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# TIME AND SPACE SCALES IN DRIVEN MAGNETIC RECONNECTION IN THE MAGNETOSPHERE

J. J. Martinell and A. Hernández-García
*Instituto de Ciencias Nucleares, UNAM,*
*A. Postal 70-543, Mexico D.F. MEXICO*

## I. Introduction

The Earth magnetosphere is the place where geomagnetic storms originate as energetic flows of solar wind plasma interact with it. One of the most common events occurring in these episodes is magnetic reconnection which accelerates charged particles that in turn give rise to the aurora. This process can take place either in the day-side or in the geomagnetic tail, when two plasma regions having opposite magnetic polarities are pushed together in what is known as driven reconnection. Since magnetospheric, as well as solar wind plasmas are effectively collisionless, the mechanism for current limitation that leads to reconnection has to come from non-ideal effects such as electron inertia. Other effects, like the Hall term in the generalized Ohm's law, may also contribute to the phenomenlogy. Many studies on collisionless magnetic reconnection have been made including various terms in Ohm's law, with and without a neutral sheet, but few are directly applicable to the magnetospheric conditions.

For the parameters at the magnetosphere, $n \sim 10^{10} cm^{-3}$, $T \sim 6eV$, $B \sim 10^{-4}G$, the ion-soumd gyroradius is $\rho_s \sim 2 \times 10\, km$ while the electron inertial skin depth is $d_e \sim 1\, km$. This regime with $\rho_s > d_e$ was not considered in [1], where driven magnetic reconnection about an X-point was studied. In contrast, this regime was addressed in [2] but for the parameters relevant for the VTF experiment, including a constant guide field perpendicular to the X-point. On the other hand one finds that $\beta \sim 1/4$, so that the relevant range to consider has $\rho_s/d_e > 1$ but $\beta < 1$. Here we study the evolution of an X-point configuration for the regime $\rho_s > d_e$ and in presence of a guide field (relevant to day-side reconnection), for a compressible collisionless plasma taking into account the Hall effect. We use the full nonlinear equations as opposed to the works of [1] and [2], where the linearized equations were analyzed analytically and numerically.

## II. Driven collisionless evolution

We consider a cartesian goemetry with a 2D dependence having an equilibrium X-point configuration in the $(x, y)$ plane and a guide field in the $z$ direction. The magnetic field is represented by $\mathbf{B} = \hat{z} \times \nabla + \psi(x, y, t) + B_z(x, y, t)\hat{z}$. The equilibrium magnetic potential is $\psi_0 = B'_\perp xy$ and is characterized by a scale length defined by $l_0 \equiv B_{z0}/B'_\perp$,where $B_{z0}$ is the guide field. The plasma velocity is written in terms of the potentials $\phi$ and $\chi$ (related to the compressibility) as $\mathbf{v} = \hat{z} \times \nabla\phi(x, y, t)\nabla\chi(x, y, t) + v_z(x, y, t)\hat{z}$. Then the two fluid equations are taken under the orderings, $\beta < 1$ and $l_0/d_i \gg 1$, where $d_i$ is the ion skin depth. In the limit $v_z \to \infty$, they can be reduced to a set of three equations for the variables $\psi, \phi$ and the density contained in $\xi \equiv l_0/d_i \ln(n/n_0)$, with $n_0$ the equilibrium density. These equations are [3]:

$$\frac{\partial U}{\partial t} = [U,\phi] + [\psi, \nabla^2\psi] \tag{1}$$

$$\frac{\partial}{\partial t}(\psi - d_e^2\nabla^2\psi) = [\psi - d_e^2\nabla^2\psi, \phi] - \rho_s^2[\psi,\xi] \tag{2}$$

$$\frac{\partial \xi}{\partial t} = [\xi,\phi] + [\phi, \nabla^2\psi] \tag{3}$$

where $U \equiv \nabla^2\phi$, $[f,g] \equiv \hat{z}\cdot\nabla f \times \nabla g$, and all variables are normalized according to $\phi \rightarrow \phi(\tau_A/l^2)$, $\psi \rightarrow \psi/(l^2 B'_\perp)$, the lenghts to the system size $l$, and the time to the Alfvén time, $\tau_A = (4\pi n_0 m_i)^{1/2}/B'_\perp$. The term proportional to $\rho_s$ in (2) is related to the electron compressibility and the ions are assumed to be cold. The Hall term is included, although the parameter $d_i$ that characterizes it does not appear under the ordering assumed.

A linearized version of this set of equations was studied in [1,2], for the case of forced reconnection by an induced electric field in an X-point, finding time-asymptotic analytical solutions that were corroborated numerically. A nonlinear simulation of a similar model was made in [4] but for an initial equilibrium of the type of a neutral current sheet and for parameters not appropriate for the magnetosphere. The linear approximation is valid for a weak forcing and it can be reduced for a single equation when an ansatz for separating the variables is made, due to the symmetry of the geometry: $\psi_1(x,y,t) = \psi(x,y)+\psi(y,t)$ and $\phi_1(x,y,t) = \phi(x,y)-\phi(y,t)$. The resulting equation for $j(x,t) \equiv \partial^2\psi(x,t)/\partial x^2$ is

$$\frac{\partial^2}{\partial t^2}[(1 - d_e^2\frac{\partial^2}{\partial x^2})j] = x^2\frac{\partial^2 j}{\partial x^2} + 3x\frac{\partial j}{\partial x} - \rho_s^2\frac{\partial^2}{\partial x^2}[x\frac{\partial}{\partial x}(x\frac{\partial j}{\partial x})]. \tag{4}$$

This can be solved using a Laplace transform method to find the time asymptotic behavior when the reconnection is driven by an influx at $x \rightarrow \infty$ of magnitude $v_\infty$. If the initial stream function at the boundary $(x, y \rightarrow \infty)$ is taken to be,

$$\phi(x,y,t=0) = \frac{v_\infty l_0}{4}\ln\left(\frac{y^2+\delta^2}{x^2+\delta^2}\right), \tag{5}$$

the solution for the current at the center (X-point) tends to a constant value [2]: $j(0,t \rightarrow \infty) \rightarrow v_\infty B_{z0}\tau_A\rho_s/2\delta^2$ and consequently, the central magnetic potential $(\psi(0,t) \sim d_e^2 j(0,t))$, that measures the reconnected flux, also approaches a constant. Here $\delta$ is a parameter used to avoid singularities and is of the order of $\rho_s$.

In order to include cases where the forcing is not small, we need to keep the nonlinear equations. We have developed a numerical code that solves the system of equations (1-3) for a finite domain, so that, the boundary conditions are applied at $(x,y) = \pm L$. In all simulations here, $L = 1$. The reconnection is driven by an induced flux at the boundaries which increases from zero at $t = 0$. The boundary conditions are:

$$\psi(\pm 1, y, t) = \pm B'_\perp y + f(t), \quad \psi(x, \pm 1, t) = \pm B'_\perp x + f(t) \tag{6}$$

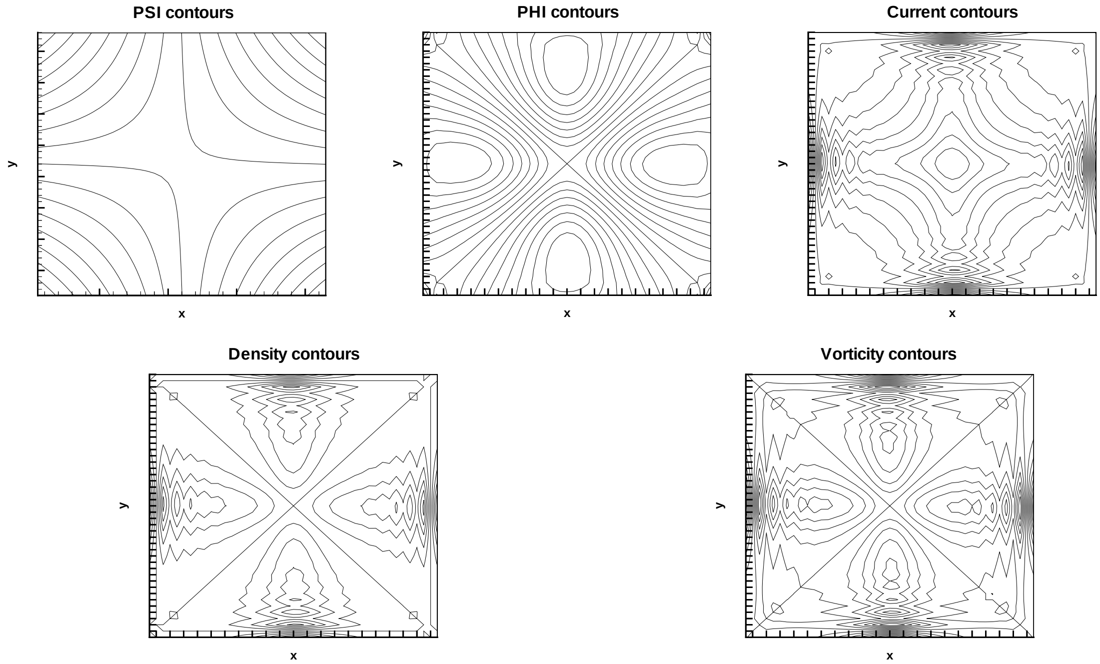


Figure 1: Contour plots for $\psi$, $\phi$, $j$, $\xi$ and $U$.

$$\phi(\pm 1, y, t) = \frac{1}{4B'_\perp}\frac{df(t)}{dt}\ln(y^2+\delta^2), \quad \phi(x, \pm 1, t) = -\frac{1}{4B'_\perp}\frac{df(t)}{dt}\ln(x^2+\delta^2) \qquad (7)$$

and $\xi(\pm 1, y, t) = \xi(x, \pm 1, t) = 0$, where $f(t) = v_\infty B_{z0}\tau_d(1-(1+t/\tau_d)\exp[-t/\tau_d])$. The initial conditions are: $\psi(x, y, 0) = B'_\perp xy$, $\phi(x, y, 0) = 0$ and $\xi(x, y, 0) = 0$.

The results of the preliminary simulations are shown in Figure (1). which show the contour plots of the relevant parameters for a typical case at a time not very advanced in the evolution. There we show the magnetic flux $\psi$ =const. (field lines), stream function $\phi$ = const. (velocity field), current $j$ =const., as well as level lines for density and vorticity, for a time corresponding to $t = 4\tau_A$. The values of the parameters used are: $\Delta t = 0.005$, $\rho_s^2 = 0.2$, $B'_\perp = 0.6$, $d_e^2 = 0.02$, $v_\infty l_0 = 0.04$ and $\tau_d = 0.5$. A small numerical dissipation was included for the three variables. The chosen magnitude of the forcing is moderate, so the linear theory is not sure to be applicable. At this stage there has not been mixing of modes, so the contours look still regular.

One important parameter to consider is the reconnected magnetic flux which is given by the value of $\psi$ at the X-point. In Figure (2) $\psi(0, 0, t)$ is shown for three different cases. In the weak forcing case, represented by the short-dashed line, a value of $v_\infty l_0 = 0.005$ was used. It is clear that the reconnected flux approaches asymptotically a constant value, which was almost reached for $t = 5\tau_A$. This is the result found above analytically and agrees with the results reported in [2], except that in [2] there is an oscillation as the asymptotic limit is reached. This may be due to the fact that here we used $\tau_d = 0.1$ meaning that the velocity drive lasts for only a short time. The other two curves are for stronger drives. The solid line is for $v_\infty l_0 = 0.04$ and $\tau_d = 0.5$, while the long-dashed line is for $v_\infty l_0 = 0.05$ and $\tau_d = 1$. While the later case starts

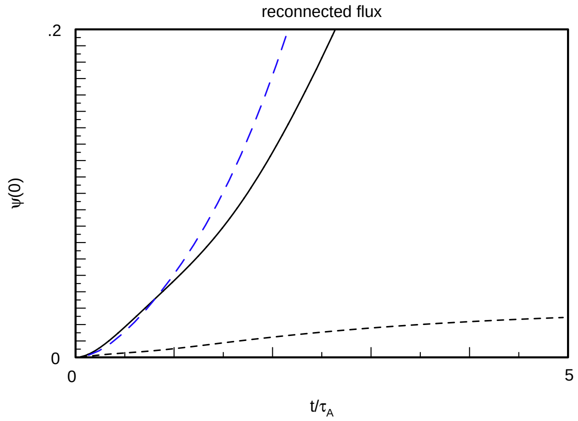


Figure 2: Time evolution of the reconnected flux for three runs having different strength and duration of the forcing.

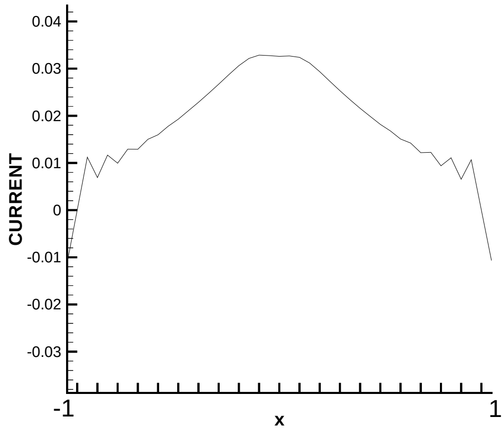


Figure 3: Spatial structure of the current.

slower it soon grows up very fast and surpasses the former. In both cases there is an exponential growth that, for the times shown, has not started to decline. However, there is no evidence (for the simulated times) of settling to a constant value as in the weak forcing case.

**Space structure:** The important point to look in our study is the way the relevant parameters vary in space. In Figure (3) we show the variation of the current along a line through the X-point for a time $t = 4\tau_A$ and a case of moderate forcing. We notice that the spatial scale near the edge is of the order of $d_e \sim 0.1$, which is in agreement with the results of [1]. They also found that the current develops a structure of a wave-packet type, for late times, but we do not obtain that, up to the times reached.

## III. Conclusions

The results obtained for the nonlinear evolution of an X-point configuration are a generalization of the linear results found previously [1,2], which should be applicable to the reconnection process in the day-side of the magnetosphere. We developed a code that solves the reduced equations, but it has the drawback that it takes a long time to run a single case. For that reason, the simulations presented here have not been followed long enough in time, such as to be able to establish the asymptotic behavior of the reconnected flux for the case of strong forcing, but there is indication that it does not approache a constant value as in the linear case.

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