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Numerical solution of continuity equation with a flux non-linearly depending on density gradient

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Abstract

An approach to integrate transport equations with fluxes being complex non-linear functions of physical parameters and their gradients, as it is predicted by theoretical models for micro-instabilities in plasma, is proposed. This approach operates without any splitting of the flux on diffusive and convective components normally involved in transport calculations.

Introduction

By considering transport processes in fusion plasma it is conventional to speak about such characteristics as particle and heat diffusivities, advection velocity etc. This approach is originated in the traditional view on the mass and heat transfer as caused by collisions of individual particles. In toroidal fusion plasmas such a situation is described by the neoclassical theory. However, diverse micro-instabilities, developing in these plasmas, lead to turbulence, tremendously enhancing mass and heat transfer [1]. The resulting anomalous fluxes are complex non-linear functions of the parameter spatial gradients. By computing profiles of the plasma parameters, these fluxes are normally splitted on diffusive and convective contributions in order to apply well developed approaches for numerical integration of the second order differential equations. Such a separation serves also as an approximate tool for interpretation of experimental data in customary concepts of diffusion and advection. However, there is not any definitive answer to the question: are the individual transport coefficients, both reconstructed from experimental measurements under usually ambiguous assumptions about the time and spatial behavior of these characteristics and obtained by a splitting of theoretically predicted fluxes, unique? Therefore, development of direct methods for integration of transport equations without flux splitting on diffusive and convective contributions would be very helpful in order to clarify this situation and to offer a firm basis for the prediction of parameter profiles in future devices. In the present contribution such an approach is elaborated and demonstrated on the example of the well known Weiland transport model [1] for the charged particle flux.

Basic equations

Time evolution of the plasma density n is governed by the continuity equation averaged over the magnetic surfaces:

$$\frac{\partial n}{\partial t}+\frac{1}{r}\frac{\partial\left(r\Gamma_{r}\right)}{\partial r}=S \tag{1}$$

where Γ_r is the density of the charged particle flux in the radial direction r and S is the density of the plasma source due to ionization of neutral particles produced by recycling and neutral beams. The transport model gives Γ_r as a non-linear function of n, $\partial_r n$ and other parameters p_j: $\Gamma_r = \Gamma_r\left(n, \partial_r n, p_j\right)$. In this study p_j are assumed as known functions of r. After replacing $\partial n/\partial t$ with $(n-n_-)/\tau$, where $n = n(t,r)$, $n_- = n(t-\tau, r)$ and τ is a small enough time step, Eq.(1) is multiplied by r and integrated from $r = 0$ with the symmetry condition $\Gamma_r(r=0) = 0$ taken into account. As a result one gets:

$$\Gamma_r = \Phi(r) \tag{2}$$

with

$$\Phi(r) \equiv \frac{1}{r}\int_0^r \left(S - \frac{n-n_-}{\tau}\right) r dr \tag{3}$$

With the known density profile at the previous time moment, $n_-(r)$, and some approximation for $n(r)$, one can compute $\Phi(r)$. The calculation of the next approximation to $n(r)$ is started at the last closed magnetic surface (LCMS), $r = a$, where the e-folding length δ is prescribed: $\partial_r n = -n/\delta$. Therefore, Eq.(2) provides a non-linear algebraic equation for $n(a)$:

$$\Gamma_r\left[n(a), -n(a)/\delta,\ p_j(a)\right] = \Phi(a) \tag{4}$$

In order to determine the density at $a-h$, where h is the spatial grid increment, the density gradient at $a-h$ is estimated as $[n(a)-n(a-h)]/h$. As a result, Eq.(2) provides the following equation for $n(a-h)$:

$$\Gamma_r\left[n(a-h), (n(a)-n(a-h))/h,\ p_j(a-h)\right] = \Phi(a-h) \tag{5}$$

When $n(a-h)$ has been found, this procedure is continued to the plasma axis, providing a new approximation for the density profile, $n_{new}(r)$. The new approximation to Φ is calculated according to the relation:

$$\Phi_{new} = (1-A_{mix})\Phi + A_{mix}\Phi(n_{new})$$

where $\Phi(n_{new})$ is determined from Eq.(3) with $n(r) \equiv n_{new}(r)$ and $A_{mix} \leq 1$ is a relaxation factor whose maximum level is restricted by the convergence of calculations.

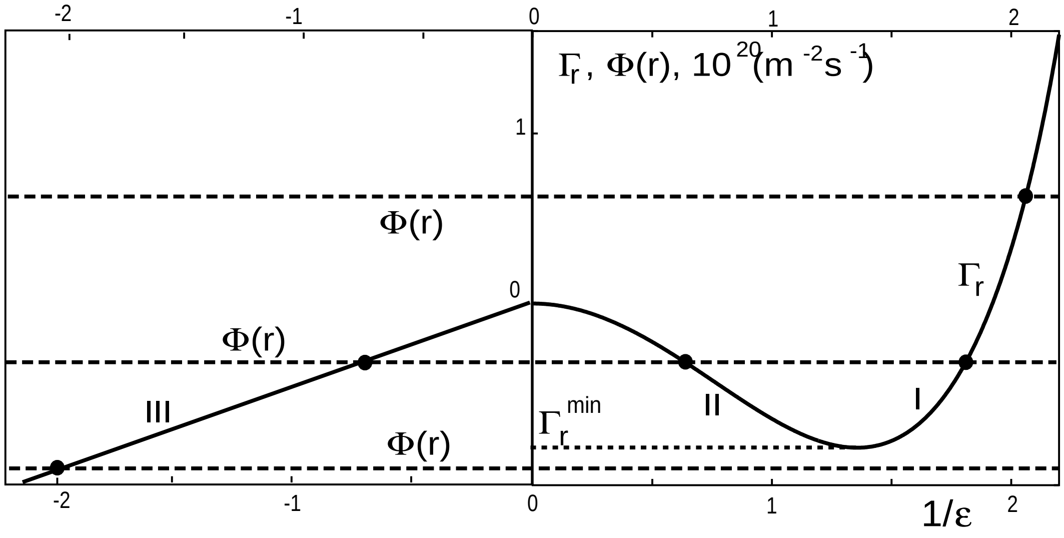

Figure 1: Particle flux density versus the parameter $1/\varepsilon = R/2L_n$ computed with the Weiland transport model [1] for TEXTOR parameters [2].

Example of application

In order to demonstrate the approach proposed, we apply this to the continuity equation with the particle flux Γ_r given by the Weiland transport model, see Eqs.(5.181), (5.182), (5.188) and (5.191) in Ref.[1]. In order to describe the case with a positive density gradient, which may be realized, e.g., on the density ramp stage, this model is extend by assuming $\Gamma_r(\partial_r n > 0) = -D_0 \partial_r n$ with $D_0 = 0.1 m^2/s$ henceforth. Figure 1 displays Γ_r versus the parameter $\varepsilon = -\frac{2}{R}\frac{n}{\partial_r n}$ computed for the conditions of the plasma interior in the tokamak TEXTOR [2]: $R = 1.75m$, $r = 0.3m$, $B_T = 2.25T$, $q = 2, s = 1$, $n = 4 \times 10^{19} m^{-3}$, $T = 500eV$, $L_T = 0.3m$. The fact that Γ_r can be directed towards the plasma axis even for a negative density gradient is normally interpreted as an inward particle pinch.

For $\Phi(r) > 0$ the equality (2) is possible at a single ε and, thus, an unique $n(r)$ can be found. For $\Gamma_r^{min} \leq \Phi(r) \leq 0$ there are, however, three possible ε and we select the one which corresponds to $n(r)$ being the closest to $n(r+h)$. An unique solution exists again for $\Phi(r) < \Gamma_r^{min}$. Since at the LCMS, $r = a$, Γ_r is always positive, $\varepsilon(a)$, $n(a)$ and, thus, the total density profile are defined uniquely. At the positions where $\Phi(r) = 0$ or $\Phi(r) = \Gamma_r^{min}$ the density gradient undergoes a sharp change between I and III branches of the $\Gamma_r(\varepsilon)$ curve. The gradient values corresponding to the unstable branch II can not be realized in the framework of the transport model applyied.

The particle source contribution from the neutral beam heating is determined by the beam power P_b, the energy of injected particles E_b, and the shape of the power deposition. The latter

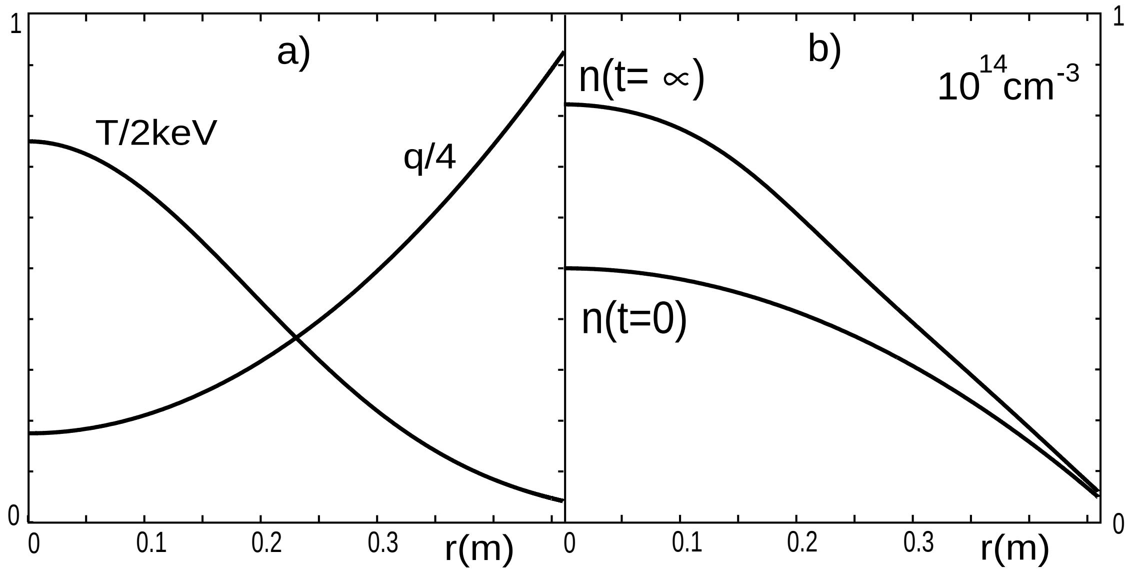

Figure 2: Radial profiles of the adopted plasma temperature, safety factor (a), initial density and found in computation steady state plasma density(b).

is assumed as a Gaussian one with the half-width r_b. The recycling source is determined by the neutral density computed in a diffusive approximation [3]. At the LCMS a certain probability R_{rec} for neutrals to recycle back into the plasma after recombination of ions and electrons on the wall is assumed. Computation have been performed for the TEXTOR parameters above and $P_b = 1.5MW$, $E_b = 50keV$, $r_b = 0.3m$, $a = 0.46m$, $R_{rec} = 0.98$, $\delta_n = 0.1m$. Figure 2a displays the adopted radial profiles of the plasma temperature T, assumed the same for electrons and ions, and of the safety factor q. Figure 2b shows the assumed initial plasma density profile, $n(t = 0)$, and that found in computations for the steady state, $n(t = \infty)$. Although any comparison with the experiment is out the scope of this contribution, the latter profile is in a good agreement with the one found under conditions of the radiation improved mode [2]

Conclusion

An approach to integrate continuity equation with fluxes, predicted by theoretical transport models and being complex non-linear functions of parameter gradients, is proposed. Computations performed with the Weiland transport model show stable convergence of iterations.

References

[1] J.Weiland, Collective Modes in Inhomogeneous Plasma, Bristol, IPP (2000).

[2] M Z Tokar et al., Plasma Phys. Control. Fusion **41**, B317-B327 (1999)

[3] M.Z.Tokar, Plasma Phys. and Contr. Fusion **36** (1994) 1819-1844.