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Nonlinear Transport Equations for Tokamak Plasmas in the Classical and in the Pfirsch-Schlüter Regimes

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Introduction

This work is an application of a general theory of the thermodynamic field (TFT) developed in previous works [1]. The characteristic feature of the theory is its purely macroscopic nature. I mean by this not a formulation based on the macroscopic plasmadynamical equations, but rather a purely thermodynamical formulation starting solely from the entropy production and from the transport equations, i.e., the flux-force relations. The law of evolution is not the dynamic law of motion of charged particles, or the set of two-fluid macroscopic equations of plasma dynamics. Rather, the evolution in the thermodynamic space is determined by *postulating* three *purely geometrical* principles (the *straightest path*, the *closeness of the thermodynamic field strength* and the *least action*). From these principles, the nonlinear Classical and the Pfirsch-Schlüter transport equations obtained by solving the thermodynamic field equations of the TFT, in the weak-field approximation, have been derived. These equations determine the nonlinear corrections to the linear ("Onsager") transport coefficients.

The Nonlinear Classical and Pfirsch-Schlüter Transport Equations

In preliminary works it was shown that the nonlinear Classical and Pfirsch-Schlüter transport equations can be cast into the form [1]

$$
\begin{aligned}
&<\hat{q}_\rho^{e(1)}>_{cl}=\Big(<\tilde{\sigma}_\perp>+\frac{2\chi_{cl}}{\pi}<\tilde{\sigma}_\perp\arctan X_{cl}>\Big)g_\rho^{(1)P}-\Big(<\tilde{\alpha}_\perp>+\frac{2\chi_{cl}}{\pi}<\tilde{\alpha}_\perp\arctan X_{cl}>\Big)g_\rho^{e(3)}\\
&<\hat{q}_\rho^{e(3)}>_{cl}=-\Big(<\tilde{\alpha}_\perp>+\frac{2\chi_{cl}}{\pi}<\tilde{\alpha}_\perp\arctan X_{cl}>\Big)g_\rho^{(1)P}+\Big(<\tilde{\kappa}_\perp^e>+\frac{2\chi_{cl}}{\pi}<\tilde{\kappa}_\perp^e\arctan X_{cl}>\Big)g_\rho^{e(3)}\\
&<\hat{q}_\rho^{i(3)}>_{cl}=\tilde{\kappa}_\perp^i g_\rho^{i(3)}
\end{aligned}
\tag{1}
$$

and

$$
\begin{aligned}
&<q_\rho^{e(1)}>_{ps}=c_{11}^e\Big(<r_e>+\frac{2\chi_{ps}}{\pi}<h_e\ \arctan X_{ps}>\Big)g_\rho^{(1)P}-c_{13}^e\Big(<r_e>+\frac{2\chi_{ps}}{\pi}<h_e\ \arctan X_{ps}>\Big)g_\rho^{e(3)}\\
&<q_\rho^{e(3)}>_{ps}=-c_{13}^e\Big(<r_e>+\frac{2\chi_{ps}}{\pi}<h_e\arctan X_{ps}>\Big)g_\rho^{(1)P}+c_{33}^e\Big(<r_e>+\frac{2\chi_{ps}}{\pi}<h_e\arctan X_{ps}>\Big)g_\rho^{e(3)}\\
&<q_\rho^{i(3)}>_{ps}=<r_i>c_{33}^i g_\rho^{i(3)}
\end{aligned}
\tag{2}
$$

where $X_{cl/ps} \equiv \frac{4R^2_{0cl/ps}|X1_{cl/ps}X2_{cl/ps}|}{R^4_{0cl/ps}-(X1^2_{cl/ps}+X2^2_{cl/ps})^2}$ and the expressions of $R_{0cl/ps}$ are given by

$$R_{0cl} = \text{Max} \mid X1_{cl} \mid = \frac{1}{L_H}\text{Max}\left| \tau_e\lambda_{+cl}\sqrt{\frac{T_e}{m_e}}\Big(a_+ + \sqrt{\frac{5}{2}}\, a_-\Big) \right| \tag{3}$$

$$R_{0ps} = \text{Max} \mid X1_{ps} \mid = \frac{1}{L_H}\text{Max}\left| \tau_e K_e\sqrt{\frac{T_e}{m_e}}\Big(\frac{B}{\beta_0} - \frac{\beta_0}{B}\Big)\Big[1.386 + 3.409\Big(1 + \frac{T_i}{T_e}\Big)\Big] \right|$$

with

$$a_\pm = \frac{\sqrt{2}\tilde{\alpha}_\perp}{\sqrt{(\tilde{\sigma}_\perp - \tilde{\kappa}_\perp)^2 + 4\tilde{\alpha}^2_\perp \pm (\tilde{\sigma}_\perp - \tilde{\kappa}_\perp)\sqrt{\Delta_{cl}}}}\Big(\frac{\tilde{\sigma}_\perp - \tilde{\kappa}^e_\perp \pm \sqrt{\Delta_{cl}}}{2\tilde{\alpha}_\perp}\Big) \tag{4}$$

$$\frac{1}{L_H} = \text{Max}\Big\{\frac{\mid \nabla_\rho P \mid}{\mid P \mid}, \frac{\mid \nabla_\rho T_e \mid}{\mid T_e \mid}\Big\} \qquad \Delta_{cl} \equiv (\tilde{\sigma}_\perp - \tilde{\kappa}^e_\perp)^2 + 4\tilde{\alpha}^2_\perp \tag{5}$$

The "scaled" variables $X1{,}2_{cl}$ and $X1{,}2_{ps}$ are defined as follows

$$\begin{pmatrix} \frac{X1_{cl}}{\lambda_{+cl}} \\ \frac{X2_{cl}}{\lambda_{-cl}} \end{pmatrix} = \mathbf{U}_{cl} \begin{pmatrix} g^{(1)P}_\rho \\ g^{e(3)}_\rho \end{pmatrix} \quad ; \quad \begin{pmatrix} \frac{X1_{ps}}{\lambda_{+ps}} \\ \frac{X2_{ps}}{\lambda_{-ps}} \end{pmatrix} = \mathbf{U}_{ps} \begin{pmatrix} \Big(\frac{B}{\beta_0} - \frac{\beta_0}{B}\Big)K_e g^{(1)P}_\rho \\ -\Big(\frac{B}{\beta_0} - \frac{\beta_0}{B}\Big)K_e g^{e(3)}_\rho \end{pmatrix} \tag{6}$$

where $\lambda_{\pm cl}$, $\mathbf{U}_{cl}$ and $\lambda_{\pm ps}$, $\mathbf{U}_{ps}$ are the eigenvalues and the eigenvectors matrixes of the contravariant Onsager matrixes $L^{\mu\nu}_{(e)cl}$ and $L^{\mu\nu}_{(e)ps}$, respectively.

$$L^{\mu\nu}_{(e)cl} = \frac{1}{\tilde{\sigma}_\perp\tilde{\kappa}^e_\perp - \tilde{\alpha}^2_\perp}\begin{pmatrix} \tilde{\kappa}^e_\perp & \tilde{\alpha}_\perp \\ \tilde{\alpha}_\perp & \tilde{\sigma}_\perp \end{pmatrix} \quad ; \quad L^{\mu\nu}_{(e)ps} = \frac{1}{c^e_{11}c^e_{33} - {c^e_{13}}^2}\begin{pmatrix} c^e_{33} & -c^e_{13} \\ -c^e_{13} & c^e_{11} \end{pmatrix} \tag{7}$$

We report, for easy reference, the relations between dimensional and dimensionless ($g^{(1)P}_\rho$ and $g^{\alpha(3)}_\rho$) thermodynamic forces and the dimensional and dimensionless ($q^{\alpha(n)}_\rho$) thermodynamic flows [2]

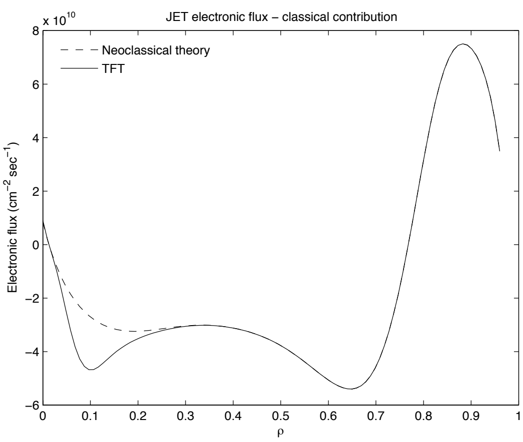
$$g^{(1)P}_\rho = -\Big(\frac{m_e}{T_e}\Big)^{1/2}\tau_e\frac{1}{m_e n_e}\nabla_\rho P \quad g^{\alpha(3)}_\rho = -\sqrt{\frac{5}{2}}\tau_\alpha\Big(\frac{T_\alpha}{m_\alpha}\Big)^{1/2}\frac{1}{T_\alpha}\nabla_\rho T_\alpha$$

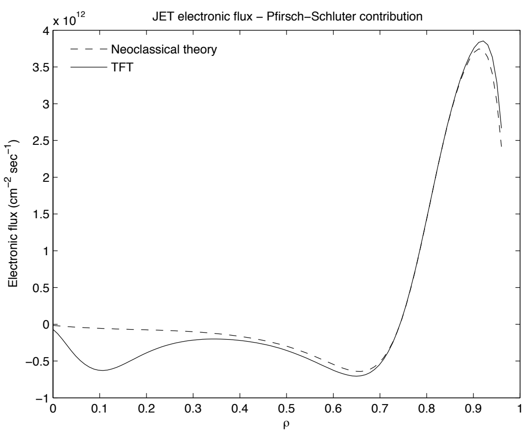
$$q^{e(1)}_\rho = \Big(\frac{m_e}{T_e}\Big)^{1/2}\frac{1}{n_e}\Gamma^e_\rho \quad q^{\alpha(3)}_\rho = \sqrt{\frac{2}{5}}\Big(\frac{m_\alpha}{T_\alpha}\Big)^{1/2}\frac{1}{T_\alpha n_\alpha}J^\alpha_\rho \tag{8}$$

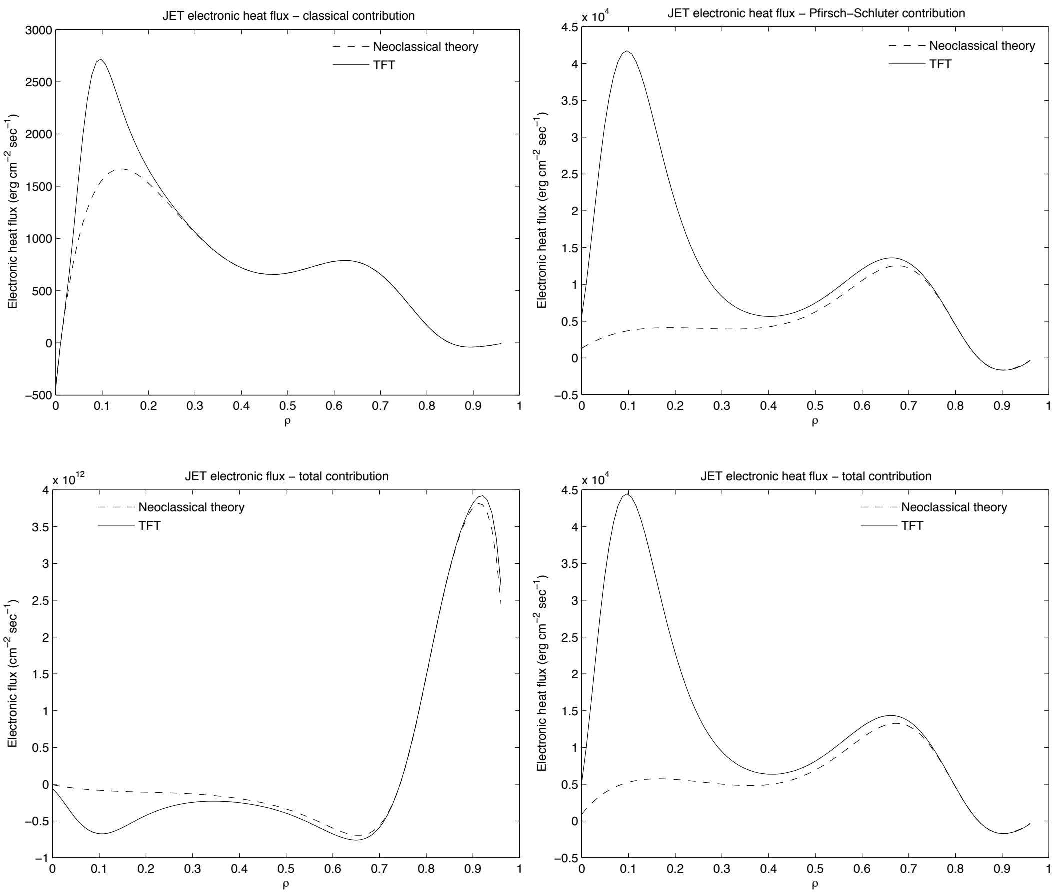
where P, T_α and J^α_ρ indicate the *total pressure*, the *temperature* and the *radial heat fluxes* of species α, respectively. Γ^e_ρ denotes the *radial flux of electrons*. n_α and m_α are respectively the *density* and the *mass* of species α. Coefficients c^α_{mn} denote the *collision matrix elements* and $\tilde{\sigma}_\perp$, $\tilde{\alpha}_\perp$ and $\tilde{\kappa}^\alpha_\perp$ are the dimensionless perpendicular component of the *electronic conductivity*, the *thermoelectric coefficient* and the *thermal conductivity* of species α, respectively (see ref. [2]). K_α are *surface quantities* defined as $K_\alpha = \frac{B_\xi}{B_\theta}\frac{1}{\Omega_{\alpha 0}\tau_\alpha}$ where B_ξ and B_θ denote the toroidal and the poloidal components of the magnetic field, respectively. $\Omega_{\alpha 0}$ and τ_α indicate respectively the *reference Larmor frequency* and the *relaxation* *time* of species α. Functions χ_{cl} and χ_{ps} can be easily computed by imposing that the Plateau diffusion coefficients matches the Banana and the Pfirsch-Schlüter results at the appropriate collision frequency. This procedure determined the value of the functions χ. We found $\chi_{cl} = 1$ and $\chi_{ps} \simeq 11$ [3]. Quantities g and h are respectively defined as $r_\alpha \equiv K_\alpha^2(\beta_0^2/B^2 - 1)$ and $h_\alpha \equiv (K_\alpha B/\beta_0)^2(\beta_0^2/B^2 - 1)^2$. In Eqs (1) and (2) we have introduced the symbol $\beta_0 \equiv < B^2 >^{1/2}$ where B denotes the intensity of the magnetic field and $< \cdots >$ the *magnetic-surface averaging operation*.

Comparison between the TFT predictions and the neoclassical results

A quite encouraging result is the fact that a dissymmetry appears between the P-S ionic and electronic transport coefficients: the latter presents a nonlinear correction, which is absent for the ions and makes the radial electronic coefficients much larger than the former. This is in qualitative agreement with the experiments. The Figures report on the profiles of the theoretical predictions of the TFT and the neoclassical results against the renormalized minor radius $\rho = \frac{r}{a}$ (a indicates the minor radius). Calculations refer to L-mode confined plasmas in the JET reactor with the charge number of ions Z equals to 1. In particular, Figs (1) and (2) compare the radial flux of electrons obtained by the Thermodynamic Field Theory (TFT) and the neoclassical theory, for L-mode plasmas in the classical and Pfirsch-Schlüter regimes, respectively. The sum of the two contributions, according to the TFT and the neoclassical theory, is shown in Fig (3). In Figs (4) and (5) we can find the comparison between the expressions of the radial electronic heat flux obtained by the TFT and the neoclassical theory for L-mode plasmas in the classical and Pfirsch-Schlüter regimes, respectively. Fig (6) compares the sums of the radial flux of electrons in these two regimes obtained by both theories. As we can see, the nonlinear Classical transport coefficients exceed the linear Classi-







cal ones by a factor 2. The electronic nonlinear Pfirsch-Schlüter transport coefficients exceed the electronic linear Classical ones by a factor, which is of order 10^2.

Conclusions

Starting from a general theory of the thermodynamic field, developed in previous works, we found that the fluxes of matter and electronic energy (heat flow) are further enhanced in the nonlinear Classical and Pfirsch-Schlüter regimes. The banana and plateau regimes will be analysed in a forthcoming paper. However, we might already anticipate that preliminary results would show this phenomenon much more amplified in these two regimes. This is in line with the experimental observations.

References

[1] Sonnino G., *Transport processes in magnetically confined plasmas in the nonlinear regime*, *Chaos*, **6**, 1, (2006); Sonnino G. 2006, *A Field Theory Approach to Transport Processes in Confined Plasmas*, sub. for publ. in the review *J. of Mathematical Physics*. **49**, 2028 (1994).

[2] Balescu R. 1988 *Transport Processes in Plasmas. Vols 1 and 2, Elsevier Sci. Publ. B.V.*

[3] Peeters Ph. and Sonnino G. 2006, *Transport Processes in L-mode Confined Plasmas in the JET Reactor* , to be submitted for publ. in the review *Plasma Phys. and Controlled Fusion*.