Effect of the ferromagnetic wall on the plasma stability

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1. Introduction

Low activation ferritic steel is considered as a leading candidate material for fusion reactors. To investigate this perspective, the AMTEX programme was established in Japan [1, 2]. In the dedicated experiments in the JFT-2M tokamak no adverse effect of ferritic steel on the plasma operation and stability was observed [1, 2]. The problem of the plasma stability in the tokamak with ferromagnetic wall has also been addressed theoretically [3-5]. For the JFT-2M, the ferromagnetic effect on the tearing mode stability was found weak [3]. On the contrary, substantial influence of the wall on the plasma stability was claimed in [4, 5] even for sufficiently saturated ferromagnetism, with a conclusion that destabilizing effect of the ferromagnetic wall would have an impact on reactor design [4]. Situation is unclear, and additional analysis is needed, especially in view of the using of ferromagnetic materials in the test blanket modules in ITER. Here the problem is analysed with the approach [6, 7] used earlier to study the Resistive Wall Mode (RWM) effects. The model is based on cylindrical approximation, allowing the mode separation. Compared to [6, 7], two new elements are included now: the finite wall thickness (also, in contrast to the thin wall approximation used in [3]) and the permeability of ferromagnetic wall, which is assumed constant.

2. Theoretical model

In the metal wall with $\sigma = \text{const}$ and $\mu = \text{const}$ (conductivity and magnetic permeability), the magnetic perturbation $\mathbf{b}$ must obey the equation

$$\mu\sigma \frac{\partial \mathbf{b}}{\partial t} = \nabla^2 \mathbf{b} \tag{1}$$

with natural boundary conditions

$$\langle \mathbf{n} \cdot \mathbf{b} \rangle = 0, \qquad \langle \mathbf{n} \times \mathbf{b} / \mu \rangle = 0 \tag{2}$$

at the wall-vacuum interfaces, with brackets $\langle ... \rangle$ meaning the jump across the surface. In the plasma-wall vacuum gap and behind the wall, $\mathbf{b} = \nabla \varphi$ with $\nabla^2 \varphi = 0$.

In the cylindrical approximation, with $\mathbf{b} = \nabla \psi \times \mathbf{e}_z$ and $\psi = \psi_{mn}(r,t)\exp(im\theta - in\zeta)$, equation (1) and the conditions (2) are reduced to (prime means the radial derivative)

$$r(r\psi'_{mn})' - (m^2 + n^2r^2 / R^2)\psi_{mn} = \sigma\mu \frac{\partial \psi_{mn}}{\partial t} r^2 , \tag{3}$$

$$\langle \psi_{mn} \rangle = 0 , \qquad \langle \psi'_{mn} / \mu \rangle = 0 . \tag{4}$$

Equation (3) describes ψ_{mn} in the wall and in vacuum regions ($\sigma = 0$) on the both sides of the wall. Usually the term n^2r^2 / R^2 is disregarded in (3) for the modes with low m and n in a system with $r / R << 1$ (large aspect ratio approximation), which is used below.

In the vacuum

$$\psi_{mn} = gr^m + hr^{-m} , \tag{5}$$

where g and h are the time-dependent constants, different in different regions. In particular, $g = 0$ behind the wall, $r > r_{out} = w + d$, if this region does not contribute to the perturbation ($m > 0$, w is the wall inner radius, and d is the wall thickness). This was the case considered in [3, 4] which corresponds to $\mathbf{b} \to 0$ at infinity. Then (4) gives us, for the outer side of the wall,

$$(r\psi'_{mn} + \hat{\mu} m \psi_{mn})_{wall} = \hat{\mu}(r\psi'_{mn} + m\psi_{mn})_{vac} = 2m\hat{\mu} g r^m_{out} = 0 , \tag{6}$$

where $\hat{\mu} \equiv \mu / \mu_0$, and μ_0 is the permeability of vacuum.

3. Integration through the wall

Integration of (3) through the wall of arbitrary thickness with condition (6) at $r = r_{out}$ yields

$$\hat{\mu}\psi_{in}(\Gamma_m - W_m) = m(\hat{\mu} - 1)(x^{-m}\psi_{mn})\Big|_{in}^{out} , \tag{7}$$

$$\hat{\mu}\psi_{in}(\Gamma_m - U_m) = m(\hat{\mu} + 1)(x^{m}\psi_{mn})\Big|_{in}^{out} , \tag{8}$$

where $\psi_{in} = \psi_{mn}(w)$, $x \equiv r / w$, '*in*' and '*out*' denote the inner and outer sides of the wall,

$$\Gamma_m \equiv -\left(\frac{\mu_0}{\mu} \frac{r\psi'_{mn}}{\psi_{mn}} + m \right)\Bigg|_{w+0} = -\left(\frac{w\psi'_{mn}}{\psi_{in}} + m \right)\Bigg|_{vac} , \tag{9}$$

$$W_m \equiv \frac{\tau_\infty}{\psi_{in}} \frac{\partial}{\partial t} \int_{in}^{out} \psi_{mn} x^{-m+1} dx , \qquad U_m \equiv \frac{\tau_\infty}{\psi_{in}} \frac{\partial}{\partial t} \int_{in}^{out} \psi_{mn} x^{m+1} dx , \tag{10}$$

and

$$\tau_\infty \equiv \mu_0 \sigma w^2 . \tag{11}$$

Either (7) or (8) can be used for deriving the dispersion relation. The first equation is more convenient because it takes the most simple form $\Gamma_m - W_m = 0$ when $\hat{\mu} = 1$, irrespective of the wall thickness. The plasma comes into these equations through the quantity Γ_m, the same that appears in the "thin-wall" equation [6, 7]

$$\tau_w \frac{\partial B_m}{\partial t} = \Gamma_m B_m - \Gamma_m^0 B_m^{ext} \qquad (12)$$

for the amplitude $B_m = im\psi_{in} / w$ of the (m,n) harmonic of the radial perturbed magnetic field at the inner side of wall. Here τ_w is the "wall time",

$$\tau_w \equiv \tau_\infty d / w = \mu_0 \sigma w d\,, \qquad (13)$$

and B_m^{ext} is the part of B_m created by the currents outside the wall ($B_m^{ext} = 0$ in [3, 4]). The meaning and properties of Γ_m are described in [6, 7]. For our purposes here we note that Γ_m is determined by the plasma parameters and does not depend on $\hat{\mu}$ explicitly.

Note also that (12) was derived assuming $\hat{\mu} = 1$. The ferromagnetic effect will be essential when Eqs. (7) or (8) will give an equation different from (12).

4. Thin wall approximation

For tokamaks, $d / w << 1$ is a natural condition. For example, in JFT-2M experiments [2], the thickness of the ferritic wall was about 10 mm, the wall position normalized to the plasma minor radius $w/a \le 1.3$, and $a \le 35$ cm, which corresponds to $d / w \approx 1/45$.

In the limit $d / w \to 0$, the right hand side of (7) and (8) is zero, the difference between ψ_{mn} and ψ_{in} can be disregarded (so-called constant ψ approximation), then

$$W_m \approx U_m \approx \frac{\tau_w}{\psi_{in}} \frac{\partial \psi_{in}}{\partial t}, \qquad (14)$$

and both (7) and (8) yield (12), which does not contain the wall permeability. Therefore, when the thin wall approximation is valid, the wall effect on the plasma stability can be analyzed, in the main approximation, disregarding the ferromagnetic properties of the wall.

This is actually the "zero wall thickness" limit. The wall permeability may enter the dispersion relation in the next approximation when the terms of the order $O(d / w)$ are retained. These corrections proportional to d / w make nonzero the right hand sides of (7) and (8) and break the $\psi = \text{const}$ approximation. Precisely, we obtain

$$\psi_{out} - \psi_{in} \approx -m\hat{\mu} \frac{d}{w} \left(\psi_{in} + \frac{\tau_w}{2m} \frac{\partial \psi_{in}}{\partial t} \right). \qquad (15)$$

This follows from

$$\hat{\mu}\psi_{in}(U_m - W_m) = m\hat{\mu}[(x^{-m} - x^m)\psi_{mn}]\Big|_{in}^{out} - m[(x^{-m} + x^m)\psi_{mn}]\Big|_{in}^{out}, \tag{16}$$

obtained from (7) and (8), the relations $x^m - x^{-m} = 2my + O(y^2)$ and $x^m + x^{-m} = 2 + O(y^2)$ with $y = (r - w)/w << 1$, and

$$U_m - W_m \approx \frac{\tau_w}{\psi_{in}}\frac{\partial\psi_{in}}{\partial t} m \frac{d}{w}. \tag{17}$$

Equations (7), (8) and (15) imply that the first-order corrections to (12) are of the order of $\varepsilon = (\hat{\mu} - 1)md / w$. In the mentioned JFT-2M experiments [1, 2], the specific permeability $\hat{\mu}$ of a low activation ferritic steel was $2 \div 4$, depending on the magnetic field. Similar range was considered in [3-5]. In these cases ε is small, and the ferromagnetic effect on the plasma stability cannot be strong. The same is true in a general case, for realistic parameters. Then, the resistivity is the only important property of the wall material in the stability problem.

5. Conclusions

The presented analysis is based on the equation (1) only, without any assumption on the plasma properties. Therefore, the conclusions are general, they can be applied without restrictions on plasma current or pressure distributions. Note that the numerical modeling in [3] was performed (in cylindrical approximation too) for the plasma with parabolic current density. Almost parabolic current and pressure profiles have been used in [5].

The obtained dispersion relation shows that, in the thin wall approximation, the wall permeability plays a negligible role and can be disregarded. This means that all the previous results for RWM obtained without care of ferromagnetic effects can be considered as quite reliable. This also explains the absence of the expected effects in the JFT-2M tokamak [1, 2].

The model allows us to show, in general agreement with [3, 4], that the increasing wall thickness results in decrease of the stability limit. However, contrary to the statements (but not the results) in [4], at reasonable wall thickness and $\hat{\mu}$ values the effect must be small.

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