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Resistive Stability of 2/1 Tearing Modes in DIII-D Plasmas With 1/1 Resonance

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I. Overview

The stability of multiple coupled resistive modes is examined using reconstructions of experimental equilibria in the DIII-D tokamak, revealing the physics involved in mode onset as discharges evolve to instability. Stability analyses of two discharges are compared, one high beta and one low beta, each with m=2/n=1 tearing mode onset and a m=1/n=1 resonant surface at a small minor radius.

The physics dominating the onset of tearing modes in high beta tokamak discharges is often in dispute, and only recently have diagnostics and equilibrium reconstructions approached the accuracy necessary to decipher the subtle effects involved [1]. A low beta DIII-D experiment was therefore designed specifically to test theories of tearing mode stability [2], with a 2/1 tearing mode being driven by an induced current gradient. The analysis of this discharge allows a point of comparison for the analysis of high beta discharges, where the physics becomes more complicated. Experimental attempts to access the highest beta in tokamak discharges, including hybrid discharges, are typically terminated by the growth of a large 2/1 tearing mode. In hybrid discharges the plasma current is significantly noninductive, intended to lengthen the discharge time, while sustaining the baseline dimensionless parameters of a burning plasma experiment such as ITER. Here we analyze a high β hybrid discharge with a 2/1 onset which terminates the high confinement, and compare with the low β results.

For the low beta plasma, a series of equilibria leading up to mode onset are analyzed. For the high beta discharge, there is little change in the equilibrium state of the discharge on approach to the mode onset. Therefore, model equilibria, based on an experimental reconstruction from the discharge just before onset, are generated varying q_{min} and pressure.

For each equilibrium the PEST3 code [3] is used to determine the ideal MHD solution including both tearing and interchange parities at all resonant surfaces. The linear resistive stability index Δ' alone is insufficient for determining the mode stability in toroidal geometry. This outer region solution must be matched to the resistive inner layer solutions at each rational surface to determine resistive mode stability. Both the outer global ideal MHD and inner layer solutions at rational surfaces are important in determining mode stability, both of which differ in high and low beta plasmas.

II. Equilibrium Reconstructions

The limited, low beta plasma was steadily heated and the plasma current raised until a 2/1 magnetic island appeared and grew. Small sawteeth oscillated within q<1, starting well before the period of interest. The discharge had three n=1 rational surfaces with m=1, 2, and 3 at approximately ρ=0.2, 0.7 and 0.9, where ρ is the normalized toroidal flux. The discharge had low toroidal flow shear, making the coupling between rational surfaces important.

Ideal MHD equilibria were reconstructed at three times that bracket the appearance of the 2/1 magnetic island at t≈1550 ms; the early t=1405 ms, the near onset t=1505 ms and the later t=1605 ms. The most important changes between equilibria are that the 2/1 rational surface moves out with time while the current gradient increases. These both contribute to an increase in the dimensionless Δ' and the onset of the 2/1 tearing mode, but the inner layer solutions and coupling must be considered to accurately predict the onset.

In the high β, long pulse discharge q_{min} approaches and remains near 1, being prevented from reducing far below 1 by current drive, some of which is anomalous. The monotonic q

profile has moderate shear throughout and reaches q_{95}~5. No sawtooth oscillations are observed, although occasional fishbone oscillations are observed. Despite the steady state nature of the discharge, the 2/1 tearing mode suddenly grows to large size and significantly reduces the confinement, without a clear indication of the root cause of the instability. Here the steady state conditions are β_N=3.1, $\beta_N/4l_i$=0.91 suggesting the discharge is not approaching the typical onset limit in β.

For this high β discharge, an accurate equilibrium reconstruction is taken just before onset of the 2/1. This is used as a basis for a "family" of equilibria where a constraint on q_0 is scanned within the uncertainty of the reconstruction, in the range between 0.98 and 1.02, with little or no change in the equilibrium near q=2 and elsewhere. The q profile has moderate shear in the core near q=1, giving a small radius of q=1 surface if it exists. For each $q_{\min}$ the pressure is varied multiplicatively, finding a new equilibrium flux solution for each pressure profile holding the q fixed. The stability of the 2/1 mode for each equilibrium is then computed.

III. Linear Stability Theory

To study the stability and growth rates of the resistive MHD 2/1 tearing mode, the PEST3 code [3] was applied to each of the equilibrium reconstructions, and the NIMROD code [4] was applied to a selective few.

There are in general two possible parities for the independent diverging solution across each rational surface. Interchange perturbations have positive local parity (even poloidal flux perturbation across the $q=m/n$ surface, corresponding to odd radial displacement ξ) and tearing type perturbations have negative local parity. Each parity mode at each resonant surface (largest leading solution) is coupled to all those at every other tresonant surfaces, of both parities, through the outer region solution. For a given toroidal eigenvalue n, PEST3 calculates the 2Nx2N matrix D′ relating the singular coefficients of the two parities that describe the ideal MHD outer region solution. For a torus with N resonant surfaces $q=m/n$ in the plasma, where $m=1,2,\ldots,N$, we write

$$D' = \begin{bmatrix} \Gamma' & B' \\ A' & \Delta' \end{bmatrix} ,$$

where the Δ' matrix is the tearing/tearing submatrix, and couples the two negative parity "tearing" components, Γ' couples the two positive parity components, and A' and B' the negative to positive and positive to negative parities, respectively.

Given the ideal outer region solution, 2/1 island stability depends on the inner reconnection region in the resistive MHD model. The resistive mode growth rate can be determined by matching the outer region matrix to an independently determined matrix describing the corresponding coefficients of the inner layer solutions at each surface. We use the resistive inner layer model of Ref. [5] (GGJ) for tearing modes in the low beta case, and the inner layer model of Ref. [6] (GTGC) for the high beta case, both of which can be expressed as a diagonal $2N$x$2N$ matrix $D(Q)$ where Q is the normalized growth rate. The simplest GGJ analytic inner layer for finite D_R is compared to the numerical result from GTGC, which solves the problem numerically, finding both $\Delta(Q)$ and $\Gamma(Q)$. GGJ solves the problem analytically for $\Delta(Q)$ alone, not $\Gamma(Q)$. In any case, the dispersion relation becomes $\det[D'-D(Q)]=0$ as a matching condition to solve for Q.

IV. Linear Stability at Low Beta

For the low beta plasma the Δ' for the 2/1 surface alone is the Δ'_{ii} matrix element 2,2 and is positive and small at all three times analyzed, as shown in Fig. 1, indicating that it is apparently not sufficient to consider only the ideal MHD solution when studying the marginal stability of the 2/1 island.

The most important effects in the dispersion relation are found to be the resistive interchange parameter D_R and the coupling to the 1/1 surface, which combine with a slowly changing ideal outer region drive to cause instability. Finite D_R, although small, has a stabilizing effect via a positive $\Delta(Q)$ value which must be overcome for instability, despite Δ'-$D(m/n)$ being positive. The coupling can to other surfaces can be combined into a single term $D(m/n)$ through the determinant. The location where each Δ'-$D(m/n)$ curve crosses the $\Delta(Q)$ curve defines the onset of instability. The uncoupled 2/1 surface solution crosses the Δ curve between 1405 and 1505 ms. When the coupling to the 3/1 surface alone is included, all times are unstable as Δ' increases, but the effect is more moderate than the coupling to the 1/1 mode. The stabilizing effect of the coupling to the 1/1 mode is clear from the decrease in the Δ'-$D(m/n)$ and only the 1605 ms case is unstable for these cases. Perhaps what is most important is that in all cases the trend is toward instability with time.

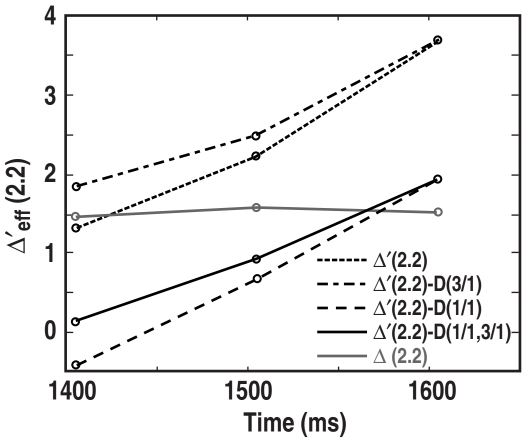

Fig. 1. The uncoupled $\Delta'(2,2)$ and the coupled $\Delta'(2,2)-D(m/n)$ where the argument to D shows which surfaces are included in the coupling, and the minimum inner layer value $\Delta'(2,2)$ of the 2/1 surface for each equilibrium: 1405 ms before onset, 1505 ms just before onset, and 1605 ms just after onset. Instability is found where $\Delta'(2,2)-D(m/n)$ is above $\Delta(2,2)$.

V. Linear Stability at High Beta

In the high beta plasma the outer layer is more sensitive to equilibrium changes [7], and the linear stability is effectively shielded from coupling by high flow shear between surfaces. The interchange parameter H is significant while the inverse beta parameter G is small, indicating that we must consider the interchange parity when analyzing the tearing stability.

From this analysis we find that the approach to q=1 resonance simultaneously causes the 2/1 mode to become unstable and the nonresonant 1/1 displacement to become large, as the ideal β limit rapidly decreases toward the experimental value. As shown in Fig. 2, for $q_{\min}$~1.02 and above the 2/1 tearing stability is weakly dependent on β_N. For $q_{\min}\leq 1.01$ the 2/1 tearing mode becomes strongly unstable as n=1 ideal limit approaches the experimental β. Thus the proximity to the q=1 resonance is critical to 2/1 tearing mode stability, and is responsible for onset of the mode. The ideal beta limit drops strongly as $q_{\min}$ approaches 1. This drives the 2/1 tearing mode linearly unstable as the Δ'_becomes large as the ideal limit approaches. Here the 2/1 mode grows to a large size, leading to loss of confinement.

However, the increasing amplitude of the nonresonant 1/1 component is strongly coupled to the 2/2 harmonic of the unstable 3/2 mode, which is thought to contribute to the current drive sustaining $q_{\min}$ above 1 in these discharges. The increased 1/1 nonresonant amplitudes are visible in Fig. 3 from the NIMROD code. Thus, this suggests the approach to q=1 resonance is self-limiting, but if $q_{\min}$ drops low enough, the 2/1 mode onsets. The current drive and evolution of $q_{\min}$ are discussed in Ref. [7].

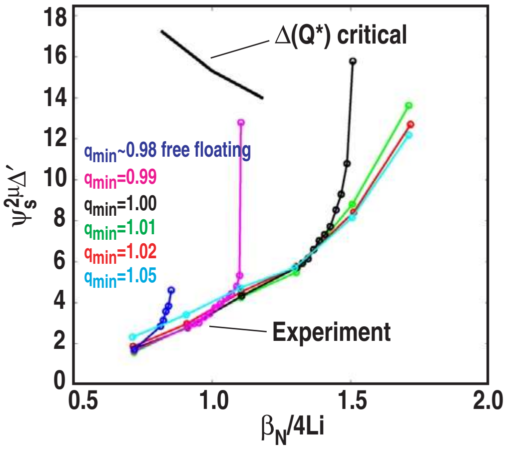

Fig. 2. The $\Delta'(2,2)$ and the minimum inner layer value $\Delta(2,2)$ of the 2/1 surface for the model high β equilibria and varying $q_{\min}$ and β. The experimental β remains fixed while the reduction in $q_{\min}$ reduces the β limit.

VI. Summary

We have shown that the ideal outer region uncoupled Δ' solution alone is insufficient for determining the MHD stability of a tearing mode in toroidal geometry. Accurate equilibrium reconstructions are taken in this study, and qualitative agreement is found between the onset point of the mode

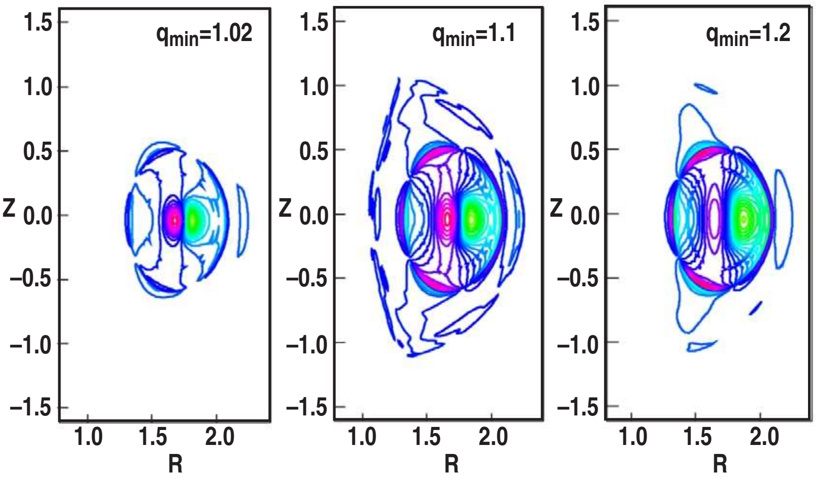

Fig. 3. Contours of the electron temperature perturbation for three high β equilibria with varying q_{min} from NIMROD. The relative amplitude of the 1/1 nonresonant component increases with decreasing q_{min}.

in experiment and that in the analysis. All equilibria considered are ideal stable (including the 1/1 mode) despite the existence of a q=1 surface. This work supports the validity and relevance of classic resistive MHD stability theory of high temperature toroidal plasmas and can lead to testing various physics effects, in the inner layer and the outer region, on mode stability in comparison with experiment.

A simple two fluid dispersion relation was also examined to gain intuition into how the results might change in a more complete, two fluid analysis of the 2/1 onset. The growth rates of the mode were significantly affected and the real frequencies approach the ω_i^* frequency, which far exceeds the MHD growth rates. The trend in the growth rates are similar to the single fluid MHD results in the uncoupled case. However, when coupling to other surfaces is included, two fluid effects are expected to be important in the evolution of the mode. Two-fluid diamagnetic effects were examined only in the uncoupled case, and modify the growth rates significantly. The large drift velocities suggest coupling would be significantly affected. Both electron and ion diamagnetic effects are important at large diamagnetic frequencies $\omega_i^* >> g_{MHD}$ and $T_e \sim T_i$.

In a low β DIII-D discharge, the coupling to the 1/1 mode and the influence of D_R are found to be the most important stabilizing effects in the single fluid picture, and significantly alter the marginal stability point of the mode. In a high β DIII-D discharge just before onset of a 2/1 tearing mode, the tearing stability at q=2 is highly sensitive to q_{min} approaching unity, as a result of the ideal _ limit rapidly decreasing toward the experimental value.

This suggests that by increasing q_{min} even slightly the 2/1 can be avoided in this case, and higher beta values may be accessible. Slowing the rate at which the q_{min} approaches unity might also allow the current drive to prevent further q_{min} reduction and 2/1 onset. In the near future we intend to test the rapid increase in 2/1 β limit with q_{min} in experiment, and investigate the role of nonlinear coupling computationally between the 2/1 and multiple n modes with 1/1 approaching resonance.

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