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Non-linear growth of marginally unstable tearing modes

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1 – Introduction

The onset and evolution of magnetic reconnection events in tokamak plasmas is a potential source for plasma disruptions. The tearing mode, both in a scenario where the drive arises from the equilibrium current density gradient and in a bootstrap current relevant scenario (neoclassical tearing mode-NTM), can play an influential role in the plasma confinement degradation, disruption onset and overall limitation of plasma performance. While significant progress has been made on tearing mode stabilisation, e.g. using schemes based on electron cyclotron or lower hybrid heating and current drive, some of the peculiarities regarding non linear tearing mode evolution have only recently regained some attention. In particular, the recent developments in the understanding of the non-linear evolution of highly unstable tearing modes [1] and the rigorous calculation of the steady state island width (width at mode saturation), for a generic magnetic equilibrium [2,3]. While in the former it is shown that modes characterised by very large $\Delta' a \gg 1$ (**a** the minor radius in a cylindrical equilibrium and Δ' being the stability parameter [4]) may exhibit explosive growth after the usual linear phase, in the latter, the saturated island width is shown to scale approximately linearly with Δ', both in the small and large island limits (relative to a critical width W_c above which temperature tens to equilibrate inside the island separatrix [5]).

In this work, we revisit the problem of non-linear ohmic tearing mode growth at the limiting scenario of a marginal unstable mode, i.e. $\Delta' a \sim 1$. In [2], current density is assumed to be a function of the flux $J \equiv J(\psi)$, i.e. plasma inertia is neglected when compared to $\mathbf{J} \times \mathbf{B}$ forces. Although reasonable for present tokamaks plasmas, characterised by a significantly low electrical resistivity (Reynolds number $\tau_R/\tau_A \gg 10^7$), generalisation to more collisional regimes and in particular for plasmas with finite anomalous perpendicular viscosity should be made with caution. In this work, through a numerical approach using reduced MHD simulations in cylindrical geometry, the existence of saturated states for the limiting scenario of a marginally unstable mode, i.e. $\Delta' a < 4$, where a is the minor radius, is investigated. Contrary to expectations, no steady state solution is obtained for plasmas with anomalous viscosity (Prandtl number $\Gamma = \tau_R/\tau_V$) and magnetic Reynolds number below a certain threshold.

2 – Non linear reduced MHD model

In the large aspect ratio limit, where toroidal effects are negligible, and considering a low-β plasma with no equilibrium rotation, the reduced MHD model [6] is an appropriate model for the study of the non-linear evolution of long parallel wavelength tearing modes. Within such a model, the magnetic field and plasma velocity are conveniently written as: $\mathbf{B} = \mathbf{B}_0 + \tilde{\mathbf{B}} = \nabla\psi_0 \times \mathbf{z} + B_{0z}\mathbf{b} + g(r)\nabla\tilde{\psi} \times \mathbf{b}$ and $\tilde{\mathbf{V}} = g(r)\nabla\tilde{u} \times \mathbf{b}$ where the helical vector $\mathbf{b} = \mathrm{nr}/\mathrm{mR}_0\theta + \mathbf{z}$ is defined and the metrical coefficient $\mathrm{g}^{-1} = |\mathbf{b}|^2 \sim 1$. The collisional Ohm's Law is assumed, i.e., $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta\mathbf{J}$, and, together with Faraday, Ampere's Law and plasma momentum equation leads to the coupled system of tearing mode equations :

$$\frac{\partial\tilde{\psi}}{\partial t} = \mathbf{B} \cdot \nabla\tilde{u} - \eta\tilde{J}_z \tag{1}$$

$$-\rho\left[\frac{\partial}{\partial \mathrm{t}}(\nabla^2\tilde{\mathrm{u}}) + \tilde{\mathbf{V}} \cdot \nabla(\nabla^2\tilde{\mathrm{u}})\right] = [(\mathbf{B} \cdot \nabla)(\mathrm{J}_z)] - \nu\nabla^2\left(\nabla^2\tilde{\mathrm{u}}\right) \tag{2}$$

where anomalous perpendicular viscosity (ν diffusion term) is added to the perturbed plasma dynamics. In this work, we will show that, for Δ′a~1 and considering anomalous plasma viscosity negligible, inertial forces (term $-\rho\tilde{\mathbf{V}} \cdot \nabla(\nabla^2\tilde{\mathrm{u}})$) are as important as electromagnetic forces (term ($\mathbf{B} \cdot \nabla\mathrm{J}_z$)) within the tearing layer and prevent conventional mode saturation.

3 – Simulation results

In our simulations, a parabolic q-profile $q(x)=q_0+(q_a-q_0)x^2$ is considered, where x=r/a with **a** the minor radius and different Δ′a values are obtained by varying q_0. Typically, for a fixed q(x) at the edge, an increasing q_0 yields a more unstable m=2,n=1 mode. The normalised stability parameter Δ′a was determined by solving $[(\mathbf{B} \cdot \nabla)(\mathrm{J}_z)] = 0$ with perfectly conducting wall boundary condition at the plasma edge and ranged from Δ′a=1.8 (q_0=0.95) to Δ′a=3.5 (q_0=1.06). In an inviscid plasma scenario ($\Gamma=\tau_R/\tau_V=10^{-4}$) and for a magnetic Reynolds number $S=8\times10^5$, a series of runs were performed for both limits in Δ′a to study the non-linear mode evolution. As expected, for a quasi-linear mode evolution, i.e., non-linear effects affect just the equilibrium flux, the mode always saturates with a given amplitude (see Figure 1). However, with full non-linear effects, whereas the more unstable mode reaches saturation, regardless of the number of harmonics considered for the evolution, the less unstable mode is unable to reach a steady state (see Figure 1).

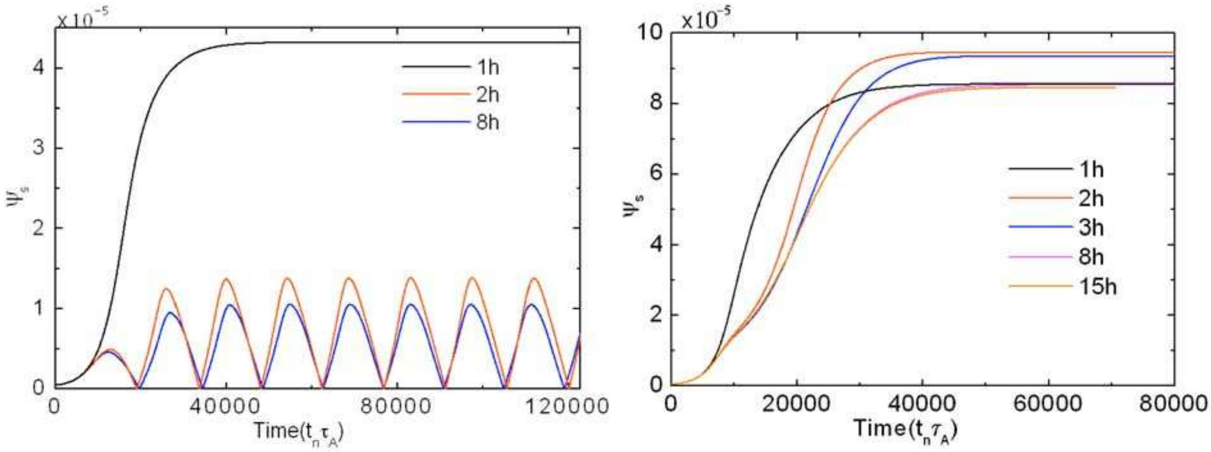

Figure 1 – Time evolution of m=2,n=1 tearing mode with q_0=0.95 (left panel) and q_0=1.06 (left panel) for $S=8\times10^5$ and $\Gamma=\tau_R/\tau_V=10^{-4}$.

In Figure 1 the absolute value of the perturbed flux (first harmonic) at the rational surface is plotted over time, hindering the true oscillatory nature (positive/negative) of the flux evolution. In a poloidal cross section such evolution would correspond to a 90º phase shift between each half cycle of oscillation. It is also worthwhile noticing that although the more unstable case reaches a steady state (right panel), the transition to the non-linear regime is rather abrupt (t_n~8500). In order to understand the nature of the oscillatory behaviour for q_0=0.95, we have performed a scan in both S and Γ. In fact, as can be observed from Figure 2 (left), an increase in S for fixed $\Gamma=10^{-4}$, above a given threshold, eventually changes the final behaviour of the reconnection process.

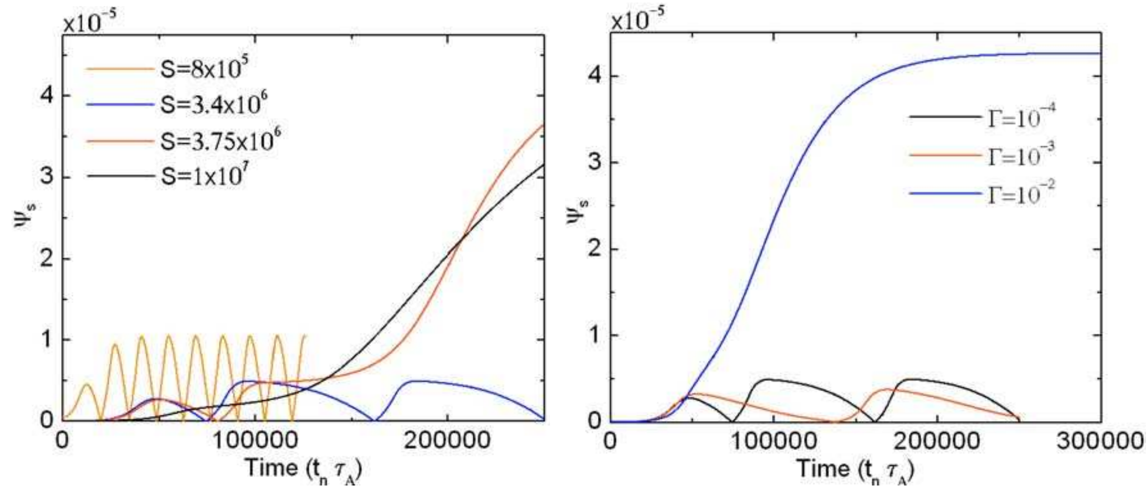

Figure 2 – Transition from oscillatory reconnection evolution to a steady state, saturated island, is favoured for increasing S (left panel with $\Gamma=10^{-4}$) and viscosity (right panel with $S=3.4\times10^6$). Simulations were done with 8 harmonics.

From $S=3.75\times10^6$ to $S=8\times10^5$ the tearing layer width, where both inertia and resistivity are important, doubles but is still significantly smaller than the steady state width using only one harmonic. To get some insight on the basis for the transition, it is instructive to recall that, just outside the layer, $\tilde{u} \propto \nabla^2\tilde{\psi}/Sk_{//}$ where $k_{//}$ is the parallel projection along the equilibrium magnetic field lines of the perturbation wave vector. Consequently, we note that inertial vortices affecting the island are reduced with increasing S, in contrast with $\mathbf{J}\times\mathbf{B}$ forces. To confirm the relevance of inertia in the regime transition, one should naturally expect that, by increasing plasma viscosity, inertial vortices should be smeared out over larger spatial scales, thereby having a reduced influence over the tearing layer. This is in fact what one observes from Figure 2 (right panel).

4 – Conclusions

In this work, through a numerical approach using reduced MHD simulations in cylindrical geometry, the existence of saturated states for the limiting scenario of a marginally unstable mode, i.e. $\Delta'a<4$, where a is the minor radius, was addressed. In contrast with conventional theory that predicts a steady state island, with a width roughly proportional to Δ', no steady state solution is obtained for plasmas with anomalous viscosity and magnetic Reynolds number below a certain threshold. Inertial forces, conventionally neglected when compared to $\mathbf{J}\times\mathbf{B}$ forces, are shown to be responsible for such behaviour.

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