Dynamics of self-gravitating dust clouds in astrophysical plasmas

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Abstract

Due to the gravitational force, clouds of dust and gas in the interstellar medium can contract and form stars and planet systems. Here we show that if the dust grains are electrically charged then the self-gravitation can be balanced by the "electrostatic pressure" and the collapse can be halted. The dynamics of self-gravitating dust clouds are investigated with hydrodynamic simulations.

We present a scenario which could lead to the formation of planets directly from interstellar dust, where the dust cloud has a mass of a satellite or of a small planet [1, 2]. If the dust particles are immersed in an ionized gas, they will be charged negative due to the attachment of electrons onto the grain surface [3]. The self-gravity in the dust leads to an instability [4, 5, 6] where the dust contracts into separate dust clouds in space. A large part of the electrons are absorbed by the dust grains so that there will be an overweight of free positively charged ions compared to free electrons. A negative potential is then set up in the cloud that balances the ion pressure and prevents the ions from escaping the dust cloud. The Coulomb force on the dust due to this potential behaves like an effective pressure force that it balances the gravitational force and halts the collapse of the dust cloud. The geometry of a dust cloud is illustrated in Fig. 1, where the gravitational force acting on the dust grains is balanced by the electric force.

In a fluid description, the dynamics of a spherically symmetric dust cloud can be described by the dimensionless continuity and momentum equations [2]

\[
\frac{\partial N_d}{\partial t} + \frac{1}{r^2} \frac{\partial (r^2 N_d v_r)}{\partial r} = 0,
\]

(1)

and

\[
\frac{\partial v_r}{\partial t} + v_r \frac{\partial v_r}{\partial r} = \frac{1}{N_d} \frac{\partial P}{\partial r} - \frac{\partial \psi}{\partial r},
\]

(2)
respectively. Here \( N_d \) is the dust particle density, \( v_r \) is the radial velocity of the dust particles, and \( r = \sqrt{x^2 + y^2 + z^2} \) is the radial coordinate. The gravitational potential \( \psi \) is given by Poisson’s equation

\[
\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial \psi}{\partial r}\right) = N_d, \tag{3}
\]

where the right-hand side represents the mass density of the dust cloud which acts as a source for the gravitational potential. The “pressure” term \( P \) in Eq. (2) arises from the electrostatic potential in the dust cloud which balances the ion pressure. In the large-scale dust cloud, there will be a quasi-neutral equilibrium \( N_i = Z_d N_d + N_e \) where the subscripts “i” and “e” denotes ions and electrons, respectively. The coefficient \( Z_d \) represents the number of electrons attached to the surface of the dust grain; it decreases with increasing particle number density of the dust grains [7]. The ions and electrons can be assumed to obey a Boltzmann distribution, viz. \( N_e = \exp(\varphi) \) and \( N_i = \exp(-\varphi) \), where \( \varphi \) is the normalized electrostatic potential. We thus have \( Z_d N_d = \exp(-\varphi) - \exp(\varphi) = -2\sinh(\varphi) \). The electric force acting on the dust fluid is related to the electrostatic pressure \( P \) as

\[
F = Z_d N_d \frac{\partial \varphi}{\partial r} = -2\sinh(\varphi) \frac{\partial \varphi}{\partial r} = -\frac{\partial P}{\partial r}, \tag{4}
\]

where \( P = 2[\cosh(\varphi) - 1] \) enters into the first term of the right-hand side of Eq. (2). The electrostatic potential \( \varphi \) is related directly to the background density \( N_d \) through the quasi-neutrality condition mentioned above and the condition that the dust grain should
be in charge equilibrium with its surrounding, so that the net electric current to the dust

grain vanishes. By balancing the ion and electron currents, $I_i + I_e = 0$, one can obtain a

condition which relates the dust charge $Z_d$ to the surface potential of the dust grain and
to the potential of the plasma surrounding the dust grain. Approximate formulas have

been derived by Havnes [7], which relate the electrostatic potential $\phi$ to the dust number
density via the rational function $\phi = (c_1N_d + c_2N_d^2)/(1 + d_1N_d + d_2N_d^2)$. The relation
between $P$ and $N_d$ (via the relation between $\phi$ and $N_d$) constitutes an equation of state
in a similar manner as in thermodynamics. For low dust particle densities, $N_d \ll 1$, the

electrostatic pressure depends on the dust density as $P = c_1^2N_d^\gamma$ with the heat ratio $\gamma = 2$,

while for large dust densities, the pressure will increase more slowly with increasing dust

densities.

Figure 3 displays the distributions of the dust density (left panels) and the gravita-
tional potential (right panels) for different masses $M$ of the dust cloud, where the total
dust mass is obtained as the integral of the dust density over the volume of the dust
cloud, $M = \int_0^R N_dr^2 dr$. Outside the dust cloud ($|r| > R$) the gravitational potential has
the exact solution $\psi = -M/|r| + \text{constant}$. The dust density is largest in the center of the
dust cloud, where the gravitational potential has its minimum. At a radius $r = R \approx 5$
we see that the dust density falls to zero (indicated in the upper left panel). We observe
from Fig. 3 that the density in the central core of the dust cloud becomes more peaked
for larger masses, while the radius of the dust cloud remains almost constant, $R \approx 5$. The
Figure 3: The time-dependent dynamics of a dust cloud with mass $M = 1.95$ (a) and $M = 4.90$ (b), showing the initial and final particle density distribution of the dust cloud (upper panels), the radial dust particle density distribution $N_d$ as a function of time (middle panels) and the value of the particle distribution at the center of the cloud, $N_c = N_d, r=0$, as a function of time (lower panel). The dust cloud with small mass reaches an equilibrium while the one with large mass collapses. (After Ref. [2].)

mass $M = 4.9$ is close to a maximum value, which we will denote the critical mass of the dust cloud, above which we could not find equilibrium solutions. The time-dependent dynamics for a dust cloud of total mass $M = 1.95$ is illustrated in Fig. 4(a). For this case, the dust cloud exhibits damped pulsations, and after some time it reaches a stable equilibrium. The pulsations have a periodicity in time of $T_p \approx 33$ corresponding to $\approx 10^5$ years for the parameters of the interstellar medium and molecular cloud [2]. The collapse of a dust cloud illustrated in Fig. 4(b), where we have taken the mass $M = 4.9$, which is slightly below its critical mass. Here, the dust cloud starts contracting slowly. At $t \approx 9$, the core of the dust cloud starts collapsing and we see a rapid increase of the dust density in the center of the cloud. Due to the inertia of the contracting dust cloud, it collapses even though its mass is slightly below the critical mass.

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