Parametric study of NTM in ITER inductive scenario

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1. Introduction

Parametric study of the Neoclassical Tearing Modes (NTMs) was done for ITER reference inductive scenario (full bore 15 MA plasma producing 400 MW of fusion power with $Q = 10$ for about 400 s). Analysis was based on the modified Rutherford equation\textsuperscript{1} (MRE) for the evolution of the magnetic island width. In the model we kept standard tearing mode stability parameter $\Delta'$, neoclassical drive of the mode, magnetic curvature effect and ECCD stabilization. Polarization current effect and mode coupling were neglected. There are lot of NTM models similar to ours which were used for interpretation of the experimental data from various tokamaks. However, careful examination shows that all of them: 1) contain series of coefficients to fit modeling result to experimental data, 2) use different expressions for particular contributions to MRE, 3) employ different models for the small island limit and NTM onset threshold. One of the principal goal of this study is to add the NTM model based on MRE maximum predictive ability for estimations of NTM influence on the discharge characteristics and specify requirements for their control in ITER. For these we reformulate MRE on the base of the standard set of assumptions but removing unnecessary oversimplifications, reorganize its structure to allow for particular contributions from plasma electrons and ions, and parameterize EC driven current characteristics obtained with use of the OGRAY code for realistic design of the EC launchers and plasma characteristics of ITER inductive reference scenario.

2. Model

In our model we use standard assumptions on the equilibrium and perturbation magnetic fields in the vicinity of the resonance surface $r = r_s$. Then, magnetic island flux function takes the form $\psi = \tilde{\psi} \cos \xi - x^2 B_0 s / 2 q_s R$, where helical perturbation $\tilde{\psi}(r, \xi) = \tilde{\psi} \cos \xi$, is taken in the $\tilde{\psi} = \text{const}$ approximation. $\xi = m \theta - n \varphi - \omega t$ is the perturbation phase, $x = r - r_s$, $q_s$ and $s$ are the safety factor and magnetic shear at the resonance surface, respectively. It is convenient to introduce nondimensional flux variable $\Omega = 8 x^2 / W^2 - \cos \xi$, with $W = 4(q_s R \tilde{\psi} / s B_0)^{1/2}$ being the island full width. Inside the island $-1 < \Omega < 1$. Assuming the island width being much smaller than island length, the integration of the Ampere law across the island yields modified Rutherford equation for evolution of the island width

$$0.8227 \frac{4 \pi \sigma_s}{c^2} \frac{dW}{dt} = \Delta' + \Delta'_{bs} + \Delta'_{mv} + \Delta'_{EC}. \quad (1)$$

Here $\sigma_s$ is the neoclassical plasma conductivity, $\Delta'$ is standard tearing mode stability parameter, $\Delta'_{bs}$, $\Delta'_{mv}$, and $\Delta'_{EC}$ are contributions from the perturbed bootstrap current, magnetic field curvature, and EC driven current, respectively.

For the above formulated assumptions, bootstrap drive of the mode takes the form

$$\Delta'_{bs} = \frac{16 \sqrt{2} R q}{c s B_0} \frac{1}{W} \sum_{\sigma_s} \int_{-1}^{\infty} d \Omega \left( \tilde{J}_{bs} \right) \frac{\cos \xi}{\sqrt{\Omega + \cos \xi}} d\xi. \quad (2)$$
Here \( \langle \ldots \rangle \approx \int d\xi (\ldots) (\Omega + \cos \xi)^{-1/2} / \int d\xi (\Omega + \cos \xi)^{-1/2} \) means averaging over the island magnetic surface, \( \tilde{j}_{bs} \) stands for the perturbation of the bootstrap current. We take for the equilibrium bootstrap current the form

\[
j_{bs} (r) = \sum_{A=n_e, T_i} j_{bs}^A = f_{bs}^A \nabla T_e / T + f_{bs}^A \nabla T_i / T + f_{bs}^A \nabla n / n
\]

(3)

Which takes into account partial contributions from the plasma density, electron and ion temperature gradients. Here \( T = 0.5(T_e + T_i), n = n_e \). Partial contributions from temperature and density gradients, \( f_{bs}^A \), we calculate according to [2] where convenient for routine calculations expressions for equilibrium bootstrap current as well as for neoclassical conductivity are presented

\[
\langle j_{bs} B \rangle = -J(\psi) p(\psi) \left[ L_{s1} \frac{\partial \ln n}{\partial \psi} + R_{pe} (L_{s1} + L_{s2}) \frac{\partial \ln T_e}{\partial \psi} + (1 - R_{pe}) \left( 1 + \frac{L_{s2}}{L_{s1}} \right) L_{s1} \frac{\partial \ln T_i}{\partial \psi} \right]
\]

with coefficients given in terms of the trapped particle fraction (only geometry dependent value), electron and ion collisionalities and plasma ion charge. Then, to calculate bootstrap drive of the mode, one should firstly calculate perturbed profiles of plasma temperatures and density. It is well known\(^1\), that for the sufficiently large island width, these profiles are flattened inside the island. For this case straightforward calculations give expression for the bootstrap drive of the mode in terms of equilibrium bootstrap current

\[
\Delta'_{bs} = 6.3 \frac{4\pi}{c} j_{bs} (r_1) \frac{Rq}{B_0 s W}
\]

(4)

In the small island limit the cross island transport can not be neglected. Different transport mechanisms are responsible for establishment of the perturbed temperature and density profiles. Then, the generalization of Eq. (4) with allowance for the incomplete flattening the plasma parameter profiles inside the island takes the form

\[
\Delta'_{bs} = 6.3 \frac{Rq}{B_0 s} \frac{4\pi}{c} \sum_{A=n_e, T_i} j_{bs}^A \frac{W}{W^2 + W_{bs,A}^2},
\]

(5)

where characteristic island widths \( W_{bs,A} \), \( \frac{1}{W_{bs,A}} = \sum_k \frac{1}{W_{bs,A,k}} \) are determined by the combination of transport mechanisms governing the establishment of the perturbed profiles (See Eqs. 14 -22 of [3]) . Similar expressions for the bootstrap drive were given in [4-6], where different forms of the equilibrium bootstrap current were considered. Equation (5) just generalizes aforementioned expressions by implicit separation for partial contributions from plasma species with allowance for the corresponding finite perpendicular transport corrections.

In general case the contribution of the magnetic curvature term (magnetic well) into MRE takes the form

\[
\Delta'_{mw} = \frac{16\sqrt{2} Rq}{c s B_0} \frac{1}{W} \sum_{\sigma_1} \int_{\Omega} \left( \Omega, \xi \right) \cos \xi \frac{d\Omega}{\sqrt{\Omega + \cos \xi}} d\xi
\]

(6)

where magnetic well current, \( j_{mw} \), should be found from the current continuity equation of the form \( \nabla \times j_{mw} = -c \nabla p \left( \nabla \times B / B^2 \right) \). According to [7], stabilizing contribution of the magnetic curvature effect for the flattened temperature and density profiles is given by
\[ \Delta_{mw}' = -6.3 \frac{U_{mi}}{W}, \]  

(7)  

where \( U_{mi} \) was called in [7] as magnetic island magnetic well. It was there suggested that this parameter should substitute widely used in MRE the resistive interchange mode stability parameter \( D_R \), as the island width for the hot plasma much exceeds that of the resistive layer, implying that the curvature effect in magnetic island problem should be ideal rather than resistive phenomena. Taking into account that curvature effect is proportional to the total plasma pressure gradient, it (similarly to Eq. (5) for the bootstrap drive) can be decomposed into the sum of the partial contributions with allowance for corrections corresponding to the small island limit  

\[ \Delta_{mw}' = -6.3 U_{mi} \sum_{A=n,T_1,T_2} \nabla P_A \frac{1}{\nabla p} \frac{1}{W + W_{mw,A}}, \]  

(8)  

where \( \frac{1}{W_{mw,A}} = \sum_k \frac{1}{W_{mw,A,k}} \) (see Eq. (22) of [3]).

The radial profiles of EC driven current were calculated for realistic design of the launchers [8] and for several positions of resonance surfaces in ITER Scenario 2 with use of the OGRAY code [9]. Different steering angles result in CD on different surfaces with different current density peak values (\( P_{EC} = 20 \text{MW} \)) and widths. Narrow radial profiles closer to plasma edge result in higher current peak in spite of lower temperature. Red line corresponds to the bootstrap current density. The parametric dependencies of driven current characteristics, such as peak current density and radial profile width, were deduced from the results of ECCD modeling and used for NTM modeling via modified Rutherford equation. The ECCD stabilization term \( \Delta_{EC}' \) in MRE is calculated according to Eq. (2) with substitution \( J_{bs} \rightarrow J_{ECCD} \). With high accuracy, the driven current profile can be approximated by the Gaussian form  

\[ j_{ECCD}(\rho) = j_{cd} \exp\left(-2(\rho - \rho_{cd})^2/w_{cd}^2\right). \]  

(9)  

Polinomial fit for the current peak value, \( j_{cd} \) [A/cm\(^2\)] (corresponding to the 20MW injected power from both upper and lower EC beam arrays in the upper launcher) and width, \( w_{cd} \), gives  

\[ j_{cd} = 97.11 - 393.35\rho + 560.15\rho^2 - 253.9\rho^3, \]  

(10)  

\[ w_{cd} = 0.1829 - 0.3303\rho + 0.1558\rho^2. \]  

(11)  

Here radial variable is the square root from the normalized toroidal flux.

3. NTM in ITER inductive scenario.

ITER inductive reference scenario (scenario 2 [10]) is characterized by the almost flat density profile, i.e. corresponding contributions from density gradient into the bootstrap drive and curvature effect are vanished, along with uncertainties caused by the absence of the reliable model for the particle transport in the small island limit. Moreover, due to the high ion temperature, the ion drift orbit width is comparable to the island width, thus,
according to kinetic analysis of [9], ion contribution to the bootstrap drive should be also neglected. However ion temperature gradient does contribute to the stabilizing magnetic curvature effect, rising its relative role in NTM stability.

Free streaming of the electron heat flux along the magnetic field line is the dominant mechanism in establishment of the perturbed electron temperature profile. Corresponding critical island width (Eq. 20 of [3]) is of the order of several cm. Principal uncertainty in its calculation caused by unknown anomaly of the transverse heat conductivity. For the ion temperature, critical island width corresponds to the smallest of given by Eqs. 16, 18 of [3]. Additional uncertainty comes from the unknown island rotation frequency. By the order of magnitude it can be taken to be equal to the ion diamagnetic drift frequency.

Curvature effect was shown to be important in estimation of the saturated island width, confinement degradation and EC power requirements for NTM stabilization in ITER. In table 1 the bootstrap drive and curvature effect for the m/n=2/1 and 3/2 modes are shown. Transport corrections do not affect much saturated island width. Confinement degradation is estimated with use of the “belt” model providing \( \frac{\Delta \tau_e}{\tau_e} = -4 \left( \frac{a_s}{a} \right)^4 \frac{w_{sat}}{a_s} \). Relative weakening of the drive due to curvature effect (4\(^{th}\) column, no transport corrections was allowed for) gives reduction in EC power needs. Transport corrections to \( \Delta_{bs}^r \) and \( \Delta_{mv}^r \) could provide further decrease (about 2 times) in EC power necessary for the NTM stabilization in ITER.

Simulation of the ECCD stabilization has shown that the steering capability of recent dogleg option is very good (substantially greater then necessary for Scenario 2). Current generated at the resonance surface \( q = 1/2 \) can be estimated as good ( \( j_{CD} / j_{BS} = 1.45 \) ) and marginal for resonant surface \( q = 3/2 \) ( \( j_{CD} / j_{BS} = 1.0 \) ).

Our calculations revealed no essential difference between amplitudes of the various forms of the curvature parameters \( D_R \) and \( U_{mi} \). As it was shown in [7], these parameters are strong functions of the magnetic shear. Variation of the local shear value, due to EC driven current will be considered in a future study.

<table>
<thead>
<tr>
<th>m/n</th>
<th>( \Delta_{bs}^r )</th>
<th>( \Delta_{mv}^r )</th>
<th>( \Delta_{bs}^r + \Delta_{mv}^r )</th>
<th>( w_{sat} )</th>
<th>( \Delta \tau_e / \tau_e )</th>
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<td>3/2</td>
<td>1.05</td>
<td>-0.4</td>
<td>0.62</td>
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<td>0.13</td>
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<td>-0.52</td>
<td>0.52</td>
<td>14.2 cm</td>
<td>0.15</td>
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References: