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Role of the electrostatic field in α^2-Dynamo

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The importance of electrostatic mechanisms in the set-up and the sustainement of a dynamo has been recently shown in the context of the Reversed Field Pinch devices [1]. Before this work, in astrophysical as well as RFP dynamos, no attention was given to the electric field, nor to its electrostatic nature, mainly because equations do not involve it explicitely. To describe the generation of large scale astrophysical magnetic fields, many mechanisms have been proposed, among which, the so called α^2 and $\alpha-\omega$ dynamos [2, 3]. Following [1], we investigate, in this paper, the role of the electrostatic field in the context of an α^2 dynamo situation. There are some important differences with RFP models: the geometry of course, but also an external uniform inductive electric field is applied in the latter, whereas a small scales forcing is imposed onto the flow in the former. It is not therefore straighforward to infer electrostatic α^2 dynamo properties from RFP results.

We inject small scale helicity in a three-dimensional resistive MHD turbulent flow in presence of a weak initial magnetic field. The magnetic dynamic of such α^2 dynamo prototype flows are such that, generally, the energy concentrates on one large scale mode [4, 5, 6]. One important ingredient in the driving of this nonlinear self-organization process is the inverse magnetic helicity cascade, leading to the large scale paradigm equation [3]

$$\frac{\partial}{\partial t}\mathbf{b}_{LS} = \alpha\nabla\times\mathbf{b}_{LS} + \beta\Delta\mathbf{b}_{LS}\,. \qquad (1)$$

Here α and β are parameters controlled by turbulent and statistical properties of the fields, and $\mathbf{b}_{LS}$ is the troncature of the magnetic field at the largest scales. However there are still some controversies about the meaning and the validity of the model equation (1) [4, 7]. Noteworthy, once large scale structures have grown up, suppression or/and sign inversion of the kinetic helicity injection rate do not modify the Fourier distribution of the magnetic helicity, indicating that the inverse cascade does not operate efficiently anymore. We find that the origin of such behavior is linked to the emergence of a robust and possibly dominant electrostatic field, inhibiting induction at forcing scales, and therefore any signed helicity inverse cascade. It follows that taking into account explicitly in dynamo models some properties of the electrostatic charges could be an interesting track.

Since $\partial\mathbf{b}/\partial t = \nabla\times\mathbf{E}$, to the extent that large scales evolve slowly, a condition for a quasistationnary state to take place is to satisfy the identities [1]

$$\eta \mathbf{j} - \mathbf{u} \times \mathbf{b} = \mathbf{E} \sim -\nabla\Phi \,, \tag{2}$$

where $\mathbf{j} = \nabla \times \mathbf{b}$ is the courant, $\mathbf{E}$ the electric field and Φ is the electrostatic potential. We should therefore expect the domination of the electrostatic drift velocity in the perpendicular direction $\mathbf{u}_{\mathrm{ES}} = (\nabla\Phi \times \mathbf{b})/b^2$, at least in zones where b is not null. Nevertheless, as noted above, on resistive time scales, the system does not reach a complete stationnary state, and we first need to provide a quantification of the electic fields in respectively the linear, the inverse cascade and the saturation phases.

Let us consider a 3D periodic (box of volume $[2\pi]^3$ and grid size is 196^3) and incompressible flow $\mathbf{u}$ with a constant plasma density, forced by divergence free fields $\mathbf{f}$ localized around a wave number $k_f \gg 1$. The ratio of injected helicity is maximal, i.e. relative helicity $H_r(\mathbf{f}) = +1$ at any time. Note that similar forcings have been used by Meneguzzi *et al* [8] and Brandenburg [6]. In usual units, the equations of incompressible MHD read

$$\frac{D}{Dt}\mathbf{u} = -\nabla p + (\mathbf{b}\cdot\nabla)\mathbf{b} + \nu\nabla^2\mathbf{u} + \mathbf{f} \,, \tag{3}$$

$$\frac{\partial}{\partial t}\mathbf{b} = \nabla \times (\mathbf{u} \times \mathbf{b}) + \eta\nabla^2\mathbf{b} \,, \tag{4}$$

where, by definition, $\mathbf{b}$ is the magnetic field, η the magnetic diffusivity, ν the kinematic viscosity and the pressure term p is the sum of the hydrostatic pressure and the magnetic pressure. Here, both fluid velocity $\mathbf{u}$ and magnetic field $\mathbf{b}$ are expressed in Alfvèn speed units. The magnetic Prandtl number is $P_m = \eta/\nu$ and we restrict ourselves to $P_m = 1$ with $\eta = \nu \ll 1$. Initial conditions are $\mathbf{u} = 0$ and $\mathbf{b} \sim 0$. The intensity of the forcing is tuned by the parameter $A_F = dE(\mathbf{f})/dt$, the density of energy injected in the flow by time unit. Another important parameter of the coloured forcing is $r_F = \frac{T_F}{T_{eddy}}$ the ratio of the characteristic time of the forcing (time to update all the random phases) over the eddyturnover time of the turbulent flow. $r_f = 0$ corresponds to a temporal uncorrelated forcing, and, noteworthy, the limit $r_f \to +\infty$ is the closest to the RFP case, the constant forcing playing the role of the uniform external electric field.

Let us consider now the situation where the flow is forced in the wavenumber range $k_F = 20 \pm 2$ with $A_F = \dot{E}(\mathbf{f}) = 2 * 10^{-4}$ and $r_F \sim 0.14$. ν and η are both set to $2 * 10^{-3}$, giving a Reynolds number based on the Taylor microscale around 2. Time evolution of kinetic (blue) and magnetic (green) energy density are plotted in figure 1. The upper graphs of figures 2 and 3 are

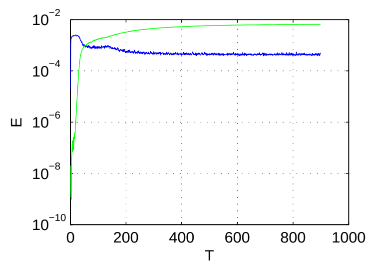

Figure 1: Energies versus time

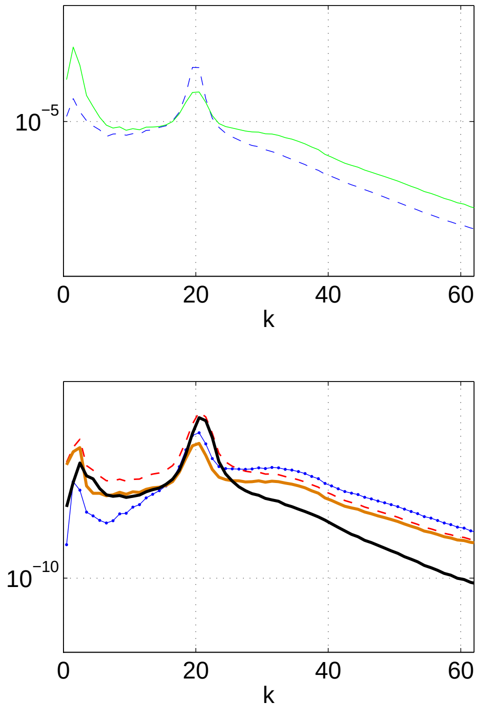

Figure 2: T=100

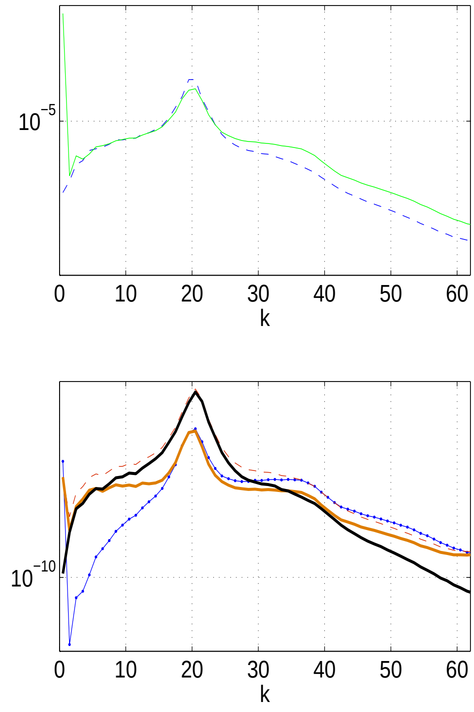

Figure 3: T=900

kinetic (blue) and magnetic (green) spectra, respectively at times $T = 100$ and $T = 900$. We use the same scales on both figures. As expected, a clear three phases dynamo process is observed, the linear phase (i.e. when Laplace/Lorentz force effect is negligeable) ending at time 40, then the inverse magnetic cascade process takes place until $T \sim 200$, to end up with saturation phase where kinetic energy is stabilized and the magnetic one slowly increases by piling up at the box scales.

The lower graphs of figures 2 and 3 focus on Ohm's law $\eta \mathbf{J} = \mathbf{u} \times \mathbf{b} + \mathbf{E}_{es} + \mathbf{E}_{ind}$ and more precisely on the associated spectra $E^2_{ind}(k)$ [thick, brown] , $E^2_{es}(k)$ [thick, black] and $\eta^2 J^2(k)$ [dotted, blue]. Here E^2_{es} and E^2_{ind} denote respectively the electrostatic and inductive electric field. There is also the total electric field spectrum $E^2(k) = E^2_{ind} + E^2_{es}$ [dashed, red]. At time $T = 100$, the spectrum of $\eta^2 J^2$ indicates that small scales ($k > 35$) are dominated by dissipative effect $\eta \mathbf{J} \sim \mathbf{u} \times \mathbf{b}$. At scales where inverse cascade occurs, the inductive electric field is strong, even though it coexists with an electrostatic one. Note also that the cascade generates a large scale flow, via Lorentz's force, disappearing in the saturation phase. Indeed, at $T = 900$, an equipartition between kinetic and magnetic spectra is approximately reached at intermediate scales ($k > k_{LS} = 1$ and $k \ll k_f$) while the largest scales ($k = k_{LS}$) are controlled by the resistive term $\eta \mathbf{J}$ in Ohm's law.

At forcing scales, the electrostatic field dominates the inductive one, and, furthermore, the inductive electric spectrum is balanced by the courant's one. This suggest the following scenario: first, dissipation of k_F-inductive structure $\eta \mathbf{J} \sim \mathbf{E}_{ind}$, second, generation of k_F-electrostatic charges via magnetic structures induced by the flow thanks to the $\mathbf{u} \times \mathbf{b}$ term. Since the inverse cascade mechanism is inefficient at forcing scales, one might wonder whether the spectral overlapping of ηJ with E_{ind} is the result of a common action of the Alfvèn waves - *a priori* generated and guided by the large scale magnetic structure - and the forcing.

During the phase of merging of magnetic structures, $E_{es}(k,t)$ becomes the dominant term at forcing scales, and, it follows an inverse cascade. This cascade is generally incomplete and depends on various parameters. Indeed retroaction of the largest scales on the intermediate ones can slow down and eventually stop the cascade. In the limit $r_f \longrightarrow +\infty$, we found a full inverse cascade until the largest scales.

Those results show that the electrostatic field is a major actor in the sustainement of the dynamo. In fact, in spatial zones far enough from a magnetic null point, electrostatic velocity $\mathbf{u}_{\mathrm{ES}}$ is defined and, as a first order approximation, it is straightforward to get $\partial_t \mathbf{b} \sim \nabla \times \alpha_{es} \mathbf{b}$ where $\alpha_{es} = E_{\parallel}^{es}/b$. This indicates that the electrostatic charge separation along the field lines originates an α effect. This is to be constrated with the classical picture of inductive mechanisms, linked to the pinch of magnetic flux tube.

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