*33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, P-2.186 (2006)*

Anomalous momentum transport due to drift waves in tokamaks

J. Weiland and H. Nordman

Chalmers University of Technology and EURATOM-VR Association
S-41296 Göteborg, Sweden

Abstract

Transport coefficients for both toroidal and poloidal momentum transport have been derived for the Weiland model. The turbulent drive also gives source terms due to Geodesic Acoustic Modes (GAM's). A rather complete model has been obtained for the shift in the eigenfunction due to flowshear. The reactive closure of the model enhances diffusivities and source terms associated with temperature perturbations. Momentum pinches are obtained for both poloidal and toroidal flows. The pinch in the toroidal flow is, however, usually rather weak, making a model with only diagonal element reasonable. Nonlinear feedback between flows, flowshear, shift of eigenfunction and turbulence level makes simulations sensitive and it frequently becomes necessary to freeze one or more variables.

Introduction

The interest in momentum transport has recently increased strongly. This is mainly because of the need to understand the formation of internal transport barriers[1]. While the poloidal momentum transport has been assumed to be neoclassical, the toroidal momentum transport has been considered to be anomalous with the transport coefficient $\chi_\varphi \approx \chi_i$ where χ_i is the ion thermal conductivity. Because of this the so called Prandtl number $P_r = \chi_\varphi / \chi_i$ has been introduced and early models even predicted $P_r = 1$ for comparison with the thermal conductivity of the slab ITG mode[2]. Recent results from JET, however, typically give $0.2 > P_r > 0.5$[3]. Recently also experimental results from JET have put the assumption that the poloidal momentum transport is neoclassical in serious doubt since rotation velocities about an order of magnitude larger than the neoclassical have been observed[4]. This also lead to modified conclusions regarding the ability of different transport models to predict transport barriers[5]. In the following we will present theories for both toroidal and poloidal momentum transport. While the toroidal momentum transport already has been compared to experiment with reasonably good result the poloidal momentum transport is still a very open question although good possibilities for dominance of turbulent poloidal momentum transport have been found.

Formulation

We start by deriving the *diagonal element* of a general variable since this is can be applied for both toroidal and poloidal momentum. ExB convection of a general variable F, using Fick's law leads to:

$$\chi = -\frac{\Gamma}{dF_0/dx} = D_B k_y^2 \frac{\gamma}{\omega_r^2 + \gamma^2} \left|\hat{\phi}\right|^2 \qquad \hat{\phi} = \frac{e\phi}{T_e} \qquad D_B = \rho_s c_s$$

With the saturation level given by[6]:

$$\gamma\delta T = \mathbf{v}_E \cdot \nabla\delta T \quad ; \quad \hat{\phi} = \frac{\gamma}{k_x c_s k_y \rho_s} \tag{1}$$

We arrive at:

$$\chi = \frac{\gamma^3 / k_x^{\,2}}{\omega_r^{\,2} + \gamma^2} \tag{2a}$$

This result was also obtained by kinetic orbit integration[7]. It is directly applicable to momentum diffusivity χ_m (toroidal or poloidal). For comparison the diagonal element of χ_i has the same form but also gets a Doppler shift due to the diamagnetic heat flow (closure term).

$$\chi_i = \frac{\gamma^3 / k_x^{\,2}}{(\omega_r - \frac{5}{3}\omega_{Di})^2 + \gamma^2} \tag{2b}$$

Thus we see that the Doppler shift has a direct effect on the Prandtl number .The result is typically for JET 0.2 < P_r< 0.5 in agreement with experiment[3,8]. We also note that the normalization of diffusivities here follows the original normalization of our model[6].

Off diagonal transport elements

A problem with deriving transport equations for momentum is that we can get several nonlinear terms in addition to the usual Reynolds stress. This is partly related to using low frequency drifts in the momentum equation. These drifts were derived from the momentum equation. For this reason it is necessary to compare with derivations of zonal flows, using some systematic perturbation scheme or to compare with nonlinear gyrokinetic theory[9]. As it turns out this also gives us a contribution from the nonlinearity in the energy equation which is particularly strong close to marginal stability.

$$\frac{\partial T_f}{\partial t} = -\vec{v}_E \cdot \nabla\delta T$$

It enters as a nonlinear source term for Geodesic Acoustic Modes.

Toroidal momentum balance

We will now, first consider the off diagonal parts of the toroidal momentum balance. We write the equation of toroidal momentum as:

$$m(\frac{\partial}{\partial t} + \vec{v}\cdot\nabla)v_\phi = e\left[E_\phi + (\vec{v}x\vec{B})_\phi\right] - \frac{1}{n}\frac{1}{R}\frac{\partial P}{\partial\phi} \tag{3}$$

We here want to rewrite (3) as a transport equation in the radial direction. There will only be average transport in the radial direction. Thus

$$\nabla\cdot(\vec{v}v_\phi) \;\rightarrow\; \frac{\partial}{\partial r}(v_r v_\phi)$$

Here the convective velocity, v_r is only the ExB drift while the convected velocity, v_ϕ includes also the diamagnetic drift and the parallel velocity. The poloidal component of the ExB drift is $v_{E\theta} = ik_r \rho_s c_s \hat{\phi}$ where $\hat{\phi} = \frac{e\phi}{T_e}$. The toroidal component is smaller by a factor $\frac{\varepsilon}{q} = \frac{B_\theta}{B_\phi}$. The same applies to the diamagnetic drift. This leads to the perpendicular contribution from Reynolds stress:

$$\Gamma_\perp = \langle v_{Er} v_{\varphi r} \rangle = -D_B{}^2 \frac{\varepsilon}{q} k_r k_\theta \frac{1}{2} \hat{\phi}^* \left[\hat{\phi} + \frac{1}{\tau} \hat{P}_i \right] + c.c \qquad (4a)$$

The contribution from the parallel velocity is usually larger. It can be written:

$$\Gamma_\perp = \langle v_{Er} v_\parallel \rangle = -iD_B k_\theta \frac{1}{2} \hat{\phi}^* v_\parallel + c.c \qquad (4b)$$

where

$$\hat{v}_\parallel = \frac{k_\parallel c_s}{\omega} \left[\hat{\phi} + \frac{1}{\tau} \hat{P}_i - \frac{\varpi + \frac{1+\eta_i}{\varepsilon_n \tau}}{\tilde{k}_\parallel \tilde{c}} \hat{A}_\parallel \right] \qquad \hat{v}_\parallel = \frac{v_\parallel}{c_s} \qquad \hat{A}_\parallel = \frac{eA_\parallel}{T_e} \qquad (5)$$

Here a complication arises because $k_\parallel$ enters only to first power. An average $\langle k_\parallel \rangle$ of $k_\parallel$ over the mode profile then gives zero in the absence of background flows. We then need this average in the presence of background flows. This was derived in Ref 10 and leads to: ($\rho = r/a$)

$$\langle k_\parallel \rangle = -\frac{1}{qR} \frac{0.5(\varpi + \frac{5}{3})K + h(\varpi - \varpi_{rk})\kappa_1}{\varpi(1+\frac{5}{3}) + \frac{1}{\varepsilon_n \tau}(\eta_i - \frac{2}{3}) + \frac{5}{3\tau}(1+\frac{1}{\tau})} \qquad (6a)$$

$$\varpi = \frac{\omega}{\omega_{De}} \; ; \; K = \frac{qR}{a} k_\theta \rho_s \frac{d}{d\rho} \hat{V}_\parallel \; ; \; \kappa_1 = \frac{L_n}{a} \frac{1}{sk_\theta \rho_s} \frac{d\hat{V}_p}{d\rho} \; ; \; h = 4k_\theta{}^2 \rho_s{}^2 \frac{\omega}{\omega_{\bullet e}} \left(\frac{q}{\varepsilon_n} \right)^2 \qquad (6b)$$

Thus we see that both gradients of toroidal and poloidal flows contribute.

Now assuming $\frac{a}{qR} \propto k_\theta \rho_s \approx 0.3$ we can estimate

$$\langle k_\parallel \rangle \propto \frac{a}{L} \frac{1}{qR} \qquad (7)$$

where L is the smallest of the length-scales of poloidal and toroidal flows. This value is typically considerably smaller than the usual estimate 1/qR in the interior but can eventually become comparable to it. Thus usually the diagonal element dominates for toroidal momentum transport.

Poloidal momentum transport

We write the equation for poloidal momentum transport as:

$$m\left(\frac{\partial}{\partial t}+\vec{v}\cdot\nabla\right)v_\theta = e(E_\theta + (\vec{v}\times\vec{B})_\theta) - \frac{1}{n}\frac{\partial P}{r\partial\theta} \tag{8}$$

We can now write a transport equation for V_θ in the form:

$$\frac{\partial}{\partial t}V_\theta + \frac{\partial}{\partial r}(v_r v_\theta) = S_v \quad ; \qquad \Gamma_v = v_{Er}v_\theta = -D_m\frac{\partial V_\theta}{\partial r} \tag{9}$$

We here consider transport of

$$v_\theta = v_{E\theta} + v_{*i\theta}$$

where $v_{E\theta}$ is the cross field drift and $V_{*i\theta}$ is the full perturbed ion diamagnetic drift including temperature gradient. This actually gives us Eq (4a) but without the factor ε / q. Thus

$$\chi_{mp} = -D_B{}^2 k_r k_\theta \frac{1}{2}\hat{\phi}^*\left[\hat{\phi}+\frac{1}{\tau}\hat{P}_i\right]\frac{L_{vp}}{V_p} + c.c \tag{10}$$

An interesting observation is here that the pure ExB part always gives a pinch which just means that the usual Reynolds stress generates zonal flows. The part due to the pressure perturbation is due to the diamagnetic drift. It introduces a dependence on the fluid resonance in the energy equation and can go in both directions. However the convective part usually leads to a more peaked equilibrium.

Discussion

Using the saturation level (1) in (4) and (10) we find that the diagonal element typically dominates for toroidal momentum transport while the off-diagonal elements tend to dominate for poloidal momentum transport. An interesting aspect is that the diagonal element of χ_i has a Doppler shift as seen in Eq (2b). Since the ITG mode typically has $\omega_r \approx 2\omega_{Di}$ this is sufficient to give us a Prandtl number between 0.2 and 0.5. This has been verified by simulations[8]. A consequence of this is that we expect Prandtl numbers larger than 1 for the trapped electron mode.

References

1. V. Parail, Plasma Phys. Control. Fusion **44**, A63 (2002).
2. N. Mattor and P.H. Diamond, *Phys. Fluids* **31**, 1180 (1988).
3. P. de Vries et. al. These proceedings.
4. K. Crombe et. al. Phys. Rev. Lett. **95**, 155003-1 (2005).
5. T.Tala et. al. Proc. 32nd EPS Conference on Plasma Phys. Tarragona, June 27 – July 1 2005, ECA Vol **29C**, P-4-044 (2005).
6. J. Weiland and H. Nordman Proc. Theory of Fusion Plasmas, Varenna Lausanne workshop, Chexbres, 1988 p 451.
7. A. Zagorodny and J. Weiland, Phys. Plasmas **6**, 2359 (1999).
8. J. Weiland, A. Eriksson, H. Nordman, P. Strand, T.Tala and P.deVries, 10th ITPA workshop, Princeton, April 2006)
9. J. Weiland, A. Zagorodny, T.A. Davydova and R. Moestam, IAEA.TCM, Trieste March 2-4 2005.
10. T. A. Davydova and J. Weiland, Phys. Plasmas **7**, 243 (2000)