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An Eulerian Vlasov code for the numerical solution of the kinetic sheath

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We present in this work a numerical study of the collisionless kinetic sheath at a plasma-wall transition in the one dimensional phase-space, using an Eulerian Vlasov code. Both electrons and ions are treated with a kinetic Vlasov equation. The Vlasov equation is solved directly in the 1D phase-space by a fractional step method previously reported in the literature [1-3]. It is well known that particle-in-cell (PIC) codes have a high level of numerical noise, especially in the low density regions of the phase-space near the wall. In the present work, we take advantage of the very low noise level of the Eulerian Vlasov code to study accurately the different kinetic and macroscopic parameters associated with the non-neutral plasma sheath at a plasma-wall transition.

The pertinent equations and results.

The pertinent Vlasov equation is written in dimensionless form for the electrons and ions in the phase-space *x*-*v* as follows:

$$\partial_t f_{e,i} + v \cdot \partial_x f_{e,i} + M_{e,i} E_x \cdot \partial_v f_{e,i} = 0 . \qquad (1)$$

The subscripts refer to electrons or ions. $f_{e,i} = f_{e,i}(x,v,t)$ is the electron (ion) distribution function and ∂ denotes partial derivatives. The Vlasov equation is coupled to Poisson's equation for the potential φ :

$$\frac{\partial^2 \varphi}{\partial x^2} = -(n_i - n_e), \quad \text{where} \quad n_{e,i} = \int_{-\infty}^{\infty} f_{e,i}\, dv , \text{ and } \quad E_x = -\frac{\partial \varphi}{\partial x} \qquad (2)$$

Time is normalized to ω_{pi}^{-1} , where ω_{pi} is the ions plasma frequency, velocity is normalized to the acoustic velocity $c_s = (T_e / m_i)^{1/2}$, where T_e is the electron temperature and m_i is the ion mass. Space is normalized to the Debye length $\lambda_{De} = c_s / \omega_{pi}$. With this normalisation, we have M_i =+1 in Eq(1) for the ions, and $M_e = -m_i / m_e$ for the electrons. The Bohm criterion requires that the ions enter the sheath region with a high velocity. We assume ions are injected at the right boundary $x=L$ at t=0 with the sound speed $v_0 = \sqrt{1 + 1/t_{ei}}$, where $t_{ei} = T_e / T_i$. The ion distribution function then takes the form of a shifted Maxwellian:

$$f_i(x,v,0) \sim Cg(v^2)\frac{e^{-t_{ei}(v-v_0)^2/2}}{\sqrt{2\pi/t_{ei}}} \; ; \quad \text{where} \quad g(v^2) = (1-e^{-2t_{ei}v^2}) \tag{3}$$

C is a constant and $g(v^2)$ is a smoothing factor chosen to minimize the deformation of f_i at v=0 to allow for a smooth transition to f_i =0 for v>0. We study a case with t_{ei}=1. In this case, the constant C in Eq.(3) is equal to 1/0.773359 to allow for an initial ion density of 1 at x=L. The electron distribution function $f_e(x,v,0)$ at the sheath entrance x=L is initially taken at t=0 as a truncated Maxwellian written in our normalized units $\sim \exp(-m_e v^2/(2m_i T_e)$, and truncated at a velocity such that its density and current are equal to the density and current obtained from Eq.(3). These distribution functions for ions and electrons are assumed to extend initially uniformly throughout the domain, so that the system is initially neutral. We assume in the present calculation that the ion and electron particles hitting a wall at x=0 are collected by a floating limiter. Then:

$$\left.\frac{\partial E_x}{\partial t}\right|_{x=0} = -(J_{xi}-J_{xe})|_{x=0} \; ; \text{ or } \; E_x|_{x=0} = -\int_0^t (J_{xi}-J_{xe})|_{x=0}\,dt, \text{ where } J_{xe,i} = \int_{-\infty}^{\infty} v f_{e,i}\,dv \tag{4}$$

Integrating Eqs.(2) over the domain (0,L), we get: $E_x|_{x=L} - E_x|_{x=0} = \int_0^L (n_i - n_e)dx = \sigma$ (5)

We assume that the plasma ions and electrons can circulate at the right boundary, i.e. that the plasma extend at the right boundary such that the point next to the last grid point is identical to the last grid point. With these boundary conditions, the distribution functions at x=L remained essentially the same during the simulation as the initial value, as shown in Fig.(1) (the full curve is the initial distribution at x=L, the broken curve is the distribution at t=15.). $E_x|_{x=0}$ the electric field at x=0 is calculated from Eq.(4). The electric field at x=L verifies Eq.(5), although it remained negligibly small. It follows from Eqs.(4,5) that the charge accumulated on the wall $E_x|_{x=0}$ is essentially equal and opposite in sign to the charge σ appearing in the system $E_x|_{x=0} = -\sigma$. We use deuterium ions. A small time-step $\Delta t = 1.25x10^{-3}$ was taken for the ions, and for the electrons this time-step was smaller by a factor of 10, our main concern being to get the results as accurate as possible. With the present value of Δt, we have about 100 time steps for the ions and 1000 time steps for the electrons in each electron plasma oscillation. The length of the system is L=40., and 400 grid points were used in space , 200 grid points were used in velocity space for the ions, and 300 grid points in velocity space for the electrons. The maximum and minimum velocities for the electrons are taken to be respectively $\pm 4./\sqrt{m_e/m_i}$. For the ions , the maximum velocity is taken to be $0.1x4./\sqrt{t_{ei}}$ (the ion distribution function is 0 for v>0), and the minimum velocity is taken to be $-1.5x4./\sqrt{t_{ei}}$. For the case t_{ei}=1, $v_0 = \sqrt{2}$ in Eq.(3). The temperature $T_e(x)$ is shown in Fig.(2) at t=15,(full curve):

$$T_e(x) = \frac{m_e}{m_i} \frac{\int_{-\infty}^{+\infty} (v - <v>)^2 f_e(x,v) dv}{\int_{-\infty}^{+\infty} f_e(x,v) dv} \quad ; \quad <v> = \frac{\int_{-\infty}^{+\infty} v f_e(x,v) dv}{\int_{-\infty}^{+\infty} f_e(x,v) dv} \qquad (6)$$

A similar definition holds for the ion temperature (without the factor m_e / m_i , see dotted curve in Fig.2). The ion and electron distribution functions at time t=15 are shown in Figs.(3,4). The bottom curves show the distribution functions plotted at (from left to right), x=0, $x=L/30.$, $x=L/10$, $x=L/2$, and $x=L$. We present in Fig.(5) the potential profile, fixed to zero at the left boundary. The electron distribution functions in the bottom curves of Fig.(4) show truncation at high velocities. These cuts appear at the velocity $v_c = \sqrt{2\Phi m_i / m_e}$, where Φ is the value of the potential at the position where the distribution functions are plotted. We can check from Fig.(5) that the potential $\Phi(x)$ is given at these positions respectively by 0., 1.32, 2.326, 2.653, and 2.672 . This corresponds to cut-off velocities in the corresponding electron distribution functions at 0., 98.5, 130.8, 139.7., and 140.23, which are observed in the distribution functions at the bottom of Fig.(4). Fig.(6) shows the plot of the electron density (full curve), the ion density (broken curve), and the dash-dot curve plots the quantity

$$n(x) = n_e(L) e^{(\Phi(x) - \Phi(L))/T_e(L)} \qquad (6)$$

almost identical to the full curve. Fig.(7) shows the electric field (full curve), and the charge ($n_i - n_e$, broken curve). Fig(8) presents the electron current (full curve), the ion current (broken curve), and the total current ($J_{xi} - J_{xe}$, dotted curve, equal to zero).

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References.

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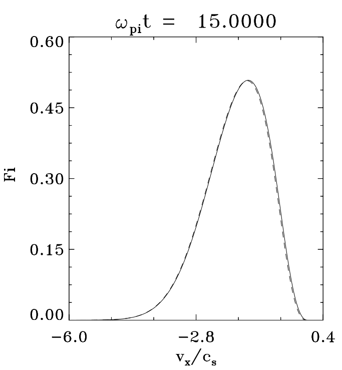

Fig.1

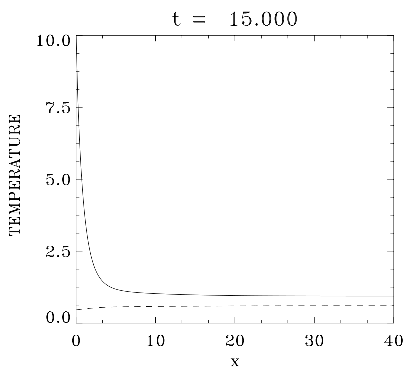

Fig.(2)

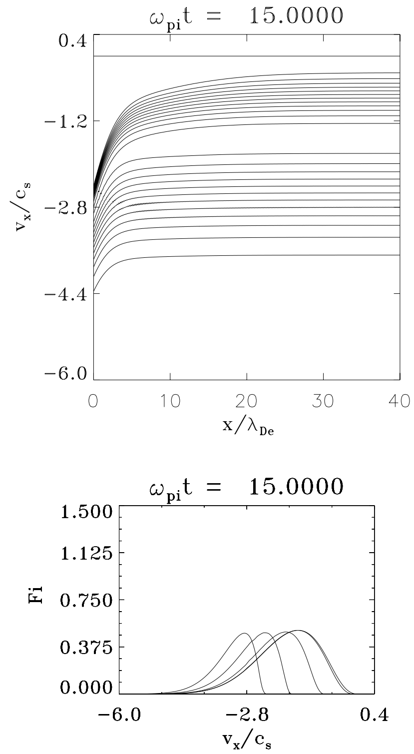

Fig.3 Ion distribution functions

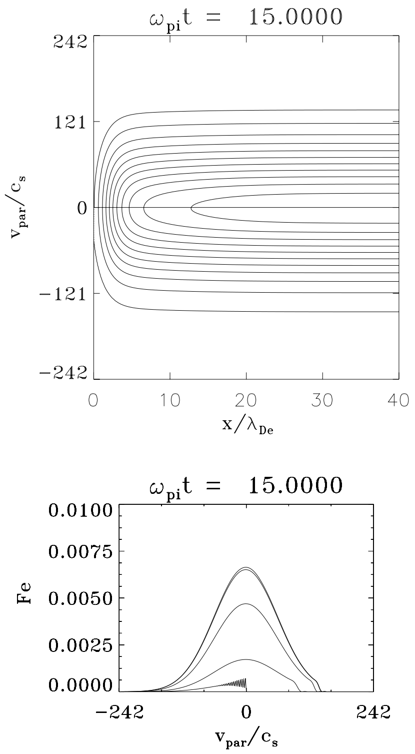

Fig.4 Electron distribution functions

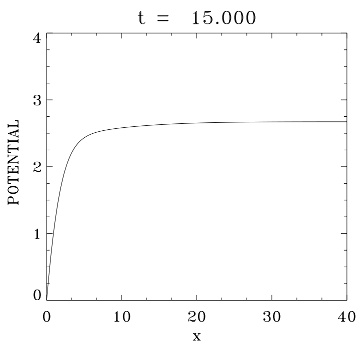

Fig.5 Potential

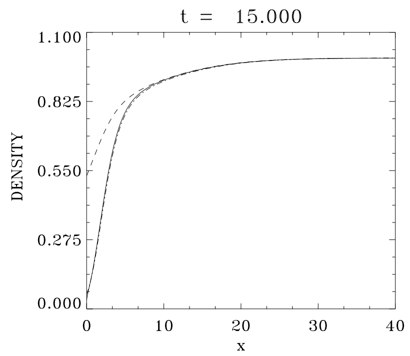

Fig.6 Ion (broken curve), electron densities (full curve), *n(x)* (dash-dot curve).

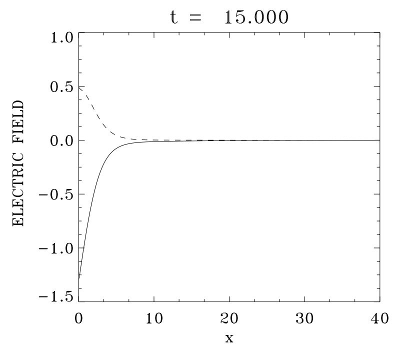

Fig.7 Electric field (full curve) and charge

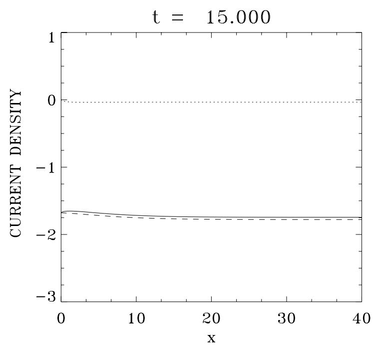

Fig.8 Current density J_{xe} (full curve) and J_{xi}