

Charge state of a spherical plasma

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Introduction

For the intensities of present day ultrashort, superintense laser pulses, the energy that the ions in a target acquire due to direct interaction with the electromagnetic fields of the laser pulse is usually small, while the energy the electrons suddenly acquire can be of the order of tens of KeV per particle. Due to their thermal "pressure", these "hot" electrons expand creating an electrostatic field around the cluster. In the mean time part of the most energetic electrons has been able to overcome this electrostatic barrier determining an overall non neutrality of the cluster.

In a time of the order of some electron plasma periods, this charging process leads to a steady state configuration (SSC) which is achieved before the ions can depart significantly from their initial configuration, due to their much longer response time. Afterwards ion acceleration takes place, as predicted theoretically [1, 2, 3, 4, 5, 6], and confirmed experimentally [7, 8].

Here we present analytical and numerical investigations of SSC [9]. One of the main results is that that SSC is uniquely dependent on the ratio R between the radius of the ion core and the initial electron Debye length. By means of simple analytical models we describe the charging process in the limit of fast thermalization time (collisional regime) and in the opposite limit of collisionless dynamics. In particular in both regimes we obtain the SSC charge. A detailed comparison with 3D PIC simulations has been performed and it provides a good agreement in the range $5 < R < 40$. An analytical fit given by a Padè approximation also reproduces very well the numerical results.

A model for charging process

We model the charging process by assuming as a starting configuration a neutral distribution of fixed ions and a thermal distribution of electrons (of temperature T_0), uniform inside a sphere of radius R_0 . We denote by N_e the time dependent number of electrons in the cluster, that are assumed to be uniformly distributed during the charging process inside the ion core. As a further simplification, we assume that, on average, the radial crossing of the electron trajectories does not lead to a relative redistribution of the charge in front and behind each electron outside the core. Hence, the condition for an electron to reach infinity is that it has a positive total

energy when it reaches the ion core surface at $r = R_0$. Such an electron is considered definitively “evaporated” from the cluster. The evaporation of the electrons changes N_e , the total energy of the system and causes an energy redistribution of the remaining electrons. We discuss the “collisional” regime and the “collisionless” one. In the first one, the electrons which have not evaporated are assumed to thermalize instantaneously at a temperature T , which turns out to be a decreasing function of time. In the second regime no thermalization occurs, and the evaporation causes a progressive depletion in the high energy tail of the electron distribution function, which on the other hand remains isotropic in velocity space.

In what follows, lengths will be measured in units of the initial Debye length $\lambda_d = (T_0/4\pi n_0 e^2)^{1/2}$, with e the absolute value of the electronic charge, time t in units of $\omega_{pe}^{-1} = (4\pi e^2 n_0/m_e)^{-1/2}$, with m_e the electron mass, energies in units of the initial electron temperature T_0 , velocities in units of the initial electron thermal speed $v_{th,0} = \sqrt{T_0/m_e}$, mass in units of the electron mass and particle numbers in units of N_0 . Since inside the ion core the electron density is taken to be uniform, with the adopted normalization the normalized electron density n_e and the normalized total number of electrons N_e are numerically equal.

Collisional regime

If collisions assure a fast thermalization process, the electron evaporation rate is obtained by calculating the flux of electrons with positive total energy through the core surface,

$$\frac{dN_e}{dt} = -\frac{3}{\sqrt{2\pi}} \frac{(1+\phi_T)}{\tau} e^{-\phi_T} N_e, \quad (1)$$

where $\phi_T = e\Phi_R/T = (1-N_e)R^2/3T$, with Φ_R the electrostatic potential at the ion core surface and $\tau = R/\sqrt{T}$ is the electron crossing time of the ion core. At the same time each evaporating electron carries away the residual energy $v^2/2T - \phi_T$ giving a total energy flux

$$\Phi_U = \frac{3}{\sqrt{2\pi}} \frac{(2+\phi_T)}{\tau} e^{-\phi_T} N_e T. \quad (2)$$

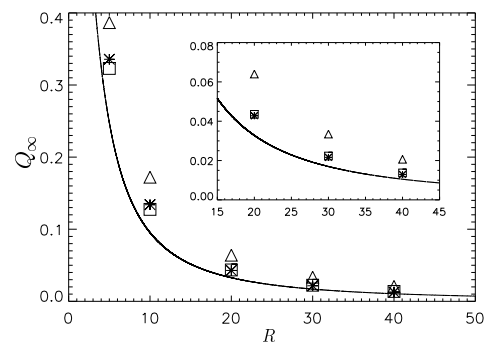


Figure 1: Comparison between the SSC value of the charge predicted by the collisional model (triangles), the collisionless model (squares), the PIC simulations (stars), and the solution of Eq. (8) (solid line).

On the other side the electrostatic energy U_Φ changes due to loosing particles:

$$\frac{dU_\Phi}{dt} = -\frac{2}{5} R^2 (1 - N_e) \frac{dN_e}{dt}. \quad (3)$$

By imposing the total energy balance we obtain an evolution equation for T :

$$\frac{d(3N_e T/2)}{dt} = -\frac{3}{\sqrt{2\pi}} \frac{(2 + \phi_T)}{\tau} e^{-\phi_T} N_e T + \frac{2}{5} R^2 (1 - N_e) \frac{dN_e}{dt}. \quad (4)$$

Therefore Eq.(1) and Eq.(4) gives the time evolution of the system.

Collisionless regime

If the plasma electrons inside the ion core are not significantly affected by collisions, we may assume that the populations of different energy $N(\mathcal{E})$ are non mixed during evolution and we can consider the evaporation process to act separately on the different populations, with the coupling between them being given only by the electrostatic potential. As a result the distribution function becomes non-Maxwellian.

We assume that the electron distribution remains homogeneous in coordinate space and isotropic in velocity space. Thus, denoting by $N_{\mathcal{E}}$ the time dependent number of electrons with kinetic energy (normalized on the initial temperature T_0) in the interval $[\mathcal{E}, \mathcal{E} + d\mathcal{E}]$, and introducing the time dependent quantity $\phi_0 = e\Phi_R = \frac{1}{3}(1 - N_e)R^2$, which differs from ϕ_T in the previous section by the normalized temperature factor $1/T$, we obtain

$$\frac{dN_{\mathcal{E}}}{dt} = -\frac{3}{2\sqrt{2}} \frac{\sqrt{\mathcal{E}}}{R} \theta(\mathcal{E} - \phi_0) N_{\mathcal{E}}. \quad (5)$$

This implies that the evaporation of the electron population with energy \mathcal{E} stops at a well defined time $t = t_{\mathcal{E}}$, where $t_{\mathcal{E}}$ is such that $\mathcal{E} = \phi_0(t_{\mathcal{E}})$. Therefore,

$$N_{\mathcal{E}} = N_{\mathcal{E}}(0)e^{-t/t_d} \quad \text{for } t \leq t_{\mathcal{E}} \quad \text{and} \quad N_{\mathcal{E}} = N_{\mathcal{E}}(0)e^{-t_{\mathcal{E}}/t_d} \quad \text{for } t > t_{\mathcal{E}} \quad (6)$$

with $t_d = \left(2R\sqrt{2}/3\sqrt{\mathcal{E}}\right)$ the \mathcal{E} -dependent decay time and $N_{\mathcal{E}}(0)$ the electron kinetic energy distribution at the initial time $t = 0$. We assume the electron velocities at time $t = 0$ to be Maxwellian distributed, hence the initial electron kinetic energy distribution $N_{\mathcal{E}}(0)$ is given by

$$N_{\mathcal{E}}(0) = 2/\sqrt{\pi} e^{-\mathcal{E}} \sqrt{\mathcal{E}}. \quad (7)$$

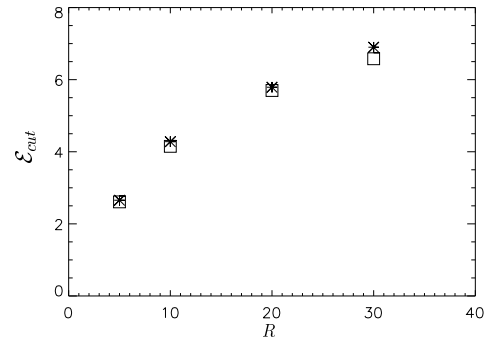


Figure 2: Comparison between the cutoff energy \mathcal{E}_{cut} as a function of the ion core radius R predicted by the collisionless model (squares) and by the PIC simulations (stars)

The electron number N_e , and therefore ϕ_0 , can thus be calculated performing numerically, at fixed t , the integral $N_e = \int d\mathcal{E} N_{\mathcal{E}}$. Note that in this collisionless model a rough estimate of the asymptotic electron number could be obtained by approximating the final electron distribution function with the initial one for $\mathcal{E} < \mathcal{E}^*$, and with zero for $\mathcal{E} > \mathcal{E}^*$. We can then determine the cutoff energy \mathcal{E}^* self consistently by equating its value to the electrostatic energy of the configuration with charge $Q(\mathcal{E}^*) = \int_{\mathcal{E}^*}^{\infty} N_{\mathcal{E}}(0) d\mathcal{E}$,

$$\mathcal{E}^* = [Q(\mathcal{E}^*) R^2/3]. \quad (8)$$

Comparison with numerical results and conclusions

Our analytical results have been compared with numerical results obtained by means of spherical PIC code [9]. The comparison between the charge value obtained numerically and that predicted analytically is shown in Fig. 1, while the results for \mathcal{E}_{cut} are shown in Fig. 2. The agreement among the numerical results and the values obtained in the collisionless regime is very good in the whole range 5 – 40. With regard to the thermal model adopted for the collisional regime we remark that for small radii it predicts a moderately larger value of Q_{∞} , but the two different regimes lead to very close values Q_{∞} in case of large radii.

In conclusion, we have investigated the charging up process of a spherically symmetric plasma configuration in vacuum in the limit of immobile ions. Two different simplified models have been presented and their results have been compared with numerical ones providing a very good agreement.

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