

Tokamak current driven by poloidally asymmetric fuelling

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The fuelling of a tokamak plasma, and the transport losses that balance it, are generally poloidally asymmetric. The transport usually peaks in the outer midplane, while much of the recycling takes place in the divertor. In a single-null magnetic configuration, the X-point region therefore contributes strongly to the refuelling of the plasma. We show that such up-down asymmetric particle transport and refuelling gives rise to a toroidal electric current. This result is similar to earlier work showing that up-down asymmetric electron cyclotron resonance heating drives a toroidal current even if the waves do not interact preferentially with electron travelling in any one particular parallel direction [1]. The current produced by up-down heating, transport or refuelling arises because of toroidal effects and can be understood by an analogy to the usual bootstrap current. The latter is calculated by solving the drift kinetic equation

$$v_{\parallel} \nabla_{\parallel} f_1 - C_e(f_1) = -\mathbf{v}_d \cdot \nabla f_0,$$

for the deviation f_1 of the electron distribution function from the Maxwellian f_0 . The drift velocity \mathbf{v}_d is approximately vertical and points upward if the toroidal magnetic field is in the direction favourable for H-mode access (and the X-point is at the bottom). Mathematically, the right-hand side of the drift kinetic equation then represents a source above the midplane and a sink below it, if f_0 peaks in the centre. In the same way, a bootstrap-like current is produced by any such up-down asymmetric source or sink, including a pure heating source such as that considered in Ref. [1] or a pure particle source considered here.

We calculate this current in various collisionality regimes from the linearised electron drift kinetic equation, averaged over turbulent fluctuations,

$$v_{\parallel} \nabla_{\parallel} f_1 - C_e(f_1) = S(v, \theta), \quad (1)$$

where $C_e(f_1)$ is the linearised electron collision operator, and S represents the up-down asymmetric part of the source and turbulent transport. For an ionisation source, S is isotropic in velocity space and localized to low energies. In the low-collisionality banana regime, we solve

Eq. (1) and find a current

$$j_{\parallel} \simeq -\frac{eqRJ(\theta_*)}{2} \int_0^{\infty} S_0(v) \left[1 + \frac{\alpha v}{v_{Te}} + O\left(\frac{v^2}{v_{Te}^2}\right) \right] 4\pi v^2 dv - (1-G) \langle V_{i\parallel} B \rangle \frac{neB}{\langle B^2 \rangle} + neV_{i\parallel}, \quad (2)$$

if S is localised to a single poloidal location, $S = S_0(v)\delta(\theta - \theta_*)$, on each flux surface. For a poloidally distributed source, the current is an integral of contributions given by Eq. (2). The first term on the right of this equation is the fuelling-driven current and the second term is the electron shielding of the ion current (last term). The shielding coefficient G has been calculated in the literature [2]. The electron thermal speed is denoted by $v_{Te} = (2T_e/m_e)^{1/2}$, and we have written

$$\begin{aligned} \alpha &= 10f_c \left\langle 1 + \frac{H(\theta)B^2}{J(\theta_*)\langle B^2 \rangle} \left(1 - \sqrt{1 - \frac{B(\theta_*)}{B(\theta)}} \right) \right\rangle, \\ J(\theta_*) &= \frac{B^2(\theta_*)}{4} \int_0^{1/B_{\max}} \frac{\langle H(\theta)\sqrt{1-\lambda B} \rangle}{\langle \sqrt{1-\lambda B} \rangle} \frac{\lambda d\lambda}{(1-\lambda B_*)^{3/2}} \\ &\simeq -\frac{\text{sign } \theta_*}{\sqrt{\varepsilon}} \int_0^1 \frac{E(\theta_*/2, k) dk}{[2 - (1 - \cos \theta_*)k^2]^{3/2} E(\pi/2, k)}, \end{aligned} \quad (3)$$

with angular brackets denoting a flux-surface average, E an incomplete elliptic integral, and

$$f_c = \frac{3\langle B^2 \rangle}{4} \int_0^{1/B_{\max}} \frac{\lambda d\lambda}{\langle \sqrt{1-\lambda B} \rangle} \simeq 1 - 1.46\varepsilon^{1/2}, \quad (4)$$

$$H(\theta) = \begin{cases} -\text{sign } \theta_*, & |\theta| < |\theta_*| \\ 0, & |\theta_*| < |\theta| \end{cases}.$$

In the approximate equalities given in Eqs. (3) and (4), we have assumed circular flux surfaces with small inverse aspect ratio, $\varepsilon \ll 1$.

The physical reason why a current arises in response to poloidally localised fuelling is that the ions and electrons spread out over the flux surface in different ways. If \dot{n} ions and electrons are added and removed in unit time and volume, this creates a current density of order $\dot{n}eqR$. If the aspect ratio is large, $\varepsilon \ll 1$, the ion current is approximately shielded by the electron current, $G \ll 1$ in Eq. (2), but a net fuelling-driven electron current remains. This current arises because an up-down symmetric source creates an imbalance between the number of co-trapped and counter-trapped particles. By a co-trapped particle, we mean a trapped particle whose instantaneous parallel velocity is in the same direction as the main plasma current. The trapped electron population thus has a net parallel flow, which is transmitted to the passing population by collisions and creates an overall parallel electron flow relative to the ions. Furthermore, it enhances the current so that it is of order $\dot{n}eqR/\varepsilon^{1/2}$. This mechanism is rather similar to that

producing the bootstrap current, where the disparity between the number of co-trapped and counter-trapped electrons is instead caused by the radial pressure gradient.

In the plateau regime of intermediate collisionality, $1 \ll \nu_* \ll \varepsilon^{-3/2}$, the current is a factor of order $1/\nu_*$ smaller than that found in the banana regime, and equals

$$j_{\parallel} = \frac{\pi \varepsilon e}{8} \int_0^{\infty} \frac{\nu f_{\text{sp}}}{f_{e0}} \langle S(\nu) \sin \theta \rangle 4\pi \nu^2 d\nu,$$

where f_{sp} is the Spitzer function defined by $C_e(\nu_{\parallel} f_{\text{sp}}) = -\nu_{\parallel} f_0$. At high-collisionality, $\nu_* \gg \varepsilon^{-3/2}$, the current changes character and becomes similar to the pressure-driven Pfirsch-Schlüter current. It can be calculated from the continuity equation

$$B \nabla_{\parallel} \left(\frac{n_e V_{e\parallel}}{B} \right) = s_p(\theta),$$

where the right-hand side represents the particle source/sink

$$s_p(\theta) = \int S(\mathbf{v}, \theta) d^3 v.$$

and vanishes on a flux-surface average, $\langle s_p \rangle = 0$, so that the source is balanced by transport in steady state. It follows that the parallel electron particle flux equals

$$n_e V_{e\parallel} = B \int_0^{\theta} \frac{s_p(\theta') d\theta'}{B \nabla_{\parallel} \theta'} + K(\psi), \quad (5)$$

where the integration constant $K(\psi)$ is a flux function. The physical reason why the Pfirsch-Schlüter-like current appears is that the poloidal variation of the source causes the electron pressure, the electron temperature and the electrostatic potential to acquire a poloidal variation, e.g., $p_e = \bar{p}_e(\psi) + \tilde{p}_e(\psi, \theta)$, where $\tilde{p}_e \ll \bar{p}_e$ if the source is weak. The accompanying parallel gradients drive a flow of the electron fluid relative to the ion fluid, i.e., a current, in accordance with the parallel Ohm's law

$$V_{e\parallel} - V_{i\parallel} = -l_{11} \left(\frac{\nabla_{\parallel} \tilde{p}_e}{\bar{p}_e} - \frac{e \nabla_{\parallel} \phi}{\bar{T}_e} \right) - l_{12} \frac{\nabla_{\parallel} \tilde{T}_e}{\bar{T}_e}. \quad (6)$$

The transport coefficients l_{jk} depend on the composition of the plasma and are not needed here. The flux-surface average of Eq. (6) multiplied by B gives $\langle j_{\parallel} B \rangle = 0$, which is characteristic of the Pfirsch-Schlüter current and implies that the current changes sign somewhere on each flux surface. Combining this result with Eq. (5) gives the local current density

$$\frac{j_{\parallel}}{B} = n_e e \left(\frac{V_{i\parallel}}{B} - \frac{\langle V_{i\parallel} B \rangle}{\langle B^2 \rangle} \right) - e \int_0^{\theta} \frac{s_p(\theta') d\theta'}{B \nabla_{\parallel} \theta'} + \frac{e}{\langle B^2 \rangle} \left\langle B^2 \int_0^{\theta} \frac{s_p(\theta') d\theta'}{B \nabla_{\parallel} \theta'} \right\rangle,$$

which is a factor $\varepsilon^{1/2}$ smaller than Eq. (2).

The bootstrap-like current (2) caused by fuelling at the X-point is in the opposite direction to the bulk plasma current (and thus stabilizing to peeling modes) if the ion drift is toward the X-point, and changes direction if the magnetic field is reversed. It is tempting to speculate that its stabilising effect is what facilitates access to H-mode when the drift is in the favourable direction. However, it appears that the fuelling current is smaller than the bootstrap current under typical H-mode conditions. The ratio of fuelling current to edge bootstrap current is

$$\frac{I_f}{I_{bs}} \sim -\frac{\dot{N}eB}{16\pi^2 a p_{ped}}, \quad (7)$$

where p_{ped} is the pedestal pressure and \dot{N} is the volume integral of \dot{n} , i.e., the total number of particles added and removed from the plasma in unit time. This appears to be a small number for pedestal parameters typical of H-modes in large tokamaks.

We close by commenting on the issue of edge plasma rotation. It has been observed in Alcator C-Mod that this rotation depends strongly on the location of the X-point and has intriguing links to the L-H transition [3]. The mechanism invoked to explain the rotation is that most of the transport across the separatrix occurs in the outer midplane, from where plasma flows to the divertor along open lines in the scrape-off layer. Depending on whether the X-point is below or above the midplane, most of this flow is either in the co-current or in the counter-current direction. It is speculated that this scrape-off layer flow causes the plasma inside the separatrix to rotate accordingly. However, our results suggest another mechanism that should be equally important. If the calculation done here for the electrons is instead performed for the ion species, one arrives at the conclusion that up-down asymmetric fuelling/transport drives plasma rotation. This rotation is comparable to that caused by plasma flows in the scrape-off layer, and it thus seems likely that the two mechanisms could operate in parallel.

References

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