33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, P-1.094 (2006)

Two-fluid equilibria of rapidly-rotating spherical tokamak plasmas

N. Savenko[1], K.G. McClements[2], A. Thyagaraja[2]

[1] *Uppsala University, Ångström Laboratory, S-751 21 Uppsala, Sweden*

[2] *EURATOM/UKAEA Fusion Association, Culham Science Centre, Abingdon, OX14 3DB, UK*

1 Introduction

Toroidal flows exceeding the local sound speed, and a pronounced variation of electron density on flux surfaces, have recently been observed in the MAST spherical tokamak (ST) during counter-current neutral beam injection [1]. In order to determine the plasma equilibrium under these circumstances it is not strictly appropriate to neglect inertial terms in the fluid momentum balance or to use ideal magnetohydrodynamics (MHD). In this paper we demonstrate that equilibria representative of rapidly-rotating ST plasmas can be readily computed analytically and numerically using simple two-fluid models.

2 General analysis

A dissipationless two-fluid model of axisymmetric equilibria with flows, first developed in [2], has recently been applied to tokamak conditions [3], with electron inertia neglected and electron temperature T_e (but not necessarily ion temperature T_i) assumed to be constant on flux surfaces. It is well-known that plasma density n and hence pressure p are not flux functions if the plasma rotates toroidally [4]. Taking p to be a function of poloidal flux Ψ and R^2, where R is major radius, the two-fluid Grad-Shafranov equation for a tokamak plasma with arbitrary toroidal flow and zero poloidal flow can be written in a form closely analogous to the familiar MHD version [3]:

$$\frac{\partial^2\Psi}{\partial Z^2} + R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\Psi}{\partial R}\right) = -ff' - \mu_0 R^2 \frac{\partial p}{\partial \Psi}, \tag{1}$$

where Z is vertical distance and $f = RB_\varphi$, B_φ being toroidal field. If T_i is assumed to be a flux function, the following expression for the ion toroidal rotation velocity can be obtained [3]:

$$\Omega_{\varphi i} = \left[\frac{2(T_i + T_e)}{m_i}\right]^{1/2} \frac{A(\Psi)}{R^2 + d}, \tag{2}$$

where m_i is ion mass, A is a flux function determined by the temperature profiles and d is a constant. The pressure p and electrostatic potential Φ are then given by

$$p = P_1(\Psi)\exp\left[-\frac{A(\Psi)^2}{R^2 + d}\right], \tag{3}$$

$$\frac{e\Phi}{T_e} = \frac{e\Phi^*(\Psi)}{T_e} - \frac{A(\Psi)^2}{R^2 + d}, \tag{4}$$

where P_1, Φ^* are flux functions and $-e$ is electron charge.

3 Rigid body rotation

In the limit $|d| \gg R^2$ we obtain from Eqs. (2) and (3) an expression for the variation of pressure on rigidly-rotating flux surfaces:

$$p = P_2(\Psi)\exp\left[\frac{m_i\Omega_{\varphi i}^2 R^2}{2(T_i+T_e)}\right], \tag{5}$$

where $P_2 = P_1\exp[-A(\Psi)^2/d]$ and $\Omega_{\varphi i}$ in this limit is a flux function. Equation (5) can also be obtained using ideal MHD when the single fluid temperature $T = (T_i+T_e)/2$ is a flux function and poloidal flows are neglected [4]. When $\Omega_{\varphi i}^2/(T_e+T_i)$ is independent of Ψ and $P_2 \propto -\Psi$, Eq. (1) has a particular integral [4]

$$\Psi \propto \frac{R_1^4}{M_\varphi^4}\exp\left[\frac{M_\varphi^2}{2}\frac{R^2}{R_1^2}\right], \tag{6}$$

where $M_\varphi = \Omega_{\varphi i}R_1 m_i^{1/2}/(T_i+T_e)^{1/2}$ is the sonic Mach number of the toroidal flow at major radius $R = R_1$. Taking RB_φ to be constant, so that $ff' = 0$, we can incorporate this particular integral into a solution of Eq. (1) which reduces in the limit $M_\varphi \to 0$ to an expression obtained by Freidberg [5] (following Solev'ev [6]):

$$\Psi = \Psi_1\left\{\frac{\gamma}{8}\left[(R^2-R_0^2)^2-R_b^4\right]+\frac{1-\gamma}{2}R^2Z^2+\frac{R_1^4}{M_\varphi^4}\left[\exp\left(\frac{M_\varphi^2}{2}\frac{R^2}{R_1^2}\right)-1-\frac{M_\varphi^2}{2}\frac{R^2}{R_1^2}-\frac{M_\varphi^4}{8}\frac{R^4}{R_1^4}\right]\right\}. \tag{7}$$

Here R_0, R_b and γ are constants that, together with M_φ and R_1, determine the plasma major radius, midplane minor radius and elongation ($\Psi = 0$ at the plasma boundary), and Ψ_1 is a constant that determines the total plasma current.

Fig 1(a) shows MAST-like flux surface contours computed using Eq. (7) for $M_\varphi = 0$ (blue curves) and $M_\varphi = 1$, $R_1 = 0.9$m (red curves). The other parameters in Eq. (7) were chosen such that the inner and outer midplane plasma boundaries, and the plasma elongations, coincided in the two cases. The toroidal flow has caused an additional outboard shift of about 5cm in the magnetic axis. Fig 1(b) shows the midplane density profile for $M_\varphi = 1$ when T_e+T_i is assumed to be constant; the dashed line indicates the location of the magnetic axis. Although the assumption of constant temperature is clearly an approximation, MAST discharges with supersonic flows have been observed to have relatively broad temperature profiles [1]. Fig 1(b) shows the inboard-outboard density asymmetry associated with toroidal rotation [4]. In this particular scenario the outboard shift of the density peak relative to the magnetic axis ($\simeq$ 10cm) is of the same order as the shift in the axis itself ($\simeq$ 5cm), suggesting that both effects should in general be taken into account when interpreting data from discharges with transonic flows.

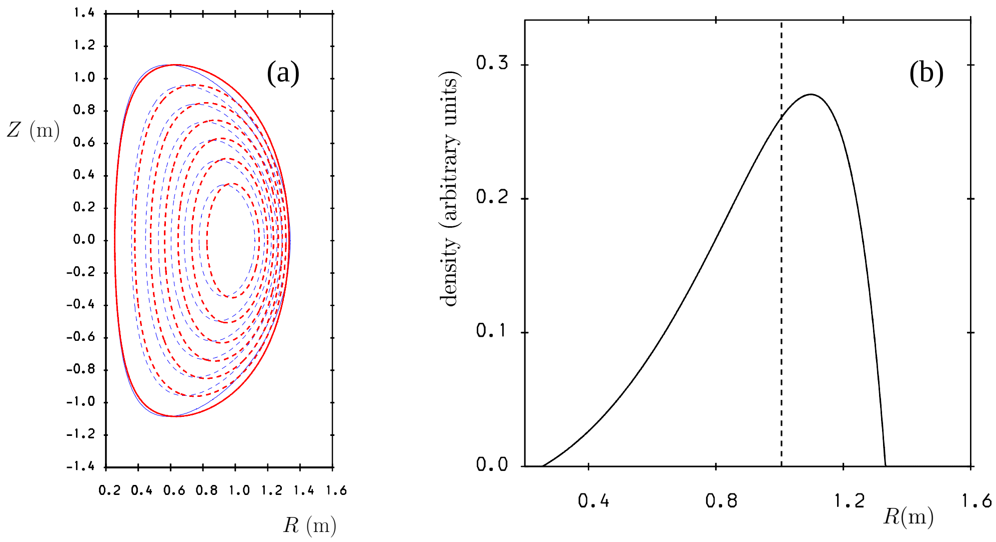

Figure 1: (a) MAST-like equilibria with no toroidal flow (blue curves) and transonic ($M_\varphi = 1$) rigid body toroidal flow (red curves). (b) Midplane density profile for $M_\varphi = 1$.

4 Keplerian rotation

The ion momentum balance equation admits solutions for the flow such that the toroidal canonical momentum $P_{\varphi i} \equiv m_i \Omega_{\varphi i} R^2 + e\Psi$ is a function of Ψ, i.e. the ion fluid on a given flux surface rotates in a Keplerian manner, with constant angular momentum [3]. This corresponds to $d \to 0$ in Eq. (2). The pressure and hence density variation on a flux surface is then qualitatively similar to that of rigid body rotation, insofar as the peak occurs outboard of the magnetic axis, but the dependence on R is of the form $\exp(-A^2/R^2)$ rather than that given by Eq. (5). Due to the large range of values of R in ST geometry, the density variation on a flux surface far from the axis is significant even for relatively modest values of M_φ when the rotation is either rigid or Keplerian. Combined measurements of density, temperature and ion toroidal velocity could in principle be used to establish which of these two limiting cases represents a better approximation.

Taking A to be constant, $d = 0$, $P_1 \propto -\Psi$ and $ff' = 0$, Eq. (1) reduces to

$$\frac{\partial^2 \Psi}{\partial Z^2} + R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial \Psi}{\partial R}\right) = CR^2 \exp\left[-\frac{M_\varphi^2}{2}\frac{R_1^2}{R^2}\right], \qquad (8)$$

where C is a constant and, as before, M_φ is the sonic Mach number at $R = R_1$. It is possible to obtain a particular integral of Eq. (8) in terms of infinite series, which are, however, rather slow to converge. For this reason we have used a simple finite difference scheme to solve Eq. (8) numerically, with a bounding flux surface passing through a set of six fixed points corresponding to the last closed flux surface of a MAST-like plasma, and a range of toroidal Mach numbers (Fig. 2). As in the case of rigid rotation, the magnetic axis R_m moves outboard as M_φ is increased, in this case by about 4cm for $M_\varphi = 1$. Similar results were obtained when Eq. (1)

was solved for the case of Keplerian rotation with a modified (quadratic) Ψ dependence in the pressure profile and a slightly different choice of points on the plasma boundary.

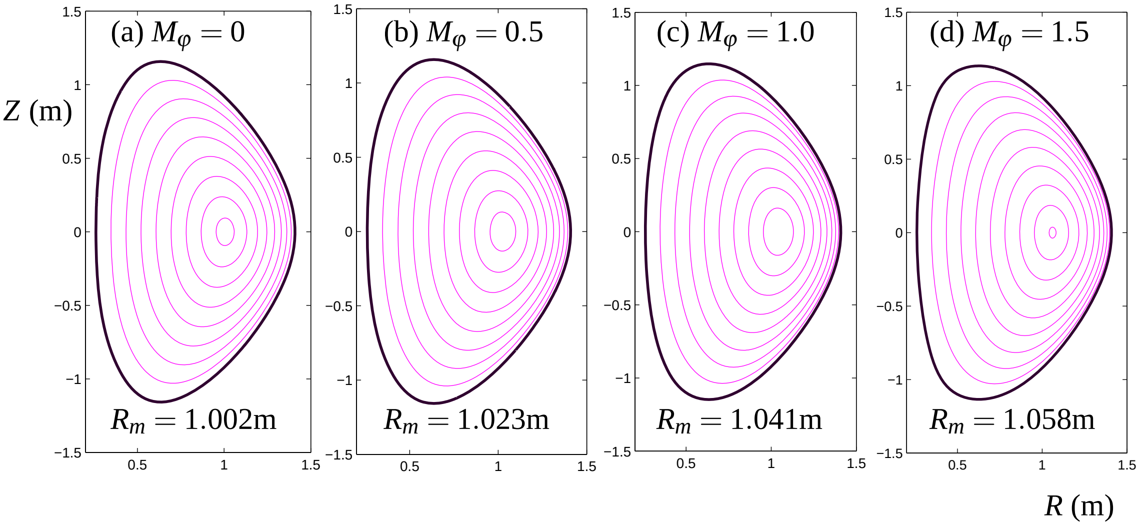

Figure 2: Equilibria with Keplerian rotation. The Mach number M_φ is evaluated at $R = 0.85$m.

5 Discussion

Equation (2) represents a two-fluid generalisation of the ideal MHD result that flux surfaces rotate as rigid bodies in the absence of poloidal flows [4]. Even in the rigid body limit Eq. (2) augments MHD by providing a specific relation between the rotation profile and the temperature profiles: in MHD the profiles can be prescribed independently. It may be expected that real tokamak plasmas will generally lie between the two extremes of rigid body and Keplerian rotation considered in Secs. 3 and 4 above. Although in our two-fluid model the density on a flux surface always increases from inboard to outboard, the precise variation with R, and also the equilibrium magnetic field, depend on whether the rotation profile is closer to the rigid body or Keplerian limits. For these reasons it is important to take full account of rotation profile measurements, if available, when reconstructing ST plasma equilibria with flows.

Acknowledgements

This work was funded jointly by EPSRC and by the Commission of the European Communities under the Contract of Association between EURATOM and UKAEA.

[1] R. Akers et al., Proc. 20th IAEA Fusion Energy Conf. (IAEA, Vienna, 2005), EX/4-4.

[2] K. G. McClements, A. Thyagaraja, Mon. Not. R. Astron. Soc. **323**, 733 (2001).

[3] A. Thyagaraja, K. G. McClements, Phys. Plasmas **13**, 062502 (2006).

[4] E. K. Maschke, H. Perrin, Plasma Phys. **22**, 579 (1980).

[5] J. P. Freidberg, Ideal Magnetohydrodynamics (Plenum, New York, 1987).

[6] L. S. Solev'ev, Sov. Phys.-JETP **26**, 400 (1968).