

Thermal Transport Effect on Stability of Drift-Tearing Mode

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1. Introduction

The magnetic island is often observed in high beta tokamak plasmas, which has the characteristics of probabilistic excitation with respect to plasma parameters and sudden growth[1]. For the achievement of high performance in fusion plasma, the control of magnetic island is one of the key issues since once it appears, the confinement degradation occurs. Up to now, the trigger condition and the sudden growth of magnetic island is still unclear from the view point of the conventional approach.

Recently the drift-tearing mode (DTM) is paid attention to, which has the characteristics of nonlinear bifurcation for the solution[2], therefore it might give some hints for the trigger mechanism of the magnetic island. In Ref.[2], the temperature evolution equation is not taken into account, however the thermal transport effects are important for the evaluation of threshold for the subcritical excitation of the magnetic island[3]. So the analysis including thermal transport effects is necessary.

In this paper, the linear analysis of the DTM based on the reduced two-fluid model is performed and thermal transport effects on the DTM are investigated. The linear analysis of the DTM is performed numerically and analytically.

2. Reduced Two-Fluid Model

We consider a plasma of major and minor radii R_0 and a with a toroidal magnetic field B_0 in the cylindrical coordinates (r, θ, z) . The model equations are derived from Braginskii two-fluid equations using reduction scheme with tokamak ordering:

$$\frac{D}{Dt} \nabla_{\perp}^2 \phi = \nabla_{\parallel} j_{\parallel} + \mu \nabla_{\perp}^4 \phi \quad (1)$$

$$\frac{\partial}{\partial t} A = -\nabla_{\parallel} (\phi - \delta p) - \eta_{\parallel} j_{\parallel} + \alpha_T \delta \nabla_{\parallel} T \quad (2)$$

$$\frac{D}{Dt} n + \beta \frac{D}{Dt} p = \beta \delta \nabla_{\parallel} j_{\parallel} + \eta_{\perp} \beta \nabla_{\perp}^2 p \quad (3)$$

$$\frac{3}{2} \frac{D}{Dt} T - \frac{D}{Dt} n = \alpha_T \delta \beta \nabla_{\parallel} j_{\parallel} + \varepsilon^2 \chi_{\parallel} \nabla_{\parallel}^2 T + \chi_{\perp} \nabla_{\perp}^2 T \quad (4)$$

and

$$j_{\parallel} = -\nabla_{\perp}^2 A, \quad p = nT, \quad \alpha_T = 0.71$$

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + [\phi,], \quad \nabla_{\parallel} = \frac{\partial}{\partial z} - [A,], \quad \nabla_{\perp} = \hat{r} \frac{\partial}{\partial r} + \hat{\theta} \frac{1}{r} \frac{\partial}{\partial \theta}$$

Equations are vorticity equation, Ohm's law, continuity equation and electron heat balance equation, respectively. For the simplicity, we assume that ion is cold and neglect the electron inertia, ion parallel velocity and toroidal effects.

The variables $\{\phi, A, n, T\}$ indicate the electrostatic potential, vector potential parallel to the toroidal magnetic field, electron density and electron temperature. The collisional transport coefficients $\{\mu, \eta_{\parallel}, \eta_{\perp}, \chi_{\parallel}, \chi_{\perp}\}$ are ion viscosity, parallel resistivity, diffusivity, electron parallel thermal conductivity and electron perpendicular thermal conductivity. Quantities $\{\delta, \beta\}$ indicate the normalized ion skin depth and plasma beta value. $(\hat{r}, \hat{\theta}, \hat{z})$ are the unit vectors and the Poisson bracket is defined by $[f, g] = \hat{z} \cdot \nabla f \times \nabla g$.

These equations are normalized as $\{\varepsilon v_A t / a \rightarrow t, r/a \rightarrow r, z/R_0 \rightarrow z\}$ where v_A is Alfvén velocity. The Hamiltonian of the system is conserved in the dissipationless limit.

3. Analysis

We pay our attention to the thermal transport parallel to the magnetic field line, because it strongly affects the temperature perturbation and then could affect the total stability of the DTM both in the linear and the nonlinear phases. Then, as the first step, we examine the effect of $\chi_{\parallel} \nabla^2 T$ on the linear stability of DTM.

The variables $f(\mathbf{x}, t)$ are assumed to vary as $f(\mathbf{x}, t) = f_0(r) + \varepsilon f_{m,n}(r) \exp[i\{m\theta + nz - \omega t\}]$ where $\varepsilon = a/R_0$ is the inverse aspect ratio and (m, n) are the poloidal and toroidal mode number. ω is complex frequency of the mode. The finite difference method in the r direction is adopted. $f_{m,n}(r)$ is assumed to satisfy the boundary conditions; $f_{m,n}(0) = 0$ and $f_{m,n}(1) = 0$. The model equations are linearized by the perturbation method. The equilibrium quantities are chosen as $q(r) = (2 - q_0)(r/r_s)^3 + q_0$, $n_0(r) = (\beta_0/\varepsilon)(1 - r^2)$, $T_0(r) = (\beta_0/\varepsilon)(1 - r^2)$ where $\{q(r), n_0(r), T_0(r)\}$ are the safety factor, the normalized equilibrium density and electron temperature. The equilibrium current $j_0(r)$ is evaluated by cylindrical equilibrium. $r_s = 0.6$ is the location of the rational surface and q_0 is the arbitrary value of safety factor at $r = 0$. The inverse aspect ratio is set to be $\varepsilon = 0.2$ and $\beta_0 = \beta / (1 - r_s^2)^2$ which gives the value of central beta.

To analyze thermal transport parallel to the magnetic field line on DTM, parameters $\{\Delta', \chi_{\parallel}, \omega_*\}$ dependence of the growth rate $\gamma = \text{Im}(\omega)$ are investigated. Δ' is the jump of the logarithmic derivative of the perturbed magnetic field, ω_* is the diamagnetic frequency and $\omega_* = \omega_{*n} + \omega_{*T}$

with $\omega_{*n} = -\delta k_{\perp} n_0'$, $\omega_{*T} = -(1 + \alpha_T)\delta k_{\perp} T_0'$ where $k_{\perp} = m/r$ is the wave number perpendicular to the magnetic field. The prime indicates the radial derivative.

Numerical approach

The linearized model equations are solved as the eigenvalue problem. In this study, the DTM with $(m, n) = (2, 1)$ is analyzed.

Figure 1 shows the Δ' dependence of the growth rate where q_0 is changed to vary Δ' . Three cases are plotted; the tearing mode ($\omega_* = 0$), the DTM (large ω_*) without parallel thermal transport ($\chi_{\parallel} = 0$) and the DTM with parallel thermal transport ($\chi_{\parallel} = 10^3$). It is shown that the DTM without parallel thermal transport is more stable than the tearing mode. On the other hand, the DTM with thermal transports is more unstable than the tearing mode. It should be noted that in this case the unstable region extends into the region of $\Delta' < 0$.

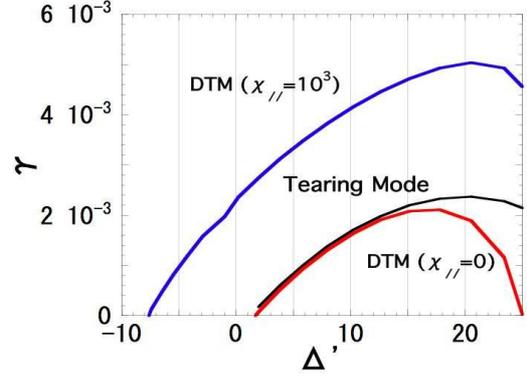


Figure 1: Δ' dependence of growth rate

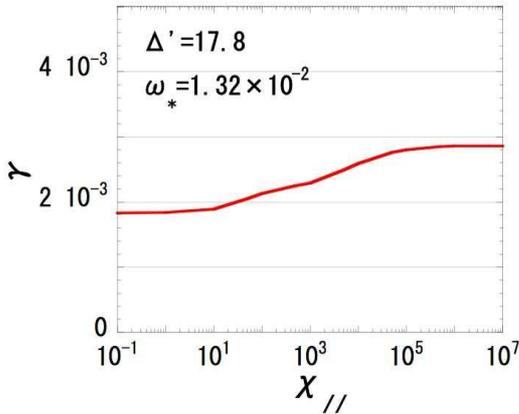


Figure 2: χ_{\parallel} dependence of the growth rate

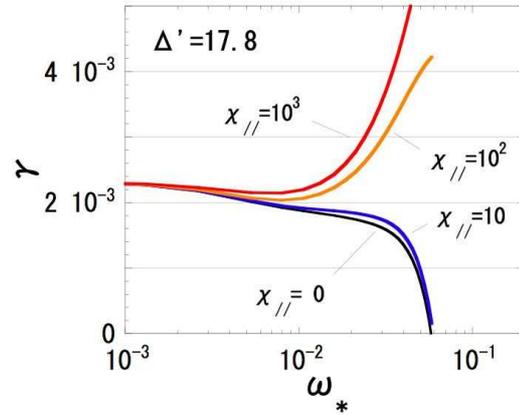


Figure 3: ω_* dependence of the growth rate

Figure 2 shows the dependence of the growth rate on χ_{\parallel} . It is found that χ_{\parallel} destabilizes the DTM and the growth rate tends to converge the constant value in the limit of $\chi_{\parallel} \rightarrow \infty$.

Figure 3 shows the dependence of the growth rate on ω_* . In this figure, both ω_{*n} and ω_{*T} are increased in proportion to ω_* . Four cases are plotted with respect to the values of χ_{\parallel} . It is found that there is the threshold value of χ_{\parallel} above which ω_* contributes to destabilize the DTM in the regime of $\omega_* \gg \gamma$.

Analytical approach

To understand numerical results qualitatively, we derive the analytical solution of DTM by boundary layer theory (local theory) around rational surface. We apply the following simplification; neglecting finite β effects, the radial gradient of equilibrium current j_0' and transport coefficients except χ_{\parallel} in the boundary layer.

The dispersion relations of DTM are obtained in two limits:

$$\omega^2(\omega - \omega_*)^3 = i\gamma_0^5, (\chi_{\parallel} \rightarrow 0), \quad \omega^2(\omega - \omega_{*n})^3 = i\gamma_0^5, (\chi_{\parallel} \rightarrow \infty) \quad (5)$$

where γ_0 is the growth rate of the tearing mode. In the limit of $\chi_{\parallel} \rightarrow 0$, ω_* contributes to stabilize the DTM[4]. The stabilizing effect of ω_{*T} disappears in the limit of $\chi_{\parallel} \rightarrow \infty$ which indicates that the thermal transport parallel to the magnetic field line weakens the stabilizing effect due to the drift frequency. The behavior of the growth rate in Fig. 2 could be understood qualitatively from this effect. The dependence of γ on ω_* (Fig.3) in the regime of $\omega_{*n} \leq \gamma_0$ is also explained by this formula. The behaviour of γ in the regime of $\omega_* \gg \gamma_0$ requires further analysis.

4. Summary

Based on the reduced two-fluid equations, the linear stability analysis of the drift-tearing mode(DTM) with thermal transport effect is performed and the parameter dependence of the growth rate is investigated. It is found that, when $\omega_* \gg \gamma_0$ holds, the DTM is destabilized by the effect of parallel thermal transport $\chi_{\parallel} \nabla^2 T$ in the large limit of χ_{\parallel} and is unstable even in the range of $\Delta' < 0$. It is also found that there is the threshold value of χ_{\parallel} above which the diamagnetic drift frequency acts as destabilization of the DTM.

The detail analysis of DTM with χ_{\parallel} is now going on in terms of destabilization mechanism in $\Delta' < 0$ and the threshold value. These issues will be reported in the conference.

References

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