*33rd EPS Conference on Plasma Phys. Rome, 19 - 23 June 2006 ECA Vol.**30I**, P-1.059 (2006)*

# Linear analysis of the Kelvin-Helmholtz instability for relativistic magnetohydrodynamics

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Abstract: A linear analysis of the Kelvin Helmholtz instability (KHI) in a two dimensional relativistic magneto-hydrodynamics is considered. As appropriate for astrophysical conditions, we neglect gravity, surface tension and viscosity, so that the relevant equations are those given by particle number and energy-momentum density conservation of a relativistic perfect fluid in flat Minkowskian metric, added by the induction equation. In a vortex sheet approximation an equation governing the dispersion relation is derived and numerically solved. Based on physical parameters, regimes of the KHI are studied and corresponding ranges of parameters are analyzed.

## Introduction

The Kelvin Helmholtz instability often accompanies flows where velocity shear is present. The original work was done in [2],[3] and generalized in [4] for compressible fluids. The effect of magnetic field has been considered in [5], the relativistic case neglecting the magnetic field was examined by [6], [7] and [8]. In [10] neglecting the current displacement the linear analysis of relativistic KHI for magnetized fluids was considered. In this paper we reexamine the problem of the KHI for fully relativistic magnetized fluids in the vortex sheet approximation, considering a more general system of equations than in [10], without neglecting the displacement current, examining also relativistic Alfvèn velocities, and analyze the stability problem depending on physical parameters.

**Main consideration**

For studying the KHI problem we consider the interface to be located in $x-z$ plane and velocities of fluids to be $\mathbf{V_0}$ and $-\mathbf{V_0}$, magnetic field has only a longitudinal component (along the x axis) which is equal to $B_0$.

We start by the special relativistic MHD set of equations: [9]

$$\frac{\partial(\gamma\rho)}{\partial t} + \nabla_i\left(\rho\gamma V^i\right) = 0 \qquad (1a)$$

$$\frac{\partial\left(W\gamma^2V^i - b^0b^i\right)}{\partial t} + \nabla_j\left(W\gamma^2V^iV^j - b^ib^j + \delta_{ij}P_t\right) = 0 \qquad (1b)$$

$$\frac{\partial\left(W\gamma^2 - b^{0^2} - P_t\right)}{\partial t} + \nabla_j\left(W\gamma^2V^j - b^0b^j\right) = 0 \qquad (1c)$$

$$\frac{\partial\mathbf{B}}{\partial t} - \nabla\times\mathbf{V}\times\mathbf{B} = 0 \qquad (1d)$$

where $b^\alpha = [\gamma(\mathbf{VB}), \mathbf{B}/\gamma + \gamma(\mathbf{VB})\mathbf{V}]$, $W = \rho h + |b|^2$, $P_t = P + \frac{1}{2}|b|^2$, $\mathbf{VB} = V_xB_x + V_yB_y$, $\gamma$ is the Lorentz factor of the fluid, $P_t$ and $P$ are respectively the total and the thermodynamic pressure and $h$ is the specific enthalpy. Equations are written down in the unit of $c = 1$, where $c$ is the speed of light. For closing the system of equations one may add also the equation of state for a perfect fluid: $h = 1 + \frac{\Gamma}{\Gamma-1}\frac{P}{\rho}$ where $\Gamma$ is the polytropic index of the gas.

The perturbation for physical quantities can be given by following: $\delta\Psi_\pm \propto exp\left[i\left(kx + q_\pm y - \omega t\right)\right]$. Here subscripts $+$ and $-$ correspond to regions with $y > 0$ and $y < 0$ respectively. Perturbing the Eqs.(1) one can get the system of equations:

$$\frac{q_+}{q_-} = \frac{\rho h\gamma^2(\omega - kv)^2 + (\omega^2 - k^2)B_0^2}{\rho h\gamma^2(\omega + kv)^2 + (\omega^2 - k^2)B_0^2} \qquad (2a)$$

$$q_\pm^2 = \frac{\widetilde{\omega}_\mp^2\left[(C_s^2 + V_A^2)\widetilde{k}_\mp^2 - \widetilde{\omega}_\mp^2\right] - C_s^2V_A^2\widetilde{k}_\mp^4}{C_s^2V_A^2(\widetilde{k}_\mp^2 + \widetilde{\omega}_\mp^2) - (C_s^2 + V_A^2)\widetilde{\omega}_\mp^2} \qquad (2b)$$

where $\widetilde{\omega}_\pm = \gamma(\omega \mp kv)$ and $\widetilde{k}_\pm = \gamma(k \mp \omega v)$.

The system Eqs.(2) was solved numerically by a C code by using the online gsl library.

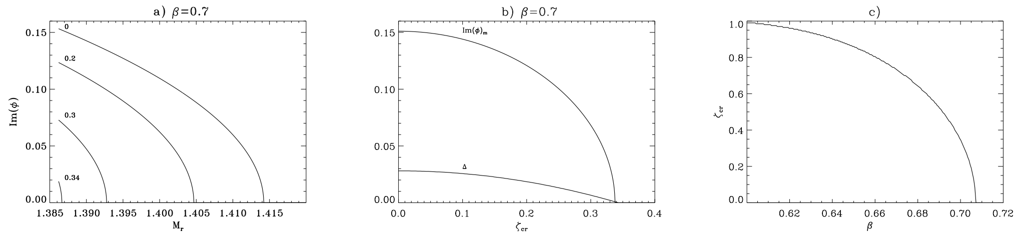


Figure 1: Plots of: a) $Im(\phi(M_r))$ for $\beta = 0.7$, labels on each curve indicate the values of $\zeta$. Set of parameters is: $\zeta = [0, 0.2, 0.3, 0.34]$, $\Gamma = 4/3$; b) $\Delta(\zeta)$ and $Im(\phi(\zeta))_m$ for $\beta = 0.7$; c) $\zeta_c(\beta)$.

**Results**

On Fig.1a we show a behaviour of the growth rate versus the relativistic Mach number $M_r \equiv \gamma M/\gamma_s$ ($\gamma$ is the Lorentz factor of the fluid, $M$-the Mach number and $\gamma_s$-the Lorentz factor corresponding to the speed of sound) for $\beta = 0.7$ ($\Gamma = 4/3$). The increase of $\zeta$ provokes the corresponding decrease of the maximum value of the growth rate and of the the range of $M_r$ in which the KHI exists. As one can see from the graph, the flow for $V_A \simeq 0.35C_s$ is already stable (see Fig.1a ). The Fig.1b shows the behaviour of the range of critical values of Mach numbers: $\Delta = M_r^{max} - M_r^{min}$ and a maximum possible value of the growth rate versus $\zeta$ for a given $\beta$. As it is clear from the graph, for a zero magnetic field, the range of $M_r$ corresponding to the appearance of the KHI is maximum, but when $\zeta$ becomes equal to $\sim 0.35$, the instability disappears. The Fig.1c shows $\zeta_{cr}$ versus $\beta$, as one can seen, for ultra relativistic velocities, $\zeta_{cr}$ is decreasing, and when $\beta = 1/\sqrt{2}$ [7] the flow becomes already stable even without the magnetic field.

**Summary**

In the present work we considered the linear analysis of the relativistic KHI problem in a two dimensional case for the vortex sheet approximation with a magnetic field directed along the flow motion. The system of algebraic equations governing the dispersion relation was derived and numerically solved. The roots of Eqs.(2) were analyzed from the point of view of a role of magnetic field, and relativistic Mach number on efficiency of the KHI. It was argued that by increasing the magnetic field, thus by increasing the magnetic pressure, the flow was becoming a more stable. Unlike the non-relativistic case, for $\beta = 0.7$ we have seen that the role of the magnetic field in the stabilization was diminishing due to the increasing role of the relativistic effects, up to the case of $\beta > 1/\sqrt{2}$ when only by virtue of them, the KHI was forbidden and the magnetic field was not any more necessary for stabilizing the flow (see Fig.1c).

The linear analysis is an approximation that is not valid always, so it is reasonable to consider a non-linear analysis of the KHI. For this reason we implement the numerical code PLUTO and depending on the physical parameters analyze the non-linear behaviour of the instability. This work is in preparation.

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