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Intermittency in laboratory and space plasmas: comparative study of multifractal statistics

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A lot of experimental evidences have been observed now in fusion devices that edge plasma turbulence is highly intermittent. In space plasmas, analysis of data recorded by the Interball-1 spacecraft revealed the scale-invariance and intermittency of magnetic field and flow in the Earth's magnetopause boundary layer over polar cusps. Magnetized laboratory and space plasmas exhibit the same intermittent behaviour and deviation from Gaussian statistics at small scales as observed in neutral fluid turbulence. Experimental investigations have highlighted slight deviations (due to strong intermittency) from Kolmogorov prediction [1] of the energy spectrum and velocity increments $<|\upsilon(x+l)-\upsilon(x)|^q>\sim l^{\zeta(q)}$, $\zeta(q)=q/3$, within inertial range $\eta << l << L$ at high Reynolds numbers, L being the integral scale of motion and η the dissipation scale [2]. This means that second order moments such as Fourier power spectra do not completely describe the wavefield and hence higher order moments must be investigated. Scaling properties in plasma turbulence should not be investigated as a function of *l*, the resolution scale, but rather as a function of the generalized scale $\xi(l,\eta,L,\ldots)$. It may be reminiscent critical phenomena in finite size systems with scaling , which depends on the size of system and correlation function diverged at critical points. The scaling of higher order moments has been interpreted in terms of multifractal processes [2,3,4]. Multifractality is related to an underlying multiplicative cascading process. Scaling properties of turbulence can be extended up to dissipative range (referred as extended self-similarity) even at moderate Reynolds number [5]. In this work, scaling properties in laboratory and space plasmas are investigated.

Edge plasma turbulence in fusion devices (Langmuir probe data from tokamak T-10, tokamak HYBTOK-II and linear plasma device NAGDIS-II [4]) and space plasma in the turbulent boundary layer based on the Interball-1 satellite data [6] are studied. In the Earth's magnetopause boundary layer over polar cusps (singular high-latitude regions of the geomagnetic trap), where the incoming solar plasma flow interacts with an indented obstacle, the analysis of space data revealed a turbulent boundary layer. In this zone of strong turbulence intermittent plasma jets appear to provide both populating of a boundary layer inside the eroded magnetic obstacle, and transient plasma transport downstream the external flow along the boundary. The signals demonstrate strong intermittency (fig.1). Several spectral indices are indicated in total power spectral density of the fluctuations (fig.2). To characterize intermittency, higher order moments of the data at different scales should be considered.

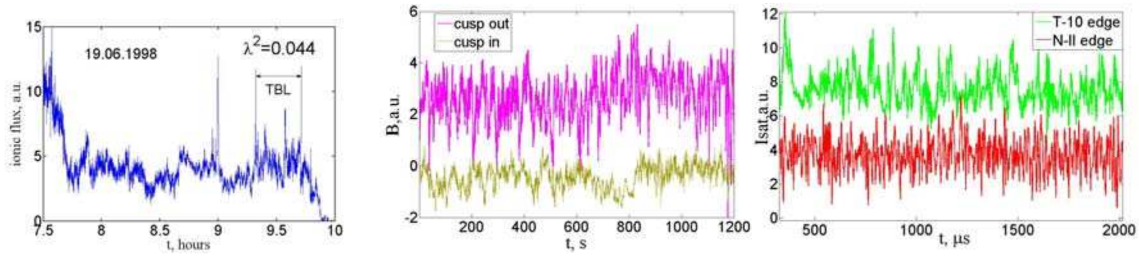
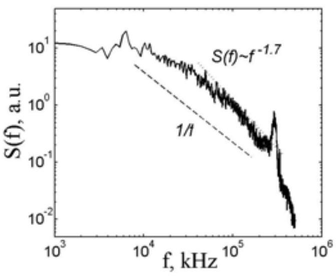


Fig.1.Ionic flux and magnetic field in space plasma, Isat in edge of fusion device.



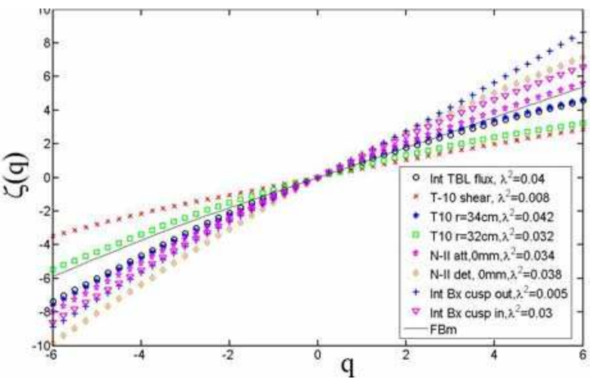


Fig.2. Typical Isat power spectrum in tokamak edge. Fig3. Scaling of moments $\zeta(q)=qH-\lambda^2q^2$.

Turbulent fluctuations both in laboratory fusion plasmas and in space plasma at turbulent boundary layer demonstrate similar multifractal statistics, i.e. the scaling behaviour of absolute moments $S(q,l)= S(q,L)\ (l/L)^{\zeta(q)}$, $S(q,l)=E(|\delta_l X(t)|^q)$, $\delta_l X(t)=X(t+l)-X(t)$ is described by a convex function $\zeta(q)=qH-\lambda^2q^2$. Using wavelet based technique [4], we have computed the scaling exponent $\zeta(q)$ (fig.3). Parabolic behaviour of $\zeta(q)$ is observed as evidence of multifractality for these time-series. For monofractal data (i.e. fractional Brownian motion, fBm) the dependence would be a linear function of q. Multifractality exponent λ^2 is estimated as relevant parameter to characterize boundary plasma turbulence. It is in the range of 0.018÷0.05 excepting monofractal (λ^2~0) cases of shear layer in tokamak and magnetic fields out of space cusp. The $\zeta(q)$ spectrum relates to the singularity spectrum D(h) (fig.4) that provides information about the statistical distribution of singularity (Hölder) exponents h.

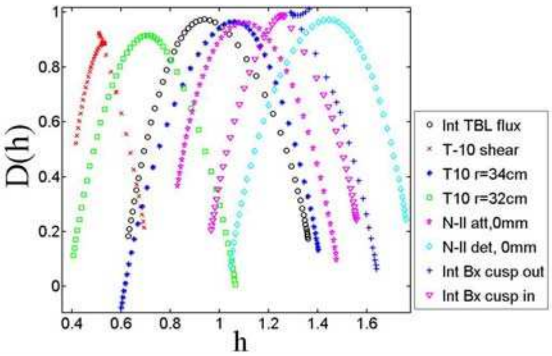


Fig.4.Singularity spectra D(h) vs Hölder exponent h.

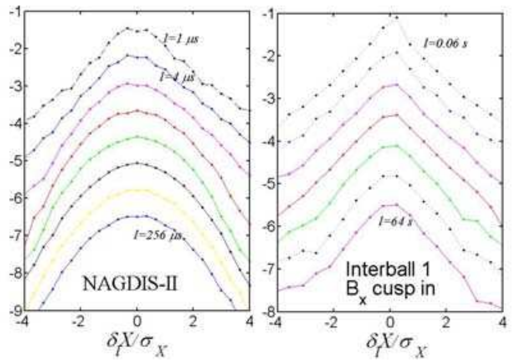


Fig.5. Standardized pdf's of increments $\delta_l X(t)$ (normalized to its standard deviation σ_X) for different time scales l (from top to bottom). Plots are arbitrary shifted for illustration.

Multifractality suggests that the probability distribution functions (PDF) satisfy cascade equation with a Gaussian kernel when going from large to small time scales [2,4]. Thus, as far as the PDF of increments at different time scales *l* are concerned, they will satisfy an evolution equation from "quasi-Gaussian" at very large scale to fat tailed PDF's at small scales. This transformation of the PDF's is shown in fig. 5. Multiplicative cascading process has «coarse» scale *T* (referred as correlation scale or inertial-range time scale) from that iterating towards finer scales. To characterize the similarity of the pdf's, we use Kullback-Liebler Divergence (KLD) called a relative entropy and a discrimination [4]. When the discrete probability distributions *P(s)* and *Q(s)* are given, the KLD is defined as follows: $D(P,Q)=\sum_{\{s\}}(P(s_i)\log(P(s_i)/Q(s_i)))$. The KLD is a good indication to evaluate how much *P(s)* and *Q(s)* resemble each other. The reference PDF *Q(s)* is Gaussian one. We have used the KLD to analyse the coarse time scale *T* illustrated multifractal process. For short times the PDF's are strongly deviated from Gaussian. At the time scale typically larger than *T*~ 50-200 μs for fusion plasma and *T*~80 sec for space plasma, the PDF's continuously transform to Gaussian shape (fig.6).

(a) (b)

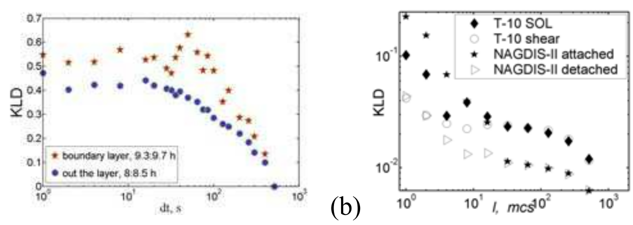
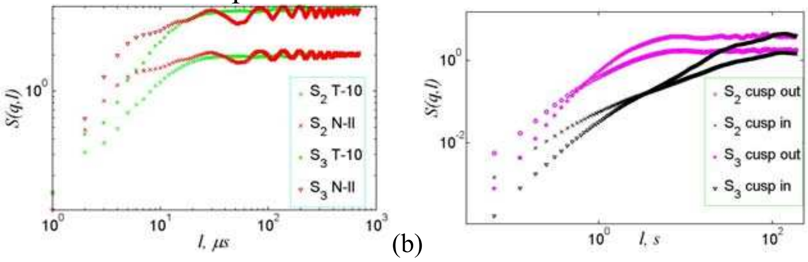


Fig. 6. KLD vs. lag l for space (a) and fusion (b) plasmas

Within the Taylor hypothesis the temporal dynamics should reflect the spatial one. So, structure functions are estimated from temporal signals of ion saturation current on Langmuir probe in fusion devices and magnetic field and ionic flux signal obtained from Interball-1 satellite data (fig.7). More robust criteria can be obtained by invoking the extended self-similarity hypothesis [5], which states that scalings of the type $S(q,l) \sim S(3,l)^{\zeta(q)/\ \zeta(3)}$ should hold for long ranges of delays $l \geq 5\eta$. It reflects generalized scale invariance of developed turbulence.



(a) (b)

Fig. 7. Structure functions of second S_2 and third S_3 orders for fusion (a) and space (b) plasmas

In fig.8,9, such ESS is demonstrated both for laboratory and space plasmas: all data scales by the same linear functions as for order q=2 as for q=4. It suggests some universality of multifractal statistics in magnetized plasmas. These data have improved our understanding and revealed some striking similarities with neutral fluids [2]. To find whether the parameters of multifractality observed in this analysis, have a more universal validity, it would be interesting to extend multifractal analysis to a broader set of plasma data.

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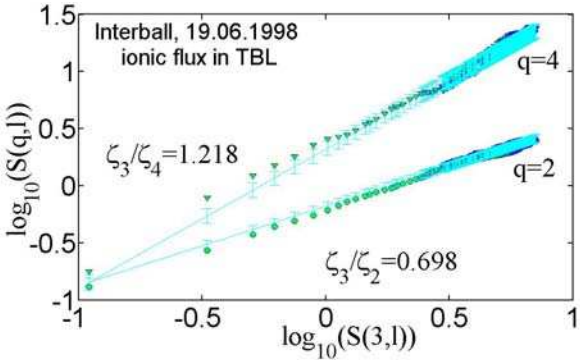

Fig.8. The dependence of second and fourth order structure functions vs. third order one for space plasma in TBL

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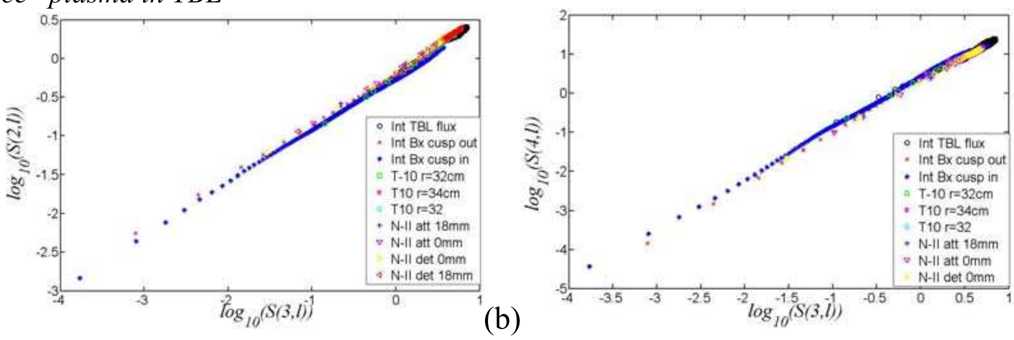

(a) (b)

Fig.9. The dependence of the second (a) and of the fourth (b) order structure functions vs. the third order one for fusion and space plasmas.