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Electron climbing a "devil's staircase" in wave-particle interaction

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Our work focuses on the experimental implementation of a specially designed Travelling Wave Tube (TWT) where a test electron beam is used to observe its non-self-consistent interaction with externally excited wave(s). This device was extensively used to mimic beam-plasma interaction because the dispersion relation closely resembles that of a finite radius, finite temperature plasma; But, unlike a plasma, the helix does not introduce any appreciable noise [1]. TWT also allowed the observation of resonance overlap responsible for Hamiltonian chaos [2], the direct exploration of nonlinear particle synchronization by a single non resonant wave [3] and of the experimental implementation of a new method to control chaos [4].

The TWT is made up of three main elements: an electron gun, a slow wave structure (SWS) formed by a 4m long helix with axially movable antennas, and an electron velocity analyzer. The electron gun creates a beam which propagates along the axis of the SWS and is confined by a strong axial magnetic field with a typical amplitude of 0.05T which does not affect the axial motion of the electrons. The central part of the gun consists of the grid-cathode subassembly of a ceramic microwave triode and the anode is replaced by a Cu plate with an on-axis hole whose aperture defines the beam diameter equal to 1mm or 3mm depending on the specific experiment. Beam currents, $I_b<1$ mA, and maximal cathode voltages, $|V_c|<200$V, can be set independently. Waves are launched by an arbitrary waveform generator with a moving probe capacitively coupled to the helix. The SWS is long enough to allow non-linear processes to develop. Finally the cumulative changes of the electron beam distribution are measured with the velocity analyzer, located at the end of the interaction region [5].

Here we consider charged test particles moving in two electrostatic waves. The equation modelling the dynamics in this case is $\ddot{x}=\sum_{i=1}^{2}\eta k_i\phi_i \sin(k_i x-\omega_i t+\varphi_i)$ where ϕ_i, k_i, ω_i and φ_i are respectively the amplitudes, wave numbers, frequencies and phases of the two waves; η is the charge to mass ratio of the particle. We focus on the experimental observation of a "devil's staircase" in such a time dependent system [6,7] considered as a paradigm for the transition to large scale chaos in the universality class of hamiltonian systems [8].

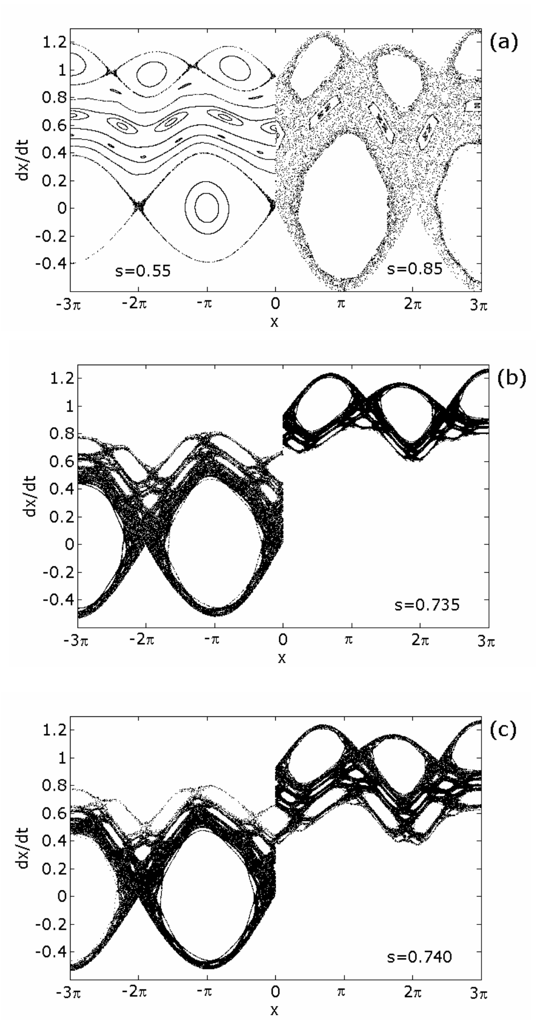

Figure 1: Poincaré surface of section. (a) Left half for $\varepsilon \simeq 0.39$ (s = 0.55) exhibits island chains of secondary resonances at rational velocities $m/(n + m)$, as seen from 15 orbits which do not mix ; right half for $\varepsilon \simeq 0.92$ (s = 0.85) exhibits large scale chaos as seen for two orbits. (b) Two orbits related to upper and lower main chaotic domains for s = 0.735, each iterated over 10^5 Poincaré periods ; to allow an easy comparison, only points with $-3\pi<(x \bmod 6\pi)<0$ are plotted for one orbit, and $0<(x \bmod 6\pi)<3\pi$ for the other.

The motion of the particles governed by this equation is a mixture of regular and chaotic behaviors mainly depending on the amplitudes of the waves and exhibits generic features of chaotic systems. Poincaré sections of the dynamics (a stroboscopic plot of selected trajectories) are displayed in Fig. 1. Taking advantage of the symmetry $x' = -x$, two different cases are superposed in Fig. 1. The strength of chaos is measured by the dimensionless overlap parameter $s=2(\sqrt{\eta\varphi_1}+\sqrt{\eta\varphi_2})/(|v_{\varphi 1}-v_{\varphi 2}|)$ defined as the ratio of the sum of the resonant velocity half-widths of the two wave potential wells to their phase velocity difference.

For intermediate wave amplitudes, nested regular structures appear as secondary resonances with a wave number $k_{nm}=nk_1+mk_2$, a frequency $\omega_{nm}=n\omega_1+m\omega_2$, a phase $\varphi_{nm} = n\varphi_1+m\varphi_2$ and an effective amplitude $\Phi_{nm} \sim \Phi_1^{|n|}\Phi_2^{|m|}$ with integer n and m, as shown in the left side of Fig. 1a for s=0.55 . The self-similar structure of phase space results from the infinitely nested higher order resonances which appear in so-called Arnold tongues between secondary resonances as predicted by the Poincaré-Birkhoff theorem [9].

For larger wave amplitudes, a wide connected zone of chaotic behavior occurs in between the primary resonances due to the destruction of the so-called Kolmogorov-Arnold-Moser (KAM) tori acting as barriers in phase space, as shown in the right side of Fig. 1a for s = 0.85 . Between these two values of s, invariant tori are sequentially destroyed and replaced by cantori. Transport in velocity (and thereby in (x,v) space) occurs through the holes of the cantori, which have a fractal structure . When a torus breaks, the flux through it is still zero, as the associated turnstile has vanishing area : trajectories leak easily through its holes only for s

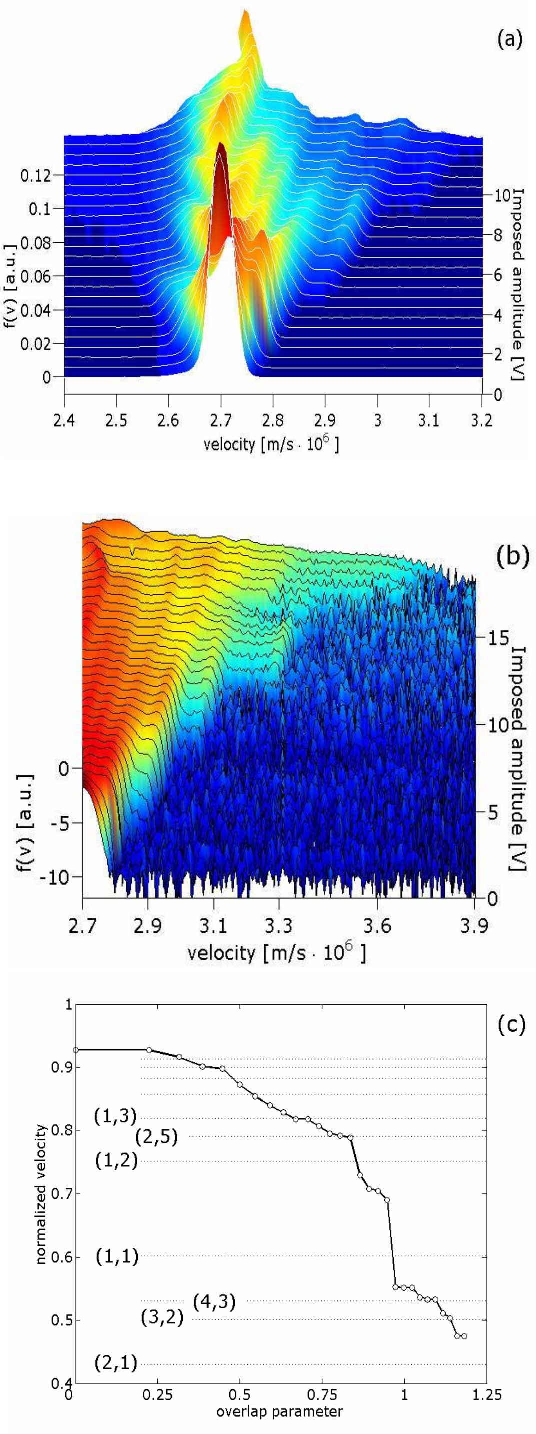

Figure 2: Experimental devil's staircase shown with a) linear and b) logaritmic scale for the measured beam velocity distribution functions $f(v)$. c) Normalized velocity frontiers of $f(v)$ versus overlap parameter s; velocity normalization is such that v=0 (1) stands for the helix (beam) mode phase velocity; secondary resonances $(n;m)$ are indicated at velocities $mk'/(n + mk')$ with $k' = v_\varphi / v_b$

values significantly above its destruction threshold. For the dynamics $\ddot{x}=-\varepsilon(\sin(x) + 0.16 k \sin k(x-t))$, with k = 5/3, which is close to our experimental conditions, the threshold for large scale chaos (destruction of the most robust torus) is near $s \approx 0.75$, as seen from the Poincaré section of orbits over long times (Fig.1,(b-c)).

If one considers a beam of initially monokinetic particles having a velocity equal to the phase velocity of one of the two waves, the chaotic zone is associated with a large spread of the velocities after some time since the particles are moving in the chaotic sea created by the overlap of the two resonances; transition exhibits a “devil's staircase” behavior for increasing excitation amplitude, due to the nonlinear forcing by the second wave on the pendulum-like motion of a charged particle in one electrostatic wave.

The experimental observation (Fig.2) was made introducing one signal at 30 MHz, corresponding to a phase velocity $v_\varphi = 4.07 \cdot 10^6$ m/s, and a monokinetic beam with velocity $v_b = 2.7 \cdot 10^6$ m/s .

In fact the applied signal generates two waves: a helix mode with a phase velocity v_φ, and a beam mode with a phase velocity equal to the beam velocity v_b. The beam mode is actually the superposition of two indistinguishable modes with pulsation $\omega = kv_b \pm \omega_b$ corresponding to the beam plasma mode.

In Fig.2a we remark the V-structure typical of a trapping behavior where the velocity bunching of the electrons around their initial velocity are obtained for amplitudes equating the interaction length to a multiple of half the trapping length [10].

Fig.2b shows how, above a certain applied signal amplitude threshold, the distribution function spreads over a much wider velocity domain, and electrons can be strongly accelerated: the transition to large velocity spread does not occur continuously but rather occurs by steps. As shown in Fig.2c we can relate this behavior to the breaking of invariant velocity barriers as expected from transition to large scale chaos in a non integrable Hamiltonian system with a good agreement with theoretical estimates.

We have thus exhibited a well-defined experiment in a laboratory device that allows a direct experimental exploration of fractal features of the complex Hamiltonian phase space. This striking result paves the road to a detailed experimental exploration of chaotic behavior in nondissipative systems.

In perspective the injection of electron packets with a prescribed phase with respect to a wave should allow to explore more details about the test particle dynamics.

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