

Bayesian design of a multichannel interferometer

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The design of plasma diagnostics is a typical task to be resolved along the preparation of fusion experiments. The design process has to meet with requirements like highest accuracy of measurements, high resolution, robustness and extensibility, as well as with constraints such as accessibility or economic restrictions. However, a general approach for diagnostic design is lacking. In this work a probabilistic ansatz based on the consistent framework of Bayesian probability theory will be discussed. First results for the multichannel interferometer at the Wendelstein 7-X stellarator, part of the "start-up" diagnostic set [1], are shown.

Bayesian Experimental Design

The approach presented here is based on decision theory and was proposed by Lindley [2]. An appropriate utility function is chosen first reflecting purpose and costs of an experiment. For the quantification of the utility of a set-up the Kullback-Leibler distance U_{KL} is used as a measure for the information gain from the experiment. It compares the knowledge - or better: ignorance - on a quantity ϕ before a measurement with the knowledge after data D are taken:

$$U_{KL}(D, \eta) = \int P(\phi|D, \eta) \cdot \log \left(\frac{P(\phi|D, \eta)}{P(\phi)} \right) d\phi$$

Probability density functions P are used to encode uncertainties. The conditional probability $P(\phi|D)$ means the probability that ϕ is true given the data D . The utility function U_{KL} depends both on the data D and the design parameters η which are the optimization parameters.

An integration over the range of expected data, where the evidence of the data is represented by the probability density function $P(D|\eta)$, yields the expected utility function EU ,

$$EU(\eta) = \int P(D|\eta) U_{KL}(D, \eta) dD,$$

only a function of the design parameters η . It is a measure of the mean information gain from the data, averaged over the expected data space.

The principle of Bayesian diagnostic design is the maximization of the EU with respect to the design parameters η .

Using Bayes theorem

$$P(\phi|D, \eta) = \frac{P(D|\phi, \eta) \cdot P(\phi)}{P(D|\eta)}$$

the EU is given by

$$EU(\eta) = \iint P(\phi)P(D|\phi, \eta) \cdot \log \left(\frac{P(D|\phi, \eta)}{P(D|\eta)} \right) d\phi dD.$$

This formulation uses the likelihood $P(D|\phi, \eta)$ and a probability density function $P(\phi)$ which reflects the range of interest and weighting for ϕ .

The likelihood can be regarded as a representation of a diagnostics model. It contains the forward calculation, which can be understood as a "virtual" diagnostic, and the error statistics of the data to be measured. The expected utility is now a function of the design parameters η only and is subject of optimization studies [3].

Multichannel interferometer at W7-X

An interferometer measures the line integrated electron density of a plasma by detecting the phase shift of a probing laser beam. It has been shown that the error statistics of the measurement has crucial impact on the expected utility [4]. For the examples presented here, a constant error (noisy background) is chosen which is in the order of a few percent of typical interferometer data, depending on the actual data value.

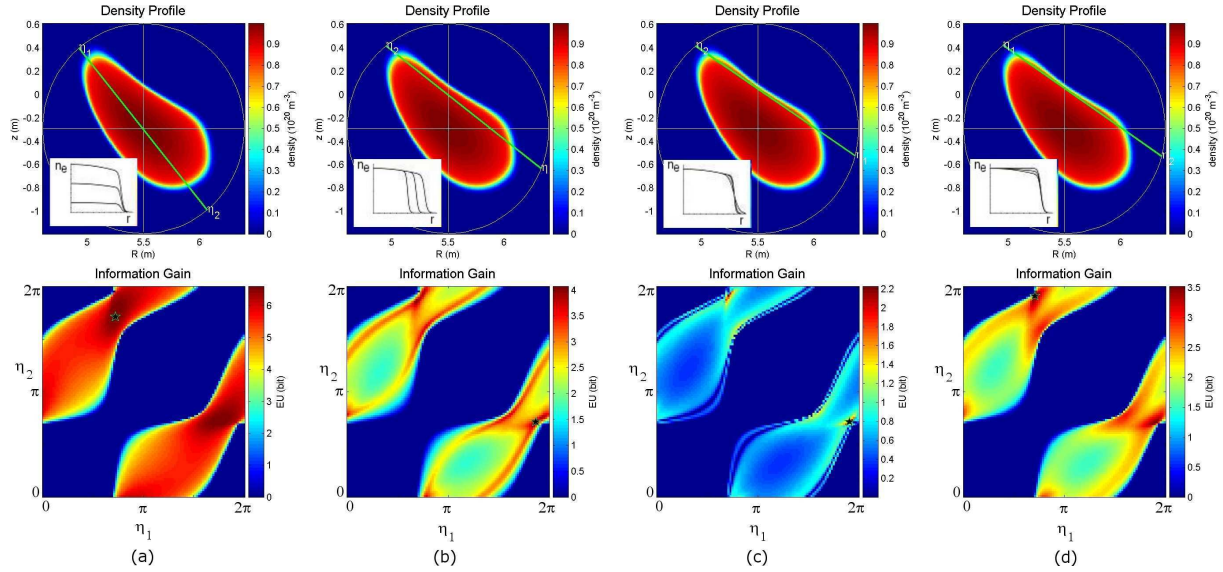


Figure 1: Optimized chords for interferometer at W7-X (toroidal angle 195°, upper row) and expected utility (lower row) for estimation of maximum density (a), gradient position (b), steepness (c) and bulge (d). The data space is generated by a variation of (a) $\phi_1 = 0 \dots 5 \times 10^{20} \text{m}^{-3}$, (b) $\phi_2 = 0.6 \dots 0.95$, (c) $\phi_3 = 1 \dots 30$ and (d) $\phi_4 = -1 \dots 0$. The insets in the upper row show the corresponding density profile variation where the maximum ordinate is $n_e = 1 \times 10^{20} \text{m}^{-3}$ and the abscissas are effective radii (r_{eff}/a).

For the creation of "virtual" data, the parametrized density function

$$n(r) = \phi_1 \cdot \frac{1 + \phi_4 \cdot (r/r_{max})^2}{1 + \left(\frac{(r/r_{max})^2}{\phi_2}\right)^{\phi_3}}$$

is used. The parameters $\phi_1 \dots \phi_4$ represent the maximum density, position of the edge gradient, steepness and bulge of the density distribution.

Figure 1 shows the design results for four single beams, optimized to estimate the parameters of interest $\phi_1 - \phi_4$ independent of the others, respectively. The result is displayed in figures of two angles η_1 and η_2 (lower row) which represent the starting and end point of a chord on a circumventing circle around the plasma (upper row).

The results indicate the different impact of shaping. The optimal beamline represents the maximum signal-to-noise ratio (SNR) chord for the respective parameter. For the estimation of maximum density (Fig. 1(a)) a beam traversing the plasma center yields best SNR. Since the effects of the other parameters are most distinct at the plasma edge, the resulting reconstruction has maximum information gain for sightlines traversing the edge region.

The result also shows a similar beamline for the estimation of steepness and bulge of the density distribution (1 (c) and (d)). This is different if more than one beamline is optimized in one step. Figure 2 shows the result of the combined optimization of two beamlines. Optimization target was the estimation of the position of the largest gradient and steepness of

the density distribution. Both parameters are not independent of each other, the steepness of the density decay is best measured in the region of the largest gradient. The combined optimization led to a slightly different result compared with the single beam optimization.

To analyze correlations between the parameters of fit functions, parametrized density functions will be replaced by distributions from predictive transport calculations [5]. This also offers the possibility to design with respect to derived quantities like the radial electric field as a design criterion.

Additional design criteria are provided by technical boundary conditions like restricted access to the plasma vessel. Fig. 3 indicates accessible chords in figures of the parametrization

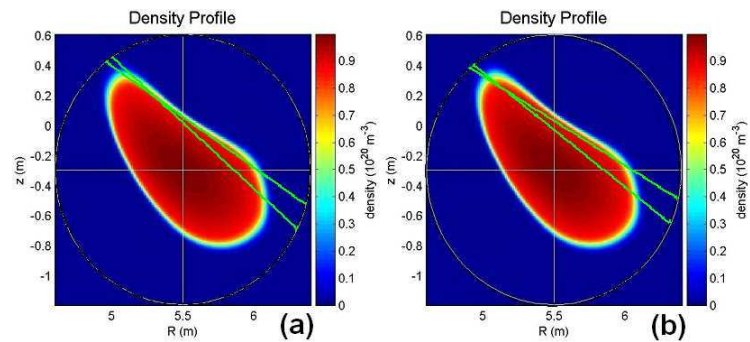


Figure 2: Optimization of two independent beamlines (a) and combined design (b). Optimization target: estimation of the position of the largest density gradient and the steepness of the density distribution.

chosen.

The effect of the technical restrictions can be quantified and compared to ideal access. A translation of the expected utility to measurement uncertainties is straightforward but depends on the forward function of the virtual instrument.

Conclusion and outlook

Bayesian diagnostics design is applied on plasma diagnostics. The design allows for a quantification of design considerations and estimates for their robustness. The reconstruction of density profiles by means of a multichannel infrared interferometer at Wendelstein 7-X shows how measuring capabilities can be detected and complicated entanglements of measurement and geometry revealed. The impact of technical boundary conditions can be quantified as well as the information gain by the inclusion of additional chords.

The combined optimization of many beamlines will be extended to four and eight chords. To avoid restrictions given by the class of parametrized functions, predictive calculations for transport modeling will be used to estimate expected density distributions.

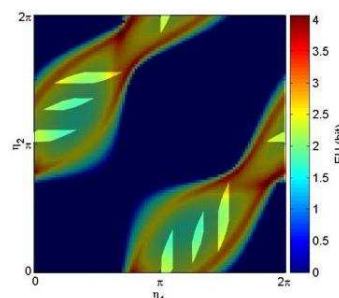


Figure 3: Restricted access (lightened areas) to the plasma due to port system in EU plot for the gradient position case.

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