33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, P-2.085 (2006)

Acoustic mode in partially ionized and dusty plasmas

J. Vranjes, S. Poedts

Center for Plasma Astrophysics, K. U. Leuven, Leuven, Belgium

The physics of dust acoustic and ion acoustic modes is discussed in a plasma with magnetized light components [electrons and ions in the case of the dust acoustic (DA) mode, and electrons in the case of the ion acoustic (IA) one], and un-magnetized heavy components (dust grains for the DA mode, and ions for the IA mode). In such situations, the acoustic mode can propagate almost at any angle with respect to the direction of the magnetic field as long as the light particles are able to move mainly along the magnetic lines to preserve quasi-neutrality.

Here we first assume a homogeneous, 4-component, weakly ionized plasma comprising protons, electrons, charged dust grains, and a heavy, immobile background of a neutral fluid. In addition, we have electron and ion currents along the magnetic field lines $\vec{B}_0 = B_0\vec{e}_z$. In a very weakly ionized plasma, i.e, in the limit $\nu_{ie}, \nu_{ei} \ll \nu_{i,e}$ the ion and electron velocities are $v_{i0} \approx eE_0/(m_i\nu_{in})$, $v_{e0} \approx -eE_0/(m_e\nu_{en})$, and $\nu_e = \nu_{ei} + \nu_{en}$, $\nu_i = \nu_{ie} + \nu_{in}$.

Perturbations are of the form $\sim \exp(-i\omega t + ik_x x + ik_z z)$ so that $|\omega - k_z v_{\alpha 0}| \ll \nu_{e,i} \ll \Omega_{e,i}$. In this limit the inertia terms in electron and ion momentum equations are negligible and the two parallel equations become

$$\frac{e}{m_e}\frac{\partial \phi_1}{\partial z} - \frac{\kappa T_e}{m_e n_{e0}}\frac{\partial n_{e1}}{\partial z} - \nu_{en}v_{ez1} - \nu_{ei}(v_{ez1} - v_{iz1}) = 0, \tag{1}$$

$$-\frac{e}{m_i}\frac{\partial \phi_1}{\partial z} - \frac{\kappa T_i}{m_i n_{i0}}\frac{\partial n_{i1}}{\partial z} - \nu_{in}v_{iz1} = 0. \tag{2}$$

From the two continuity equations we have

$$\frac{n_{e1}}{n_{e0}} = \frac{iek_z^2}{m_e\nu_{en}\omega_{e1} + ik_z^2\kappa T_e}\phi_1, \quad \frac{n_{i1}}{n_{i0}} = -\frac{iek_z^2}{m_i\nu_{in}\omega_{i1} + ik_z^2\kappa T_i}\phi_1. \tag{3}$$

Here $\omega_{(i,e)1} = \omega - v_{(i,e)0}k_z$ and the quasi-neutrality is used both in the equilibrium and in the perturbed state, the latter reading $n_{i1} = n_{e1} + Z_d n_{d1}$. The grains are un-magnetized, therefore, there is no preferential direction of the proposed dust acoustic perturbations. For an arbitrary ratio ω/β, where $\beta = 4m_n n_{n0} r_d^2 v_{Tn}/m_d$ is the grain-neutral collision frequency and r_d is the grain radius, the fluid equations for grains may become inappropriate [1] and the grain density perturbation is determined from

$$n_{d1} = \int_{-\infty}^{+\infty} f_{d1} d^3\vec{v}. \tag{4}$$

Here f_{d1} is the perturbed grain distribution obtained from the kinetic equation with the Krook's collisional term $f_{d1} = eZ_d\vec{E}_1(df_{d0}/d\vec{v})/[m_d(\omega - \vec{k}\cdot\vec{v} + i\beta)]$.

Eq. (4) yields

$$\frac{n_{d0}Z_d^2}{n_{i0}\kappa T_d}\left[1-J_p\left(\frac{\omega+i\beta}{kv_{Td}}\right)\right]+\frac{n_{e0}}{n_{i0}}\frac{ik_z^2}{m_e\nu_e\omega_{e1}+ik_z^2\kappa T_e}+\frac{ik_z^2}{m_i\nu_i\omega_{i1}+ik_z^2\kappa T_i}=0,$$

$$J_p\left(\frac{\omega+i\beta}{kv_{Td}}\right)=\frac{1}{(2\pi)^{1/2}}\frac{\omega+i\beta}{kv_{Td}}\int_c\frac{\exp(-\xi^2/2)d\xi}{\frac{\omega+i\beta}{kv_{Td}}-\xi},$$

where $\xi=v/v_{Td}$, and the integration goes along the Landau contour c. Here, we use the expansion $J_p(\chi)\approx 1+1/\chi^2+3/\chi^4+...-i(\chi/2)^{1/2}\chi\exp(-\chi^2/2)$, which is valid for $|\chi|\gg 1$ and $|\mathscr{R}\chi|\gg|\mathscr{I}\chi|$, where $\mathscr{R}$ and $\mathscr{I}$ denote the real and imaginary parts of χ. This yields

$$(\omega+i\beta)^2=-i\frac{n_{d0}Z_d^2k^2}{m_dk_z^2}\frac{\delta_1\delta_2}{n_{e0}\delta_2+n_{i0}\delta_1}-\left(\frac{\pi}{2}\right)^{1/2}$$
$$\times\frac{(\omega+i\beta)^3}{kv_{Td}}\frac{Z_d^2n_{d0}}{\kappa T_dk_z^2}\frac{\delta_1\delta_2}{n_{e0}\delta_2+n_{i0}\delta_1}\exp\left[-\frac{(\omega+i\beta)^2}{2k^2v_{Td}^2}\right]. \quad (5)$$
$$\delta_1=m_e\nu_e\omega_{e1}+ik_z^2\kappa T_e,\quad \delta_2=m_i\nu_i\omega_{i1}+ik_z^2\kappa T_i.$$

In the case of an *electron-depleted* plasma, where $n_{i0}\simeq Z_dn_{d0}$, setting $\omega=\omega_r+i\omega_i$, Eq. (5) can be easily analyzed. This yields approximately the spectrum of the dust acoustic mode $\omega_r\simeq kc_d$, $c_d^2=\kappa T_iZ_d/m_d$, and the increment/decrement

$$\omega_i\simeq-\beta-\nu_i\frac{Z_dm_i}{2m_d}\frac{k^2}{k_z^2}\left(1-\frac{v_{i0}}{c_d}\frac{k_z}{k}\right)-\omega_r\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{Z_dT_i}{T_d}\right)^{3/2}\exp\left(\frac{-T_iZ_d}{2T_d}\right). \quad (6)$$

Hence, the mode is unstable provided that

$$\frac{v_{i0}}{c_d}>\frac{k}{k_z}\left\{1+\frac{2m_d}{Z_dm_i}\frac{k_z^2}{k^2}\frac{1}{\nu_i}\left[\beta+kc_d\left(\frac{\pi}{8}\right)^{1/2}\left(\frac{Z_dT_i}{T_d}\right)^{3/2}\exp\left(\frac{-T_iZ_d}{2T_d}\right)\right]\right\}. \quad (7)$$

As a demonstration, we take $B_0=5\cdot10^{-4}$ T and $T_i=T_n=5T_d$, $T_d=30$ K. The other parameters are chosen similar to Ref. [2], i.e., $m_n=4.8\cdot10^{-26}$ kg, $m_i=8\cdot10^{-26}$ kg, $n_{n0}=5\cdot10^{17}$ m^{-3}, $\sigma_{in}=2.5\cdot10^{-18}$ m^{-2}, $r_d=7\cdot10^{-9}$ m, $\rho_d=10^3$ kg/m^3. Taking $k=0.5$ m^{-1}, i.e., wave lengths that are of the order of 10 meters, applicable to space plasma situations, like noctilucent clouds, in Fig. 1 we plot the threshold ion velocity in terms of the ratio k_z/k for the two values of the grain charge $Z_d=2$ and $Z_d=12$. The two corresponding values of c_d are 1.7 and 4.2 m/s, respectively. From Fig. 1 it is seen that (depending on the grain charge) the instability has the lowest threshold for k_z much different than k, i.e., for modes propagating at large angles with respect to the direction of the ion current. Note that for $Z_d=2$ the threshold is equal 61.7 at parallel propagation ($k_z/k=1$), while its minimum is 16.1 at $k_{z,min}/k=0.12$, i.e., for the angle $\psi=83°$. Due to the model used here which implies $|(\omega-k_zv_{i0})/k_z|<v_{Ti}\sim160$ m/s, there is a lower limit $k_z\sim0.01$, below which there is no sense to go. So the excitation of the mode at the angle ψ is very likely. The increased grain charge number reduces

the threshold v_{i0}/c_d, in the present case for $Z_d = 12$ its minimum becomes 6.4 around $k_{z,min}/k = 0.31$ (i.e., at $\psi = 72^\circ$), and at $k_z/k = 1$ it is around 11. The behavior presented in Fig. 1 is valid in general as long as the grain Landau damping term is small. In that case the shape of the curves does not change by changing the grain mass, which is due to the fact that $\beta \sim 1/m_d$. The same holds for the change of the ratio ω_r/ν_i. This unusual excitation of the mode at a large angle ψ is due to the interplay of the two terms k_z/k in (7), where the first one appears due to the oblique propagation, causing the curve growth at $k_z \to 0$, while the second one appears due to the kinetic/collisional effects. For higher values of the charge number the angle ψ decreases while in the same time the threshold minimum becomes less pronounced and finally vanishes.

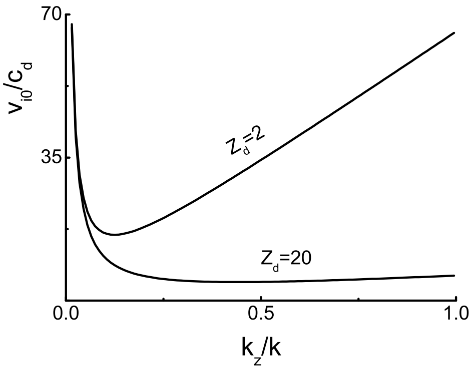

Figure 1: The threshold of the ion current for the instability of DA mode in an electron-depleted plasma, for two charge numbers on grains. The unstable values are above the line.

From the full dispersion equation (5) it may be seen that, in the absence of any collisions and neglecting the kinetic terms, the frequency becomes

$$\omega_d = kv_d = k\left(\frac{\kappa T_i Z_d^2 n_{d0}}{m_d n_{i0}}\right)^{1/2}\left(1+\frac{n_{e0}T_i}{n_{i0}T_e}\right)^{-1/2} \to kc_{d,[n_{e0}=0]}. \quad (8)$$

Clearly, in the presence of electrons $\omega_d < \omega_r$. We normalize (5) to ω_d and rewrite it in terms of dimensionless quantities, setting in addition $\omega \to \omega_d$ in the kinetic terms, which is allowed in view of its very small contribution to the dispersive properties of the mode. Specifically we set $n_{e0}/n_{i0} = s$, $n_{d0}/n_{i0} = (1-s)/Z_d$, and T_i/T_d, m_i/m_e, m_d/m_i are given before. Here we also take $\sigma_{en} = \sigma_{in}/4$ and $T_e/T_i = 10$. In the case $Z_d = 12$ the frequencies become $\omega_d = 4.16k[(1-s)/(1+s/10)]^{1/2}\,\mathrm{s}^{-1}$, $\nu_i/\omega_d = (48/k)[(1+s/10)/(1-s)]^{1/2}$, $\beta/\omega_d = (0.16/k)[(1+s/10)/(1-s)]^{1/2}$. The effect of electrons is presented in Fig. 2 for the case $Z_d = 12$. Here, the imaginary part of the frequency, normal-

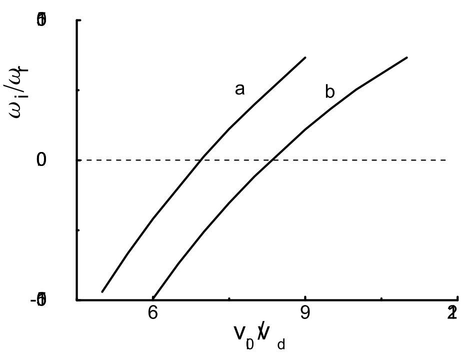

Figure 2: The increment/decrement of DA mode (5) for $k_z/k = 0.35$ and for two electron concentrations $s = 0.1$ (line a), $s = 0.4$ (line b)

ized to ω_d, for $k_z/k = 0.35$ is given for $s = 0.1$, 0.4 (lines *a* and *b*, respectively). It changes the sign at $v_{i0}/v_d = 7$ (line a) and $v_{i0}/v_d = 8.3$ (line b). The corresponding frequencies are also calculated showing slow increase within $0.93\omega_d - 1.09\omega_d$ for line *a* and within $0.8\omega_d - 1.01\omega_d$ for line *b*. The shift in the threshold for the increasing amount of electrons is due to the negative sign of the electron current with respect to the assumed direction of the wave.

A similar behavior is obtained for an ordinary electron-ion plasma with a parallel electron current. The obliquely propagating ion sound (at an angle ψ relative to the magnetic field lines and the electron current vector), in the similar parameter limit $|\omega - k_z v_{e0z}| \ll \nu_{en} \ll \Omega_e, \Omega_i \ll \omega$ yields

$$\omega_{re}^2 \approx k^2 c_s^2 \left(1 + \frac{3}{\tau}\right) - 6\frac{\nu_{en}\nu_{in}}{\tau}\frac{m_e}{m_i}\frac{k^2}{k_z^2}\left(1 - \frac{\vec{k}\cdot\vec{v}_{e0}}{kc_s}\right). \qquad (9)$$

$$\omega_{im} \approx -\nu_{in}\left[1 + \frac{3}{\tau(1+3/\tau)^2}\right] - \nu_{en}\frac{m_e}{2m_i}\frac{k^2}{k_z^2}\left[1 + \frac{3}{\tau(1+3/\tau)}\right]\left[1 - \frac{\vec{k}\cdot\vec{v}_{e0}}{kc_s(1+3/\tau)^{1/2}}\right]$$
$$-kc_s\left(\frac{\pi}{8}\right)^{1/2}\tau^{3/2}\left(1 + \frac{3}{\tau}\right)\exp\left[-\frac{\tau}{2}(1+3/\tau)\right]. \qquad (10)$$

where $\vec{k}\cdot\vec{v}_{e0} = kv_{e0}\cos\psi$, $\vec{v}_{e0} = v_{e0}\vec{e}_z$, $\tau = T_e/T_i$, $c_s^2 = \kappa T_e/m_i$. Similar to (7) the instability threshold is obtained from

$$\frac{v_{e0z}}{c_s} > \frac{\delta^{1/2}}{\cos\psi}\left\{1 + \left[\frac{\nu_{in}}{\nu_{en}}\left(1 + \frac{3}{\tau\delta^2}\right) + \frac{kc_s}{\nu_{en}}\left(\frac{\pi}{8}\right)^{1/2}\tau^{3/2}\delta\exp\left(-\frac{\tau\delta}{2}\right)\right]\frac{2m_i}{m_e}\frac{k_z^2}{k^2}\frac{1}{1+3/(\tau\delta)}\right\}. \qquad (11)$$

Here $\delta = 1 + 3/\tau$.

To conclude, the propagation of an acoustic mode in a weakly ionized plasma in which collisions of light species with neutrals far exceed the mode frequency, has an angle of preference at which the instability threshold is minimum and the increment is maximum. This is valid as long as the inertial species (grains for the DA mode, and ions for the IA mode) remain un-magnetized, while the pressure-restoring species (ions and electrons for the DA case, and electrons for the IA case) are magnetized and highly collisional. The increased charge on grains in the case of DA mode is shown to reduce the instability threshold as well as the angle dependence. In the same time the increased concentration of electrons shifts the threshold towards higher values, and reduces the mode increment.

References

[1] P. Kaw, Phys. Lett. A **44**, 427 (1973).

[2] N. D'Angelo, Phys. Lett. A **336**, 204 (2005).