

Spatiotemporal investigation of nonlinear coupling and energy transfers in drift wave turbulence

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Several recent studies have shown that intermittent coherent structures are a common feature of edge magnetized plasmas [1]. Such structures, large compared to typical turbulent scales, can significantly contribute to the high cross-field particle and energy transport in fusion experiments. In this contribution we will focus on the particular issue of energy transfers in drift wave turbulence. Since a net transfer of energy must be accompanied by a finite phase coherence between the involved structures, a bicoherence analysis, which provides a direct measurement of nonlinear phase coupling, is often used to characterize it [2, 3]. Although the physical process happens in k and not in ω space [4], experimental diagnostics usually suffer from a very limited spatial resolution. Thus, frequency-based bicoherence techniques are generally performed along with attempts to connect k and ω representations [2]. In this contribution we will present results obtained from a direct wavenumber-based bicoherence analysis, performed on measurements from an azimuthal array of 64 probes, in the linear magnetized helicon device VINETA [5]. In order to refine the description of spatiotemporal intermittent behaviour, a wavelet bicoherence [6] is used in both k and ω space. The autobicoherence is a normalized function, which is close to unity when three waves satisfy the resonance condition $\omega_1 + \omega_2 = \omega$, $k_1 + k_2 = k$, and $\Phi_1 + \Phi_2 = \Phi + \text{const}$. The k -autobicoherence reads:

$$[b^W(a_1, a_2)]^2 = \frac{|B^W(a_1, a_2)|^2}{[\int |W_s(a_1, X)W_s(a_2, \tau)|^2 dX][\int |W_s(a, X)|^2 dX]} , \quad (1)$$

where B^W is the wavelet bispectrum defined as:

$$B^W(a_1, a_2) = \int W_s^*(a, X)W_s(a_1, X)W_s(a_2, X)dX . \quad (2)$$

In these equations, $W_s(a, X)$ is the spatial wavelet transform of the signal $s(x)$, and a , a_1 and a_2 are wavelet scale lengths, which satisfy the relation $1/a = 1/a_1 + 1/a_2$. In the present analysis, a Morlet wavelet is used.

In the following we focus on the analysis of the weakly turbulent state depicted in Fig. 1. The spatiotemporal plot reveals irregular structures, non-periodic in time. From the wavelet k -power spectrum, various mode numbers are found to co-exist or alternate in time. The dynamics is dominated by mode numbers $m = 4-6$ (Fig. 2). Averaging the wavelet ω -spectrum over the

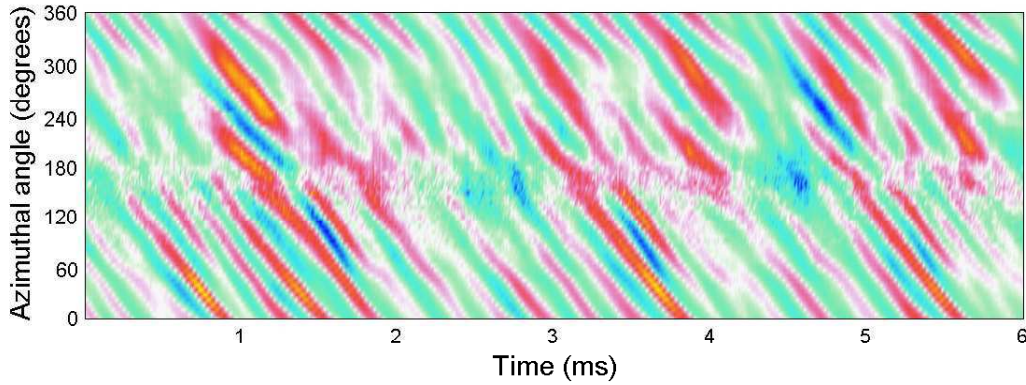


Figure 1: Spatiotemporal density fluctuations of drift waves turbulence, measured with an azimuthal 64 probe array. Discharge parameters are $B=69$ mT, $P_{Ar}=0.3$ Pa, $P_{RF}=2.6$ kW.

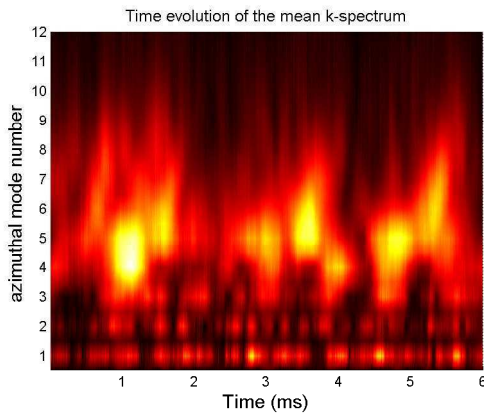


Figure 2: Temporal evolution of the mode number for the regime depicted in Fig 1.

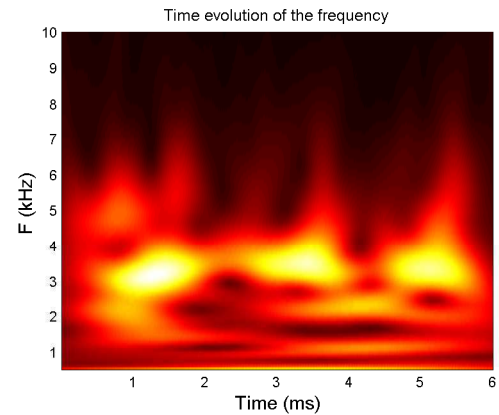


Figure 3: Temporal evolution of the frequency for the regime depicted in Fig. 1.

64 probes, a similar time evolution is found in the frequency domain, with a main frequency in the range 3-4 kHz (Fig. 3). Since several modes and frequencies exist simultaneously, the bicoherence will be used in order to detect whether phase coupling occurs between wave triplets.

Working with the whole data set leads to an average bicoherence, which is unlikely to give much information regarding the turbulent state under investigation (see e.g. Ref [6]). The total k -bicoherence is obtained by integrating the autobicoherence over all scales at each time instant. Its numerical value is not fundamental, since it depends on the chosen calculation grid.

Fig. 4 clearly shows strong temporal variations of the total bicoherence. We notice that the locations of the maxima and minima of the total bicoherence follow an evolution very similar to the temporal evolution of the frequency, Fig. 3, and to a less extent to the evolution of the mode number, Fig. 2. The dashed line in Fig. 4 represents the total statistical noise due to the wavelet decomposition. It is calculated using Eq.(7) of Ref. [6], which is an upper bound. Thus, a larger

bicoherence level can be regarded as a reliable signature of phase coupling.

In the following we focus on a smaller time window from $t = 1$ to 2 ms in order to investigate how the bicoherence level is increased at this time interval (similar results are found as well, e.g., in the interval 3.4-4 ms). In this interval, a first analysis based on the averaged f - and k - spectra shows the existence of phase coupling in both spectra (Fig. 5). More precisely, the k -bicoherence shows couplings between $m = 1$ and $m = 2 - 6$ (horizontal structure), between $m = 6$ and $m = 1 - 3$ (vertical structure), and between $m = 2$ and $m = 3$. The f -bicoherence shows couplings between 4kHz and 2, 3, 6, and 8kHz, and also between 2kHz and lower frequencies. However, these figures don't take into account the statistical noise, leading to an overestimation of the bicoherence level, especially at low scales as it will be shown later.

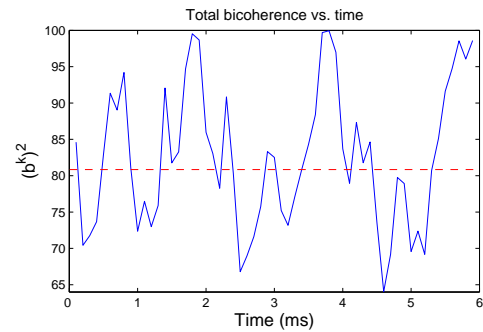


Figure 4: Time evolution of the total k -bicoherence. The dashed line indicates the maximum noise level.

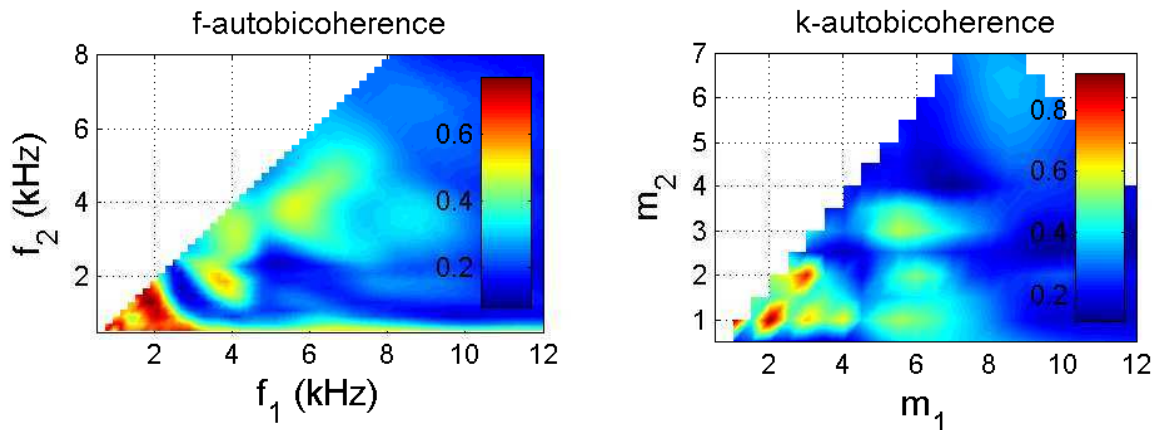


Figure 5: f - and k -autobicoherence for the time interval $t = 1 - 2$ ms.

To get insight into the dynamics of the coupling, it is necessary to further decompose the time interval. In the following, the k -bicoherence is calculated at each time instant, and averaged over short time intervals. The summed bicoherence at three selected intervals is plotted in Fig. 6, along with the corresponding wavenumber spectra. Although the maximum noise level is very high, especially at small wavenumbers, several bicoherence peaks can be seen exceeding the noise level. Looking at the first time interval ($t_1 = 1.3 - 1.45$ ms), one notices that while the wavenumber spectrum is dominated by mode numbers 4, 5, and 6, the main bicoherence level is observed at wavenumbers corresponding to $m=1, 2$, and 3. Although the bicoherence level

does not look significant with respect to the noise level, especially in the case of $m=1$, it is noticeable that the amplitudes of these modes are increased in the wavenumber spectrum when calculated in the following time interval ($t_2 = 1.45 - 1.6$ ms). Considering the whole figure, it seems reasonable to conclude that small wavenumbers grow due to an energy transfer from higher mode numbers: $m = 1$ is fed by the interactions between $m = 4$ and $m = 5$, and/or between $m = 5$ and $m = 6$, $m = 2$ by the interactions between $m = 4$ and $m = 6$, according to the phase coupling criterion $k_1 + k_2 = k$. While the spectra broaden, the energy is diffused into a wide range of scales, which is a characteristic of a transition to turbulence.

In conclusion, it is worth noting that these results are in very good agreement with the hypothesis of an inverse cascade of energy in drift wave turbulence, predicted by theoretical work [7]. Although the present paper focuses on the k -bicoherence, preliminary results show a good agreement with frequency-based analysis, as well as with a complementary analysis based on 3D fluid simulations. Direct cascading will be investigated in further steps.

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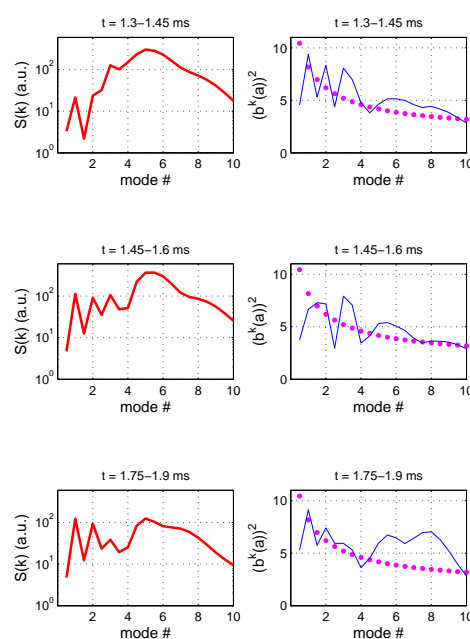


Figure 6: k -power spectra (left column) and summed bicoherence (right column) at three selected time intervals. Dots indicate the maximum statistical noise level.