33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, P-1.063 (2006)

Kinetic Alfvén wave in plasma with inelastic collisions

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In this work we present a model for the kinetic Alfvén wave propagating in a partially ionized three component plasma composed of singly charged ions, electrons and neutral atoms. The geometry and assumptions typical for a simple kinetic Alfvén wave will be used, i.e., we are dealing with perturbations in a magnetized plasma which induce bending of a magnetic field initially oriented along the z-axis. The dynamics of electrons in the direction perpendicular to the magnetic field is neglected while the dynamics of ions takes place mainly in the perpendicular plane.

We use the plasma model and effects observed in an experiment [1] in which apart from the bulk plasma species, the plasma also contains a relatively low density flux of mono-energetic electrons with energies slightly above the ionization threshold. In laboratory conditions [1], such an electron flux can be obtained in a double layer configuration in the presence of a large amount of neutrals. In the space, such a flux may appear in reconnection process in regions with weak magnetic fields which along a bottle-neck magnetic configuration creates a potential difference $e\phi_0 = \varepsilon_{pi}(1-1/\gamma)\tau/(1+\tau)$. Here, γ is the magnetic mirror ratio (the ratio of the maximum and minimum magnetic field values), $\tau = T_e/T_i$, and $\varepsilon_{pi} = m_i v_{i0}^2$ is the ion parallel kinetic energy. This results in an acceleration of electrons along the magnetic field lines.

The ionization cross section depends on the energy of ionizing particles and for singly ionized atoms it may be represented by the following fitting formula

$$\sigma(x) = \frac{\sigma_{00}}{\varepsilon_i^2}\frac{1}{x}\left(\frac{x-1}{x+1}\right)^{3/2}\left\{1+\frac{2}{3}\left(1-\frac{1}{2x}\right)\log\left[2.7+(x-1)^{1/2}\right]\right\}. \tag{1}$$

Here: $x = \varepsilon/\varepsilon_I$, $\sigma_{00} = 6.56\cdot 10^{-14}$ eV^2 cm^2, ε_I is the binding energy of an orbital electron, and ε is the kinetic energy of a bombarding electron.

In the experiment [1], the mono-energetic electrons had the energy slightly above the ionization threshold which corresponds to $x \sim 1$, i.e., to the region where the curve for $\sigma(x)$ sharply increases. In this case, even very small variations in energy of ionizing electrons arising from some accidental electrostatic perturbations in the plasma, result into significant changes in the ionization cross section. In electrostatic perturbations such as the ion sound [1] the change of the plasma density is in phase with the change of the potential, i.e., the electron energy. Hence, an accidental increase of plasma density may be further enhanced owing to the resulting increment of ionization due to the increased cross section. This phenomenon was clearly demonstrated experimentally in Ref. [1] for initial potential perturbations exceeding some critical value of about 0.1 V.

To include the effects of varying ionization cross section into calculations, we expand the expression (1) for $\sigma(x)$ in terms of $x = \varepsilon/\varepsilon_I$ as follows:

$$\sigma(x) \approx \sigma_0(x) + (d\sigma_0/dx)x_1 + (d^2\sigma_0/2dx^2)x_1^2 + ..., \tag{2}$$

where $\sigma_0(x)$ is the cross section at the given energy x of the bombarding electrons; x_1 is the linear perturbation of x. In further calculations, we shall need the ratio $[d\sigma(x)/dx]/\sigma(x)$ presented in Fig. 1.

With inelastic collisions taken into account, the electron continuity equation becomes

$$\frac{\partial n}{\partial t} - \frac{1}{e}\frac{\partial j_{ez}}{\partial z} = S. \tag{3}$$

Here $S \equiv \sigma n_n f - a_l n$ is the perturbed source/sink term as taken in Ref. [1]. The term $a_l n$ represents all losses, and it may include radiative and three body recombination, as well as losses due to transports across the magnetic field lines claimed to be dominant in the experiment. The quantity f denotes the flux of energetic electrons created by an external source. In the process of linearization, we keep only small terms of the first order in the expansion (2).

For mainly perpendicular ion motions, we have the following recurrent formula in a standard notation:

$$\vec{v}_{i\perp} = \frac{1}{B_0}\vec{e}_z \times \nabla_\perp \phi + \frac{1}{\Omega_i}\vec{e}_z \times \frac{\nabla_\perp p_i + \nabla_\perp \cdot \mathscr{P}}{m_i n_i} - \frac{\nu_i}{\Omega_i}\frac{\nabla_\perp \phi}{B_0}$$
$$-\frac{\nu_i}{\Omega_i}\frac{1}{\Omega_i}\frac{\nabla_\perp p_i + \nabla_\perp \cdot \mathscr{P}}{m_i n_i} + \frac{1}{\Omega_i}\frac{\partial}{\partial t}\vec{e}_z \times \vec{v}_{i\perp}. \tag{4}$$

The ion sound response in the parallel direction is ignored for $v_{ti} \ll \omega/k_z$, and we shall work in the limit of wavelengths much longer than the ion gyro radius. As a result of such ordering, the viscosity tensor contributions can be omitted while the contribution of the gyro-viscous part of the stress tensor to the convective derivative in the ion polarization drift, is cancelled out by the diamagnetic drift term. The collisional term in the ion momentum, in the limit of stationary neutrals is a sum of the elastic collision rate and various inelastic collision terms, with the charge exchange rate that usually dominates the others.

Taking this into account and using the Ampère law which yields $v_{ez1} = \nabla_\perp^2 A_{z1}/(\mu_0 e n_0)$, for perturbations of the form $\sim \exp(-i\omega t + ik_\perp x + ik_z z)$, from the quasi-neutrality we obtain $\phi_1 =$

$\Omega_i B_0 k_z v_A^2 \rho_s^2 e A_{z1}/[(\omega + i\nu_i)\kappa T_e] - \rho_L^2 \Omega_i B_0 n_1/n_0$. Here $\rho_L = v_{Ti}/\Omega_i$, $v_{Ti}^2 = \kappa T_i/m_i$. Combined electron continuity and parallel momentum equations yield

$$\frac{e\phi_1}{\kappa T_e} - \frac{\omega}{k_z}\frac{eA_{z1}}{\kappa T_e} + i\left(\frac{1}{k_z D_z} - \frac{k_z}{i\omega - \frac{Q_0}{n_0}}\right) v_{ez1} = -\frac{1}{i\omega - \frac{Q_0}{n_0}} \frac{Q_0}{n_0}\frac{\sigma'}{\sigma_0}\frac{e\varphi_1}{\kappa T_e}. \quad (5)$$

The notation introduced here is: $D_z = \kappa T_e/(m_e \nu_e)$, $Q_0 = \sigma_0 n_n f$, $\sigma' \equiv d\sigma_0/d\Phi$, $\Phi = e\phi_0/(\kappa T_e)$, and the perturbed source/sink term is written as $S_1 = Q_0[(\sigma'/\sigma)e\varphi_1/(\kappa T_e) - n_1/n_0]$. The perturbed energy of the bombarding electrons is given by $\mathscr{E}_1 = e\varphi_1 = e(\phi_1 + A_{z1}v_{f0})$, where v_{f0} denotes the starting velocity of the electrons in the flux which may be both positive and negative, and whose absolute value must exceed the ionization velocity $v_I = (2\varepsilon_I/m_e)^{1/2}$. This follows from the expression for the change of the kinetic energy of the ionizing electrons, with the given velocity v_{f0}, when they enter the area of the perturbed electromagnetic field.

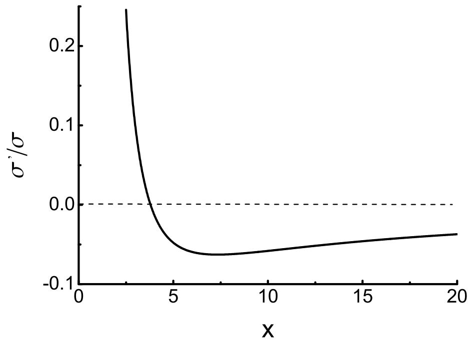

Figure 1: The ratio $\sigma'(x)/\sigma(x)$ in terms of the normalized energy x of bombarding electrons.

Formally, it is obtained from the electron momentum equation after taking the dot product with $\vec{v}_{fe}$,

$$\frac{d}{dt}\left(\frac{m_e v_{fe}^2}{2}\right) = -e\vec{E}\cdot\vec{v}_{fe} = e\nabla\phi\cdot\vec{v}_{fe} + e\frac{\partial\vec{A}}{\partial t}\cdot\vec{v}_{fe}. \quad (6)$$

Here, the subscript f is added to denote that we are dealing with the electrons from the flux. Further, applying the previously adopted model in which the electron flux is directed in the parallel direction, and the perturbed vector potential has the parallel component only, and linearizing the corresponding expressions one finds the above given perturbed kinetic energy of the ionizing electrons.

From these equations we obtain the following dispersion equation:

$$\omega^2 - k_z^2 v_A^2\left[1 + k_\perp^2\left(\rho_s^2 + \rho_L^2\right)\right] - \nu_i\left(\frac{Q_0}{n_0} + \frac{Q_0}{n_0}\frac{\rho_L^2}{\rho_s^2}\frac{\sigma'}{\sigma_0} + \frac{k_\perp^2\rho_s^2 v_A^2}{D_z}\right) - \frac{k_\perp^2 v_A^2}{D_z}\frac{Q_0}{n_0}\left(\rho_s^2 + \rho_L^2\frac{\sigma'}{\sigma_0}\right)$$

$$-\frac{Q_0}{n_0}\frac{v_{f0}k_z\nu_i}{\omega}\frac{\sigma'}{\sigma_0}\left(1 + \frac{\rho_L^2}{\rho_s^2}\right) + i\left\{\omega\left[\frac{Q_0}{n_0}\left(1 + \frac{\rho_L^2}{\rho_s^2}\frac{\sigma'}{\sigma_0}\right) + \nu_i + \frac{k_\perp^2\rho_s^2 v_A^2}{D_z}\right] + k_z v_{f0}\frac{Q_0}{n_0}\frac{\sigma'}{\sigma_0}\left(1 + \frac{\rho_L^2}{\rho_s^2}\right)\right.$$

$$\left. -\frac{1}{\omega}\left[\nu_i\left(k_\perp^2 k_z^2 v_A^2\left(\rho_s^2 + \rho_L^2\right) + \frac{k_\perp^2\rho_s^2 v_A^2}{D_z}\frac{Q_0}{n_0}\left(\rho_s^2 + \rho_L^2\frac{\sigma'}{\sigma_0}\right)\right) + k_z^2 v_A^2\frac{Q_0}{n_0}\left(1 - \frac{\sigma'}{\sigma_0}\right)\right]\right\} = 0. \quad (7)$$

An instability condition can be easily obtained in the limit of negligible ion collision and thermal terms in which case Eq. (7) yields:

$$\omega_r^2 \approx k_z^2 v_A^2 \left(1 + k_\perp^2 \rho_s^2\right) - \frac{k_\perp^2 v_A^2}{D_z} \frac{Q_0}{n_0} \rho_s^2, \tag{8}$$

$$\omega_i \approx -\frac{1}{2}\left(\frac{Q_0}{n_0} + \frac{k_\perp^2 \rho_s^2 v_A^2}{D_z}\right) + \frac{k_z^2 v_A^2}{2\omega_r^2} \frac{Q_0}{n_0}\left(1 - \frac{\sigma'}{\sigma_0}\right) - \frac{k_z v_{f0}}{2\omega_r} \frac{Q_0}{n_0} \frac{\sigma'}{\sigma_0}. \tag{9}$$

In the case $v_{f0} > 0$, or $|v_{f0}|/v_A \ll 1$, the mode becomes unstable if

$$\frac{\sigma'}{\sigma_0} < \frac{1}{1 + \dfrac{\omega_r v_{f0}}{k_z v_A^2}} \left[1 - \frac{\omega_r^2}{k_z^2 v_A^2}\left(1 + \frac{n_0}{Q_0} \frac{k_z^2 \rho_s^2 v_A^2}{D_z}\right)\right]. \tag{10}$$

The right-hand side of (10) is negative and, consequently, the instability occurs in the domain where the ionization curve is descending. This is similar to the purely collision-less case of the kinetic Alfvén wave when the the density and the potential have opposite phases: $n_1/n_0 = -k_\perp^2 \rho_s^2 e\phi_1/(\kappa T_e)$, i.e. $n_1 \sim \exp(i\pi)\phi_1$. Hence, at locations of increased density, the potential (and the energy of the ionizing electrons) is decreased, and vice versa. According to the shape of the ionization curve, this results into an increased ionization and into a growing mode. It may be noted that the maximum value of $|\sigma'/\sigma_0|$ from Fig. 1 is around 0.06, and it changes slowly for larger values of x, while in the same time $k_\perp^2 \rho_s^2 \ll 1$. The instability is therefore possible in the long wavelength limit for a wide energy range of ionizing electrons.

A different situation occurs for a large and negative velocity v_{f0}. In the limit $|v_{f0}| \gg v_A$, Eq. (9) yields the following instability criterion:

$$\frac{\sigma'}{\sigma_0} > k_\perp^2 \rho_s^2 \left(1 + \frac{n_0 v_A^2}{Q_0 D_z}\right) \frac{v_A}{|v_{f0}|}. \tag{11}$$

Depending on the wave and plasma parameters the instability may here develop at very small values of σ'/σ_0, and in the domain where the cross section curve rises.

In conclusion, the present work is the first attempt to describe the instability of an electromagnetic mode in a partially ionized plasma, caused by inelastic collisions. Such collisions are responsible for the creation and loss of plasma particles, therefore the term reacting plasma used sometimes in the literature. The applications of our results are numerous as space plasmas are often only partially ionized, and the complexity and importance of physical phenomena related to the presence of neutrals are very much underestimated in the literature dealing with wave phenomena.

References

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