phi, \quad \nabla \cdot (n \nabla \phi / \partial t) + \partial n / \partial y = - \nabla^2 \phi, \quad \frac{dn}{dt} = \hat{D} \nabla^2 n. \tag{5}
\]
In our numerical study we seed the Gaussian density blob as an initial condition with the normalised spatial scale, $\delta$, on the background density $n_{bg} = 0.001$. We purposely choose different blob size (smaller, ~equal and larger than unity) to see the phenomena when the inertia effect goes from strong to weak.

III. The “SOL” blob simulation. Here we study the impact of the Boussinesq approximation, $\nabla \cdot (n \nabla \phi / \partial t) = n \nabla^2 \phi / \partial t$, on the “SOL” blob dynamics. This approximation, which simplifies numerical algorithms, is widely used in turbulent studies in both core and edge/SOL plasmas. While in the core relatively small amplitude of density fluctuations justifies such simplification it cannot, strictly speaking, be applied for edge/SOL plasma where the relative fluctuation level is ~1.

A. The SOL blob simulation in the Boussinesq approximation. The results of numerical solutions of the SOL blobs in the Boussinesq approximation are shown in Fig. 1.

![Fig. 1](image)

For the $\delta=0.2$ blob, the initial coherent structure evolves into a mushroom-shaped structure very quickly due to inertia effects. In the $\delta=1$ blob motion, the inertia effect is getting smaller. Again, we see the slight mushroom pattern because the inertia term is still comparable to the other two terms in scale. However, the $\delta=1$ blob coherently propagates to the right hand side of the domain. We find that, in the normalized case, the $\delta=2$ blob is the most stable one and coherently moves to a very long distance. For the $\delta=5$ blob, in which the inertia term is much smaller than the other two terms, we never see mushroom shape. Instead the RT instability brought by the driving force $\partial n / \partial y$ term becomes dominant and the corresponding fingering effect appears later to break up the original coherent structure. All these typical motion modes and instabilities have been discussed in the literature.

B. The SOL blob simulation with full inertial (FI) term. In Fig. 2 we show the simulation results of four different scale length blobs in the FI-SOL case. We clearly see that, in the FI case, smaller blobs are more structurally stable and coherently propagate to a larger distance than in the Boussinesq approximation. However, larger blob like $\delta=2$ is less stable than its counterpart. Comparing with the results of Boussinesq approximation at the same time stage, the $\delta=0.2$ and $\delta=1$ FI blob motions have mushroom shapes suppressed. We don’t clearly see
mushroom shape in the $\delta=1$ blob motion. The $\delta=2$ blob is the most stable one in the Boussinesq approximation simulation but it goes to finger effect here, which we think can be due to the mushroom suppression. Meanwhile, without surprise, the $\delta=5$ blob motion doesn’t change much between the Boussinesq approximation and FI results. It is because the inertia term is negligibly small in this large size case.

In addition, we show the difference of velocity for the $\delta=0.2$ and $\delta=1$ blobs, Fig. 3. The small FI blob keeps the constant velocity better than the small blob in the Boussinesq approximation because the speed is slowed down by stronger mushroom effect. This is coincident with the mushroom suppression when we take the density variation in the inertia term into account.

Over all, the FI case gives similar results than the Boussinesq approximation case. But, the most stable scale length shifts from 2 to 1.

IV. The “high $\beta$” blob simulation. The results of numerical simulation of “high $\beta$” blob dynamics are shown in Fig. 4. We notice significant difference of these “high $\beta$” blobs and the SOL blobs. Looking at Fig. 4 (b)-(d), where blobs have the same scale lengths as in Fig.1 (a), (b), (d), we don’t see the mushroom shape or fingering shape in the blob evolutions. The dynamics of “high $\beta$” blob doesn’t obviously change when the blob scale increases from $\delta=0.2$ to $\delta=5$. Because in the “high $\beta$” case, the inertia effect would change slightly since the ratio between inertia term and the other two terms is only $\delta^{-1}$. However, for some extremely small blob, such as the 0.01 blob shown in Fig.4 (a), the inertia term becomes, nevertheless, dominant. We also observe fingering effect in the “high $\beta$” case for large blob.
As we know, the velocity of the SOL blob is proportional to $\delta^{-2}$ [3]. However, the “high $\beta$” blob velocity is predicted to be independent of blob scale [4] and here we confirm it (Fig. 5).

IV. Summary. Our numerical simulation shows that, for the SOL model, the Boussinesq approximation case in general agrees with the FI case. However, there is a shift of the most stable scale length. The modelling of the “high $\beta$” blob [4] shows that, in agreement with our expectation, the “high $\beta$” blobs have a much wider gap of structural stability around $\delta \sim 1$ than the SOL blobs. We also confirm numerically that the propagation velocity of the “high $\beta$” blobs is not sensitive to the blob size [4].

Fig. 5.

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