Assessment of physical models for the scaling of global confinement energy in W7-AS

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Introduction

A strategy to explore scalability toward reactor-relevant plasmas and the validity of physical models consists of an assessment of performance deviations from empirical scaling laws, such as ISS04 (see [1]). Deviations are found to occur both in general trends and single discharges which do not comply with the general plasma performance in similar experiments, as for example the density variation of ECRH discharges in W7-AS at high temperatures (where neoclassical transport determines the core confinement). On the other hand, H-mode discharges and HDH plasmas are examples which are found to scale as suggested by ISS04.

A particular issue for reactor assessment is the confinement dependence in stellarators on plasma beta. ISS04 indicates a small degradation of global stellarator confinement with beta ($\tau_E^{ISS04} \propto \beta^{-0.17}$) with the caveat that high-beta data suffer from clustering in device-dependent regions of the collisionality. Consequently subsets of data should be tested against physical models. In a previous approach, a subset of W7-AS $\iota \sim 1/3$ data (see circles in Fig. 1) documented in the international stellarator data base used for ISS95 [2] has been analyzed [3, 4]. Due to the machine conditions at that time the data base was mainly of low-beta character. Among four models derived according to the Connor-Taylor invariance principle [5], the collisional low-beta model was identified to be the most probable one. Furthermore, the data-analysis method used was capable of predicting the outcome of single variable scans not contained in the data base. Since then, several experimental campaigns in W7-AS have explored the high-beta regime. Those data are submitted to the International Stellarator Confinement Data Base (ISCDB) [6]. In this paper we present the results for a model comparison of six Connor-Taylor models where we re-examined the old data subset of the ISS95 data base and a high-beta subset of the newly added data. Bayesian probability theory was employed for measuring the model probability.

Connor-Taylor models

We choose to scale over the confinement energy, since the diamagnetic energy $W_{dia}$ is a measured quantity. Considered are W7-AS data only (constant major radius), so the control parameters are the electron density $n$, toroidal magnetic field $B$, absorbed power $P$ and the...
Table 1: Results for the model comparison. The last column shows the number of terms in the respective models.

<table>
<thead>
<tr>
<th>CT-model</th>
<th>$\xi_1$</th>
<th>$\xi_2$</th>
<th>$\xi_3$</th>
<th>low-beta</th>
<th>high-beta</th>
<th>$N_{term}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Collisionless low-beta</td>
<td>x</td>
<td>0</td>
<td>0</td>
<td>$10^{-14}$</td>
<td>$10^{-24}$</td>
<td>1</td>
</tr>
<tr>
<td>Collisional low-beta</td>
<td>x</td>
<td>y</td>
<td>0</td>
<td>0.998</td>
<td>$10^{-19}$</td>
<td>2</td>
</tr>
<tr>
<td>Collisionless high-beta</td>
<td>x</td>
<td>0</td>
<td>z</td>
<td>$10^{-3}$</td>
<td>$10^{-23}$</td>
<td>2</td>
</tr>
<tr>
<td>Collisional high-beta</td>
<td>x</td>
<td>y</td>
<td>z</td>
<td>$10^{-4}$</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>Ideal fluid</td>
<td>x</td>
<td>0</td>
<td>1-x/2</td>
<td>$10^{-75}$</td>
<td>$10^{-20}$</td>
<td>1</td>
</tr>
<tr>
<td>Resistive fluid</td>
<td>x</td>
<td>y</td>
<td>1-x/2+y</td>
<td>$10^{-29}$</td>
<td>$10^{-15}$</td>
<td>2</td>
</tr>
</tbody>
</table>

effective minor radius $a$:

$$W^{theo} \propto n^{a_0} B^{a_0} P^{a_0} d^{a_0}.$$  \hspace{1cm} (1)

Following Connor and Taylor’s invariance principle [5], constraints on the exponents of the above scaling law can be derived by examining the linear transformation behavior of basic model equations. Here, invariance scalings of the Fokker-Planck/Vlasov equation or Maxwell equations are considered yielding the following scaling law ansatz

$$W^{theo} \propto n a^4 B^2 \left( \frac{P}{n a^4 B^2} \right)^{\xi_1} \left( \frac{a^3 B^4}{n} \right)^{\xi_2} \left( \frac{1}{n a^2} \right)^{\xi_3} = cf(\vec{\xi}).$$  \hspace{1cm} (2)

$f(\vec{\xi})$ comprises the terms with the scaling exponents $\vec{\xi} = (\xi_1, \ldots)$. The assignment of the scaling exponents to the specific model is shown in table 1. Note that the number of multiplicative terms (i.e. scaling exponents) varies between one and three, e.g. in the simplest case of the collisionless low-beta model there is only one scaling exponent left. Obviously the more parameters a model offers the better will be the fit to the data with respect to least squares deviation. However, a model with less parameters is more probable if it intrinsically describes the data better without starting to fit noise. This principle, Occam’s razor, is intrinsic to Bayesian probability theory.

Still one can exploit the invariance principle one step further and overcome a known shortcoming of common scaling laws, i.e. the failure to mimic the saturation of confinement with $n$ or $P$. Since a sum of scaling terms is used, Eq. (2) still possesses the same linear transformation properties and we expand the scaling law over several functions $f(\vec{\xi}_k)$ resulting in scaling
Figure 1: Examined W7-AS confinement data as a function of beta and collisionality (low-beta subset: circle, high-beta subset: squares, additional data in ISCDB: plus signs).

approaches like

$$W^{theo} = \sum_{k=1}^{E} c_k f\left(\xi_k\right).$$

(3)

Qualifying of data and results

Since the uncertainty of the data enters directly the equations being evaluated in model comparison (see e.g. [4]), it is necessary to weight the data by their uncertainties [7]. A further issue is the large variation of the plasma energy as a function of the rotational transform [8]. This necessitates identifying regions in $\bar{\iota}$ with small changes in the absolute value of $W_{\text{dia}}$ (a variation of 10% of the total value was considered as tolerable). For the low-beta case, W7-AS shots with $\bar{\iota}$ between 0.33 and 0.35 were chosen. All data sets entering ISS95 for low $\bar{\iota}$ happened to occupy this range resulting in $N = 153$ data with a broad distribution in the settings of the other control parameters $n$, $B$, $P$ and $a$. In order to test the procedure in the high-beta range as well, the shot files of W7-AS were subjected to a high-beta survey. The search criteria were to consider the shot files covering the high-beta campaigns with shot number $> 50000$, a coil correction current less than -1kA, an absolute value of the toroidal field less than 1.5T without magnetic ramp, an absolute value of the total plasma current less than 500 A and neutral beam injection power larger than 2 MW. The shots satisfying these criteria were plotted and examined with respect to stationarity, i.e. within $\pm 50$ms around shot time the variation of the control parameters was less than 10%. Although the influence of $\bar{\iota}$ is expected to be less for the high-beta case, a range
for t between 0.45 and 0.49 was identified containing N=86 high-beta data with still a modest variation of $W_{dia}$ according to [8].

The two subsets are shown in Fig. 1 together with the complete data set of W7-AS contained in the ISCDB. The result of the model comparison is given in the second- and third-to-last columns of table 1. The figures derive from a summation over all relevant expansion orders in Eq. (3). As can be seen, for the two different subsets in the low- and high-beta regime, two different physical models, i.e. collisional low-beta and collisional high-beta, respectively, are the most probable ones.

**Conclusion**

The results of the model comparison indicate that the high- and low-beta data sets obey different physical models. The existent beta variation in W7-AS is sufficient to change the confinement model from a low-beta to a high-beta (inclusion of Ampere’s law) model. This result confirms quantitatively findings of the ISS04 study [1] that different configurations and parameter regimes in stellarators can hardly be expressed by a simple unified scaling law. To obtain further insight, predictive transport calculations are being carried out to account for physical differences in the subsets [9].

**Acknowledgment**


**References**

[9] Y. Turkin et al., this conference