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Destabilization of magnetosonic-whistler waves by a relativistic runaway beam

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The presence of a magnetosonic-whistler wave instability may be the reason for the observation that the number of runaway electrons produced during disruptions in large tokamaks depends sensitively on the magnetic field strength. A linear instability growth rate of these waves in the presence of a runaway avalanche is calculated both perturbatively and by numerical solution of the dispersion equation.

During the current quench in a tokamak disruption, a large toroidal electric field is induced, which sometimes generates a beam of highly relativistic runaway electrons with energy of order 20 MeV. These energetic electrons can cause damage to the wall, which is a potentially serious problem for reactor-scale tokamaks with large currents. In such devices, it is expected that the runaway beam may form particularly easily because of the efficacy of the so-called "runaway avalanche" mechanism [1]. In a runaway avalanche, existing runaway electrons generate new ("secondary") ones in collisions at close range with thermal electrons. The strength of the avalanche increases exponentially with the plasma current, and will be the dominant mechanism in the next generation of tokamaks. In present experiments, up to about a half the pre-disruption current can be converted to runaways, and it is feared that this fraction will rise in future devices because of avalanching. However, it is observed the number of runaway electrons generated varies widely between different disruptions for reasons that are not understood. In particular, several tokamaks have reported that no runaway generation occurs unless the magnetic field exceeds 2.2 T [3, 4]. In the present paper, we explore a possible reason for this observation.

The runaway beam has a strongly anisotropic velocity distribution function and may excite various kinetic instabilities. Previous work has analyzed a number of such possible instabilities using simple models for the runaway distribution function. We analyze the linear instability of magnetosonic-whistler waves destabilized by secondary runaways at Doppler-shifted harmonics of the cyclotron frequency. The unstable wave frequency will be shown to be well below the non-relativistic electron cyclotron frequency ω_{ce} but above the ion cyclotron frequency ω_{ci}. The most important resonant interaction occurs when $\omega - k_{\|} v_{\|} = -\omega_{ce}/\gamma$ (anomalous Doppler resonance), where ω is the wave frequency, $k_\parallel$ and $v_\parallel$ are the wave number and particle velocity parallel to the magnetic field, and γ is the relativistic factor. The free-energy source driving the instability is the anisotropy of the electron distribution caused by the electric field accelerating the runaway electrons. When the degree of anisotropy exceeds a critical level, unstable waves are excited at the anomalous Doppler resonance, and the interaction with these waves leads to pitch-angle scattering of resonant electrons. We assume that the dominant damping mechanism in the cold post-disruption plasma is collisional damping, which then determines the instability threshold.

Runaway distribution function

The distribution of secondary runaways is obtained by solving the kinetic equation and using the Rosenbluth-Putvinskii [1] avalanche growth rate as a boundary condition:

$$f(p_\parallel, p_\perp, t) = \frac{C}{p_\parallel} \exp\left(\frac{(E-1)t/\tau - p_\parallel}{c_Z} - \frac{\alpha p_\perp^2}{2p_\parallel}\right), \tag{1}$$

where $p = \gamma v/c$ is the normalized relativistic momentum, $\alpha = (E-1)/(1+Z)$, $E = e|E_\parallel|\tau/m_{e0}c$ is the normalized parallel electric field, $n_r = 2\pi c_Z C \alpha^{-1} \exp[(E-1)t/\tau c_Z]$ is the runaway density, $\tau = 4\pi\varepsilon_0^2 m_{e0}^2 c^3 / n_e e^4 \ln\Lambda$ is the collision time for relativistic electrons, Z the effective ion charge and $c_Z = \sqrt{3(Z+5)/\pi}\ln\Lambda$.

Stability analysis

Assuming $|k| \gg |k_\parallel|$, $\omega_{ci} \ll \omega \ll \omega_{ce}$, $k_\parallel^2 c^2/\omega_{pi}^2 \gg 1$, $k_\perp^2 v_{Te}^2 \ll \omega^2$, $\omega\omega_{ci} \ll k^2 v_A^2$, the dispersion relation of the fast wave can be simplified to

$$k^2 v_A^2 \left(1 + \frac{k_\parallel^2 c^2}{\omega_{pi}^2}\right) - \omega^2 \left(1 + \frac{k^2 v_A^2}{\omega_{ci}\omega_{ce}}\right) = \frac{\omega_{ci}^2 \omega^2}{\omega_{pi}^2}\left[\left(1 + \frac{k^2 v_A^2}{\omega_{ci}^2}\right)\chi_{11}^r + \left(1 + \frac{k_\parallel^2 v_A^2}{\omega_{ci}^2}\right)\chi_{22}^r - 2i\frac{\omega}{\omega_{ci}}\chi_{12}^r\right],$$

where $v_A = c\omega_{ci}/\omega_{pi}$ is the Alfvén velocity and the superscript ^{r} denotes the runaway contribution to the dielectric tensor. The instability growth rate for a small perturbation due to runaways $\omega = \omega_0 + \delta\omega$, where $\gamma_i = \mathrm{Im}\delta\omega$, is given by $\gamma_i/\omega_0 = -(k^2 v_A^2/2\omega_{pi}^2)\mathrm{Im}\chi_{11}^r$ where the runaway contribution to the susceptibility is

$$\chi_{11}^r = 2\pi \frac{\omega_{pr}^2 \omega_{ce}^2}{n_r k_\perp^2 \omega c^2} \int_0^\infty dp_\perp \int_{-\infty}^\infty dp_\parallel \sum_{n=-\infty}^\infty \frac{n^2 J_n^2(z)\left[\frac{\partial f}{\partial p_\perp} + \frac{k_\parallel c}{\omega\gamma}\left(p_\perp \frac{\partial f}{\partial p_\parallel} - p_\parallel \frac{\partial f}{\partial p_\perp}\right)\right]}{\gamma(\omega - k_\parallel c p_\parallel/\gamma - n\Omega)}.$$

where $\omega_{pr} = \sqrt{n_r e^2/m_{e0}\varepsilon_0}$ is the non-relativistic runaway electron plasma frequency, J_n is the Bessel function of the first kind and of order n, and $\Omega = eB/m_e = \omega_{ce}/\gamma$, $z = k_\perp c p_\perp/\omega_{ce}$. Substituting the runaway electron distribution from Eq. (1) into the expression for γ_i and evaluating the derivatives we obtain:

$$\frac{\gamma_i}{\omega_0} = \frac{\hat{C}}{4a_{-1}^2}\left\{K_\perp^2 K_\parallel (1-y) I_0(\lambda) + \left[2a_{-1}b_{-1} + K_\perp^2 K_\parallel (1-y)\right] I_1(\lambda)\right\} e^\lambda \tag{2}$$

where $y = \omega_0/k_\parallel c$, $K_\perp = k_\perp c/\omega_{ce}$, $K_\parallel = k_\parallel c/\omega_{ce}$, $\lambda = K_\perp^2/(2a_{-1})$, $a_{-1} = \alpha K_\parallel(y-1)/2$, $b_{-1} = \alpha(1-y) - K_\parallel(1-y) - 1/c_Z$, I_n are modified Bessel functions of the first kind and of order n, and $\hat{C} = (1-y)(\pi\alpha\omega_{pr}^2/2c_Z\omega_{pi}^2)(k^2v_A^2/\omega_0^2)(k_\parallel^2/k_\perp^2)\exp\{-1/[c_Z K_\parallel(1-y)]\}$. Assuming that $\lambda \ll 1$ and $\alpha \gg K_\parallel$ the growth rate can be simplified to

$$\gamma_i(\omega_0, k, k_\parallel) = \frac{\pi}{4c_Z}\frac{\omega_{pr}^2}{\omega_{pi}^2}\frac{k^2v_A^2}{\omega_0}\exp\left[\frac{-\omega_{ce}}{(k_\parallel c - \omega_0)c_Z}\right] \tag{3}$$

to the lowest order in λ. At low frequencies the growth rate increases monotonically, and at higher frequencies we can use the whistler branch of the dispersion relation ($\omega_0 = kk_\parallel v_A c/\omega_{pi}$). The growth rate of the fastest growing wave can be obtained using the condition $\partial\gamma_i(k_\parallel, k)/\partial k_\parallel = 0$, leading to $c_Z k_\parallel c(1 - kv_A/\omega_{pi}) = \omega_{ce}$. This condition can be used with (3) to derive the maximum growth rate. The $\gamma_i(k_\parallel)$ function also has a maximum, determined by the $\partial\gamma_i/\partial k_\parallel = 0$ condition, giving $k_\parallel c = 2\omega_{ce}/c_Z$ so that $\gamma_i^{\max} = 1.3\cdot 10^{-9} n_r/B_T$ (where B_T denotes B in Tesla) is the growth rate of the most unstable mode. The most unstable wave has wave number $kv_A/\omega_{pi} = 1/2$, parallel wave number $k_\parallel c = 2\omega_{ce}/c_Z$, and wave frequency $\omega_0 = \omega_{ce}/c_Z$.

In the cold post-disruption plasmas collisional damping dominates and the wave is destabilized when the growth rate is larger than the electron-ion collisional damping rate: $\gamma_d = 1.5\tau_{ei}^{-1}$ [2]. The threshold $\gamma_k \equiv \gamma_i - \gamma_d > 0$ can be written as

$$\frac{n_r}{n_e} > \frac{Z^2 B_T}{20 T_{eV}^{3/2}}. \tag{4}$$

where T_{eV} is the background plasma temperature in eV and n_r/n_e is the fraction of the runaways. This inequality is the local threshold for the instability of the magnetosonic-whistler wave, where every quantity is to be understood at a given location.

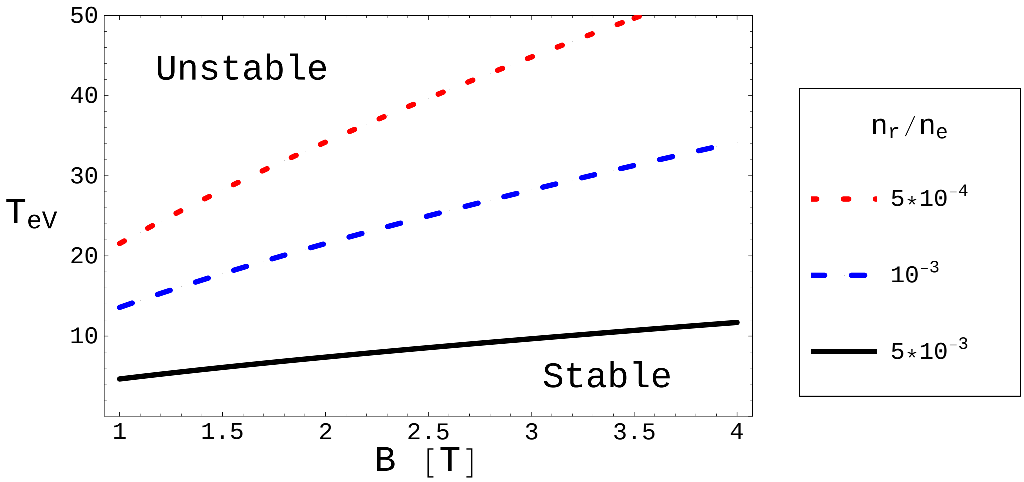

Figure 1. Stability threshold from Eq. (4) for $Z = 1$ and different runaway fractions.

The results of the perturbative stability analysis are confirmed by the numerical solution of the dispersion relation.

Quasi-linear analysis

The distribution of the relativistic runaway electrons evolves in time according to

$$f(p_\perp, p_\|, t) = \frac{C}{(2\alpha\hat{\tau}(t) + p_\|)} \exp\left(\frac{(E-1)t/\tau - p_\|}{c_Z}\right) \exp\left(-\frac{\alpha p_\perp^2}{2(2\alpha\hat{\tau}(t) + p_\|)}\right) \quad (5)$$

Thus, pitch-angle scattering increases the mean perpendicular energy linearly with time. Assuming a narrow spectrum of unstable waves centered around $k_c = \omega_{pi}/2v_A \simeq 3\cdot 10^3 n_{20}/B_T$ and $k_\| \simeq \omega_{ce}/c \ln\Lambda \simeq 30 B_T$ and assuming that the time-evolution of the spectral energy is $W_k(t) = \frac{1}{2} T_e e^{2\gamma_k t}$, the time scale for perpendicular energy increase can be estimated to be $t = (1/2\gamma_k)\ln\left(\hat{\tau}\gamma_k m_e^2 c^3/\pi^3 e^2 T_e k_c^2\right) \simeq 3\cdot 10^{-7}$ s, for $\gamma_k \simeq 10^8$ s^{-1}, $\gamma = 40$, $T_e = 10$ eV and $\hat{\tau} \simeq p_\|/(2\alpha) \simeq 0.1$.

Conclusions

The threshold of the instability depends on the fraction of runaways, the magnetic field and the temperature of the background plasma. The quasi-linear analysis shows that the main result of the instability is rapid pitch-angle scattering of the runaway electrons. It appears possible that this instability could explain why the number of runaway electrons generated in tokamak disruptions depends on the strength of the magnetic field [3, 4]. One reason for the absence of runaways could be that the whistler instability scatters runaways and prevents the beam from forming. The observed critical field is of the same order of magnitude as our instability threshold. Taking, for instance, a plasma density $n_e = 3\cdot 10^{19}$ m^{-3} and a runaway current density $j_r = n_r ec = 2$ MA/m^2 typical of JET, gives a threshold temperature $T_e = 17$ eV (for $Z = 1$). Experimentally, the post-disruption temperature is highly uncertain, but is commonly believed (on various grounds) to be around 10 eV. It is thus clear that the instability threshold is in the right parameter range.

References

[1] M. N. Rosenbluth, S. V. Putvinski, Nuclear Fusion, **37**, 1355, (1997).
[2] M. Brambilla, Phys. Plasmas, **2**, 1094, (1995).
[3] R. D. Gill, B. Alper, M. de Baar *et al.*, Nuclear Fusion, **42**, 1039 (2002).
[4] R. Yoshino, S. Tokuda, Y. Kawano, Nuclear Fusion, **39**, 151, (2000).

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