

Axisymmetric Mode Stability in Tokamaks with Reversed Current Density

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The "current hole" tokamak regimes with nearly zero or negative toroidal current in the central region feature good plasma confinement properties and may be naturally attained in the advanced tokamak (AT) regime.

Resistive $n = 0$ stability of the circular cylinder with reversed current was investigated in [1]. Non-linear MHD modeling of the toroidal plasma evolution including off-axis current drive, show that the $n = 0$ activity leads to cyclic reconnection events which clamp the central current density at approximately zero [2]. These results could be due to the non existence of axisymmetric equilibria with nested flux surfaces [3]. In this paper we investigate the stability properties of reversed current density equilibria with axisymmetric islands.

The stability analysis of equilibrium configurations with islands requires the development of new numerical tools capable to deal with arbitrary topology of the magnetic surfaces. In particular, the most general and promising approach is to renounce the idea of using of magnetic coordinates and projections of the plasma displacement vector onto magnetic surfaces. For that purpose, a code for ideal MHD stability calculations on unstructured triangular grids was developed. The new code is coupled to an unstructured grid MHD equilibrium code [4] capable to treat equilibria with axisymmetric magnetic islands.

The code was verified against the KINX code results [6] for free boundary $n = 0$ instability. Preliminary results for reversed current equilibria with islands show a possibility of ideal internal $n = 0$ instability – a novel property not possible for tokamak plasma with nested flux surfaces.

1. Ideal MHD stability of force-free configurations: the problem formulation

For stability modelling without use of special coordinates and magnetic surface projections the original form of plasma potential and kinetic energy functionals can be used:

$$W_p = \frac{1}{2} \int \left\{ |\vec{Q}|^2 + \vec{j} \cdot \vec{\xi} \times \vec{Q} + (\nabla \cdot \vec{\xi})(\vec{\xi} \cdot \nabla p) + \gamma p (\nabla \cdot \vec{\xi})^2 \right\} d^3r, \quad \vec{Q} = \nabla \times (\vec{\xi} \times \vec{B}), \quad (1)$$

$$K_p = \frac{1}{2} \int \rho |\vec{\xi}|^2 d^3r. \quad (2)$$

In the force-free case, $\vec{j} = \alpha \vec{B}$, $p = 0$, they can be rewritten in terms of electric field perturbation amplitude $\vec{E} = i\omega \vec{e}$, $\vec{e} = -\vec{\xi} \times \vec{B}$ (time dependence $e^{i\omega t}$ is assumed for eigenvalue problem):

$$W_p = \frac{1}{2} \int \left\{ |\nabla \times \vec{e}|^2 - \alpha \vec{e} \cdot \nabla \times \vec{e} \right\} d^3r, \quad K_p = \frac{1}{2} \int \rho |\vec{e}|^2 / B^2 d^3r, \quad (3)$$

combined with the requirement $(\vec{e} \cdot \vec{B}) = 0$.

In vacuum region an analogous potential energy functional

$$W_v = \frac{1}{2} \int \left\{ |\nabla \times \vec{e}|^2 + \sigma |\vec{e}|^2 \right\} d^3r, \quad (4)$$

is supplied with regularizing term with small parameter σ making vacuum subproblem well defined. The boundary condition is evident from continuity of tangential projection of \vec{e} : the projection must vanish on ideal wall. So the MHD stability problem is reduced to minimization of the Rayleigh quotient $\omega^2 = \text{inf}(W_p + W_v) / K_p$ with quadratic functionals (3),(4) over all admissible functions \vec{e} satisfying the mentioned constraints.

For tokamak modelling we use the standard equilibrium magnetic field representation, $\vec{B} = \nabla\psi \times \nabla\phi + f\nabla\phi$, with poloidal field function ψ being the solution of Grad-Shafranov equation:

$$-R^2\nabla \cdot \left(\frac{\nabla\psi}{R^2} \right) = R^2p' + ff' \quad (5)$$

with $p' = 0$ and piece-wise constant/linear ff' for the first experiments.

2. Approximation on unstructured grids

To approximate unknown vector \vec{e} on triangular grid we choose the following longitudinal and poloidal projections: $\vec{e} = e_\phi \nabla\phi/|\nabla\phi| + \vec{e}_{pol}$ and different types of finite elements for them: standard node-based "hat"-functions W_i for e_ϕ and so called edge-based Whitney elements [5], $W_{mn} = W_m \nabla W_n - W_n \nabla W_m$, for \vec{e}_{pol} . The last function has continuous tangential projection on the edge (m, n) and zero tangential projections on all other edges. The choice looks well suited with the goal to approximate the operator " $\nabla \times$ " and corresponding boundary conditions: just set the homogeneous Dirichlet conditions for all nodes and edges on the ideal wall.

For approximation of the terms without derivatives we exploit the idea of hybrid finite elements and use the scalar and vector functions which are piece-wise constant in cells and have the same integral as the original ones.

Triangular grid with N nodes contains approximately $2N$ cells and $3N$ edges. So we have N degree of freedom for e_ϕ and roughly $3N$ degree of freedom for e_{pol} .

To provide the constraint, $(\vec{e} \cdot \vec{B}) = 0$ in plasma region, one of the possible approaches is to introduce Lagrange multipliers in grid nodes; it extends the vector of unknowns with additional N elements;

The method of inverse iterations with the complete LU decomposition is used to find several main eigenvalues/functions. The postprocessing step includes reconstruction and plotting of the displacement vector, $\vec{\xi} = \vec{e} \times \vec{B}/B^2$.

The equilibrium and stability calculation use the same grids which are optionally adapted to the solution features (e.g. jump in the current density) [4]. For tests we used also triangulated structured grids taken from CAXE/KINX calculations [6].

3. Axisymmetric modes: tests and benchmarks

Study of $n=0$ mode stability is a natural non-trivial step. This case corresponds to minimal number of nonzero terms to be approximated and allows to work in real arithmetic.

The following tests were performed:

1) Vacuum eigen-value problem, $-\nabla \times \nabla \times \vec{e} = \lambda \vec{e}$, in circular cylinder with no constraints. In this case the components e_ϕ and \vec{e}_{pol} are decoupled and the exact solution is known. The proposed approximation seems free of spectral pollution and shows the second order convergence on uniformly refined grids.

2) Stable spectrum for tokamak equilibrium with constant current density and circular shape, fixed boundary. Comparison with KINX: no numerical destabilization found.

3) Vertical instability of elongated tokamak equilibrium, free boundary. Constant current density, $q_{bou} = 1$, elongation=1.6, similar wall at $r_w = 2.5a$. The most unstable function is close to rigid vertical displacement. There are also several other negative eigenvalues (numerically destabilized?) showing tendency to vanish on finer grids.

4. Reversed current density: internal $n=0$ instabilities

Preliminary results on ideal $n = 0$ stability are obtained for configurations with reversed current density. The first series of the configurations are based on the eigen-functions of Grad-Shafranov operator which can be considered as force-free equilibria with a toroidal current density $j_\phi = \lambda\psi/R$ and different topology of the magnetic surfaces [3].

The configurations with not-nested surfaces (both "central" and "dipole" type) were found to be unstable against axisymmetric modes even for fixed boundary (ideal wall at plasma boundary)(Fig.1,2).

The typical values of the negative eigenvalue ω^2 lie between -0.05 and -2.0 and scale with squared poloidal Alfvén frequency (the value of $(\max|\psi| - \psi_{bou})^2/a^4 \simeq 1$ for all the cases considered).

In the presented calculations aspect ratio $A = 3$, and toroidal magnetic field was chosen typically to provide the safety factor value of order 1 at the "central" magnetic axis; the instabilities look rather insensitive to the value. Increasing the aspect ratio up to infinity shows that the instability of the configuration with circular shape is specifically toroidal, but for shaped cylindrical plasma there exist analogous unstable eigenfunctions.

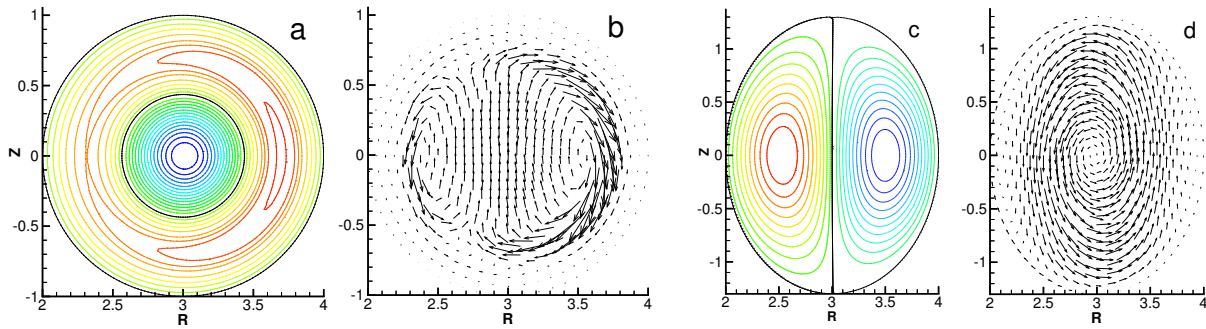


Fig.1. Magnetic surfaces and unstable displacements for the equilibria with $j_\phi = \lambda\psi/R$. Shown in black are the lines where the flux ψ and current density change the sign; Aspect ratio $A=3$. a,b)reversed current equilibrium of "central" type with circular boundary; c,d)reversed current equilibrium of "dipole" type with boundary elongation $E=1.3$

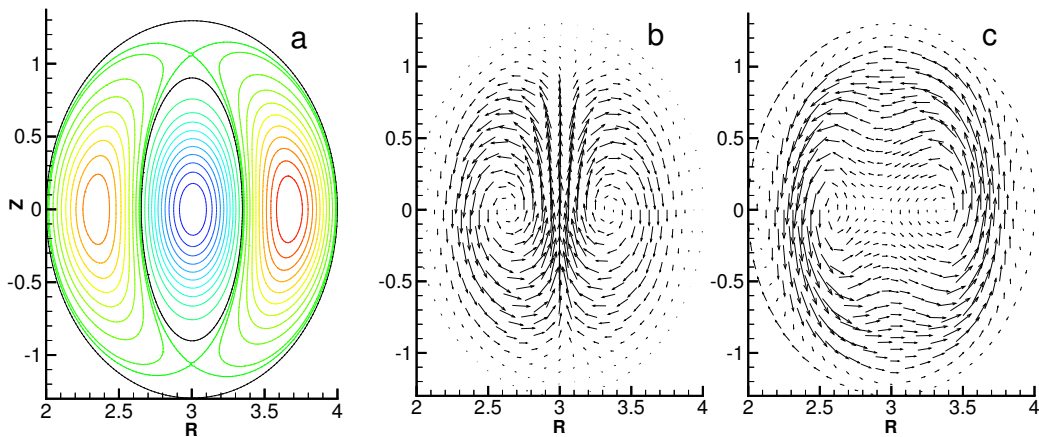


Fig.2. a) reversed current equilibrium of "central" type with $j_\phi = \lambda\psi/R$; boundary elongation $E=1.3$. b,c) two unstable eigenfunctions for the equilibrium.

Another series of calculations dealt with the configurations with piece-wise constant profiles of $f f'$ of different signs [3]. Elongated plasma shapes were chosen giving reversed current holes of "central type", Fig.3. The strong enough internal instability exists corresponding to horizontal displacement of the current hole. The eigenfunction is mostly concentrated inside and in the neighborhood of the delimiting surface, no remarkable activity is observed at the separatrix.

Some other unstable functions are much more sensitive to the grids and parameters of approximation and most likely are numerically destabilized.

Further validation is needed to confirm the results.

