

Positively charged dust in Tokamaks

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Introduction

In this paper we consider the effect of electron emission mechanisms on the floating potential of a dust grain, which is particularly relevant for dust in tokamaks. Imagine a spherical dust particle which is at its floating potential in a plasma, when we suddenly turn on a flux of emitted electrons, Γ_{em} . In order to understand what happens, we have to consider the relative magnitudes of the plasma electron flux, Γ_e , and Γ_{em} . We assume singly charged ions.

Limiting Cases

If $\Gamma_{em} \ll \Gamma_e$, the floating potential remains negative. The emitted electrons do not significantly affect the electric field structure around the grain, and are accelerated into the plasma. The flux balance at the surface becomes $(1 - \delta)\Gamma_e = \Gamma_i$, where $\delta = \Gamma_{em}/\Gamma_e$.

The planar case has already been discussed [1].

We can modify the OML theory to include this effect, resulting in the following expression

$$(1 - \delta) \exp(V_d) = \left(\frac{\theta}{\mu}\right)^{\frac{1}{2}} \left(1 - \frac{V_d}{\theta}\right), \quad (1)$$

where $V_d = e\phi_d/(k_B T_e)$, ϕ_d is the dust surface potential, and $\theta = T_i/T_e$ and $\mu = m_i/m_e$. Figure 1 shows V_d as a function of δ for different values of θ . Electron emission decreases the magnitude of V_d with increasing emission current. The model breaks down as δ approaches unity, predicting negative V_d (positive ϕ_d), which is inconsistent with the normal OML assumption that the electron density obeys the Boltzmann law.

If $\Gamma_{em} \gg \Gamma_e$, the dust grain charges positive. Each emitted electron creates a positive charge on the grain, and a potential barrier forms. Once the barrier is equal in magnitude the energy of the emitted electrons, they are trapped, and a steady state can be reached. Thus we can estimate the normalised potential by $V_d = V_{OML} - eU_{em}/(k_B T_e)$, where V_{OML} is the OML potential excluding electron emission and U_{em} is the energy of emitted electrons in eV.

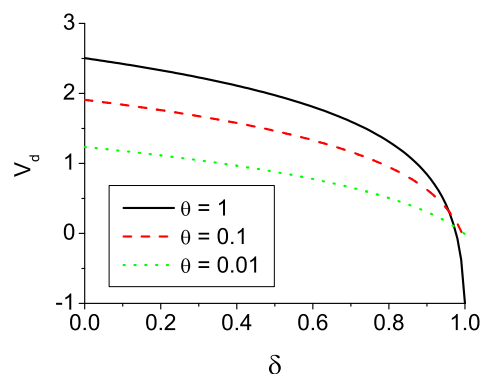


Figure 1: Floating potential for $\delta < 1$.

Intermediate Case

Whilst it is not strictly accurate, it is convenient to assume that the emitted electrons have a well defined temperature. As we have two populations of electrons, we have two Debye lengths, a plasma electron Debye length λ_D , and an emitted electron Debye length λ_{Dem} . We decide which of the two populations of electrons is responsible for the shielding by comparing the magnitudes. If $\lambda_{Dem} \ll \lambda_D$, the emitted electrons perform almost all the shielding, and are trapped near the dust grain. This means a potential well is formed, which has been observed in simulations for thermionic emission [2]. The converse situation would imply the emitted electrons all escape from the field structure.

For our simple model we assume $\lambda_{Dem} \ll \lambda_D$, implying that a potential well exists (figure 2). The floating condition is $I_e(a) + I_i(a) + I_{em}^{in}(a) - I_{em}^{out}(a) = 0$, where $I_{em}^{in}(a)$ and $I_{em}^{out}(a)$ correspond to the inward (returning) and outward currents of emitted electrons respectively, and a is the grain radius. We assume there is a minimum in the potential profile at $r = b$, where $b > a$. Emitted electrons moving away from the grain which reach b escape. Thus $I_{em}^{out}(a) - I_{em}^{in}(a) = I_{em}^{out}(b)$. The emitted electrons are assumed to obey the Boltzmann law between a and b as they are in a repulsive potential, and therefore $I_{em}^{out}(b) = \gamma \exp(\Delta V \sigma) I_{em}^{out}(a)$, where $\gamma = b^2/a^2$, $\sigma = T_e/T_{em}$ and $\Delta V = (\phi(b) - \phi(a))/(k_B T_e)$ (hence $\Delta\phi$ is positive, whereas ΔV is negative). Using the relation $I_{em}^{out}(a) = \delta I_e(a)$ and combining with the previous equations, we find $I_i(a) = \delta \gamma \exp(\Delta V \sigma) I_e(a) - 1$.

As the emitted electrons usually have a lower temperature than the primary electrons, the overall potential is still negative (figure 2). Thus we can safely assume that all ions that get to b have enough energy to get to a . We assume that all are collected, hence $I_i(a) = I_i(b)$.

The plasma electrons are in an attractive potential for $a < r < b$. We use modified OML considerations for this region as electrons come not from infinity, but a finite distance [3]. For a Maxwellian distribution of electrons

$$I_e(a) = I_e(b) \left[1 - \left(1 - \frac{1}{\gamma} \right) \exp \left(\frac{\Delta V}{\gamma - 1} \right) \right]. \quad (2)$$

Using equation 2, the fact that $I_i(a) = I_i(b)$, with the previous equations we find

$$\frac{I_i(b)}{I_e(b)} = \left[1 - \left(1 - \frac{1}{\gamma} \right) \exp \left(\frac{\Delta V}{\gamma - 1} \right) \right] (\delta \gamma \exp(\Delta V \sigma) - 1). \quad (3)$$

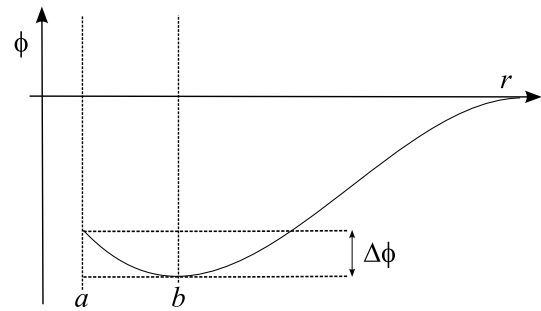


Figure 2: A positive grain with a negative surface potential, having a potential minimum at a distance $r = b$.

To solve for ΔV we need to make an assumption about the potential at b , and the relative magnitude of a , λ_D and λ_{Dem} . We take $\lambda_{Dem} \ll \lambda_D$ for simplicity.

Assuming $a \gg \lambda_D$, we have planar geometry, and can set $a \approx b$ ($\gamma = 1$). We use the Bohm condition to find $I_e(b)$ and a Maxwellian one way flux to find $I_e(b)$ which yields

$$\Delta V = \frac{1}{\sigma} \ln \left[\frac{1}{\delta} \left(1 - \sqrt{\frac{2\pi(1+\theta)}{\mu}} \right) \right]. \quad (4)$$

To find the potential relative to the plasma, we require the potential at b .

Since we have assumed $\lambda_{Dem} \ll \lambda_D$, the emitted electrons dominate the shielding process close to the dust grain forming a sheath of negative charge around the grain of width $\approx \lambda_{Dem}$. Most of the emitted electrons are trapped. Thus the plasma sees a neutral object of radius $a + \lambda_{Dem}$, and proceeds to shield it in the normal way. An estimate for $V(b)$ is therefore the OML potential, which for $\theta = 1$ is 2.5.

Figure 3 shows the overall potential $V_d = V(a)$ plotted for various values of $\sigma = T_e/T_{em}$ and δ . The potential difference increases as σ decreases, due

to there being more kinetic energy available to the emitted electrons relative to the plasma population. Similarly, it increases for increasing δ , as a result of there being more secondary electrons produced. However, one of the most striking things to notice is that as σ increases, ΔV very quickly becomes small. In a typical tokamak plasma, $T_e \approx 100$ eV, $T_{em} \approx 5$ eV, therefore $\sigma = 20$. This part of the model deals with values of δ above 1, but not *much* greater, so we take a value $\delta = 10$, and this results in $|\Delta V| \approx 0.12$, a small potential difference relative to $V(b)$.

Note that if $\lambda_{Dem} \approx \lambda_D$, the point b will be at a potential less in magnitude than the OML floating potential. It is difficult to estimate this potential.

If $a \ll \lambda_D$ we have to consider the spherical geometry of the problem. The inward electron flux at b can be written in terms of the potential at b using the Boltzmann relation. For simplicity, we use OML to estimate the ion flux from infinity to b , and we find $\Gamma_i(b) = n_0(1 - V(b)/\theta)\bar{v}_i$. Inserting the electron and ion fluxes into equation 3 results in

$$(1 + V(b)\theta^{-1}) \exp(V(b)) \sqrt{\frac{\theta}{\mu}} = \left[1 - \left(1 - \frac{1}{\gamma} \right) \exp\left(\frac{\Delta V}{\gamma - 1}\right) \right] (1 - \gamma\delta \exp(\Delta V\sigma)). \quad (5)$$

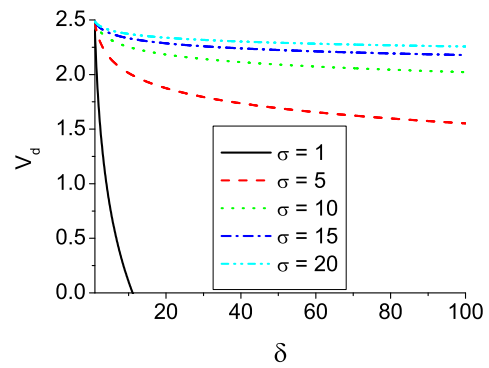


Figure 3: Floating potential for $\delta > 1$ for $a \gg \lambda_d$.

$V(b)$ and $\gamma (= b^2/a^2)$ both need to be specified before we can solve for ΔV . γ is probably related to the radius of the dust grain, as it is determined by λ_{Dem} . Hence if we assume $\lambda_{Dem} \ll \lambda_D$, we can assume that the plasma sees a neutral particle of radius $a + \lambda_{Dem}$. For this analysis, we will simply choose a value of γ , and proceed. Once we have chosen this parameter, we can test various values of $V(b)$ and solve for ΔV to see if the results are consistent with the assumptions. For example, we have assumed a negative potential, so we must satisfy the criterion $V(a) > 0$ (V has the opposite sign to ϕ).

We again take $\sigma = 20$, and $\delta = 10$. Figure 4 shows $V(a)$ against $V(b)$ for various values of γ . The equation yields positive values of $V(a)$ for only selected values of $V(b)$, and that as γ increases in magnitude, $V(a)$ becomes more negative. The curve is singular when $V(b)$ is equal to the OML potential, in this case 2.5. Due to computational difficulties, results for values of γ greater than 13 cannot be computed, however, it seems clear from the graph that if the particle is very small, the assumption that the grain is at a negative potential (positive V) will be incorrect. If we approximate b as $a + \lambda_{Dem}$, then $\gamma = (1 + a/\lambda_{Dem})^2$. Thus $\gamma = 13$ corresponds to $a/\lambda_{Dem} = 0.38$, and we wish to find out about far smaller grain sizes than this. In experiments, the dust found is $< 0.1\lambda_D$, hence it is likely to be $< 0.01\lambda_{Dem}$ given our assumption $\lambda_{Dem} \ll \lambda_D$.

If we change δ by a large amount, we find that the difference between the curves is actually quite small, suggesting that the geometry is far more important than the magnitude of the current of secondaries.

Which point on the curves in figure 4 we select is the subject of further work.

References

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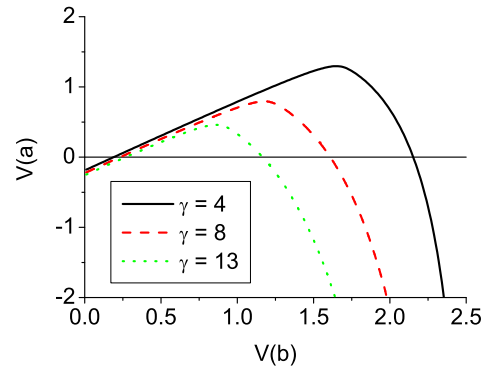


Figure 4: Floating potential $V(a)$ against $V(b)$ with $\delta > 1$ and $a \ll \lambda_d$ for fusion parameters.