Fluctuation-Induced Cross-Field Electron Transport and Dynamo Effect Accompanied by Turbulent Diffusion in an Electron Beam-Plasma System

R. Sugaya, T. Maehara and M. Sugawa

Department of Physics, Faculty of Science, Ehime University
Bunkyo-cho 2-5, Matsuyama 790-8577, Japan

Abstract

It has been proved theoretically and numerically that the cross-field electron transport and dynamo effect accompanied by turbulent diffusion are induced by the unstable localized electrostatic waves in an electron beam-plasma system.

1. Introduction

Fluctuation-induced cross-field electron transport and dynamo effect accompanied by turbulent diffusion in an electron beam-plasma system were investigated theoretically and numerically on the basis of the transport equations and magnetohydrodynamic equations\(^1\). It was verified that in the first stage where the unstable localized electrostatic waves are growing linearly or nonlinearly the cross-field electron transport and the generation of cross-field electric field are dominant\(^4\), and in the second stage where the electrostatic waves saturate in the amplitude the turbulent diffusion becomes dominant in the formation of the plasma profile. In the first stage the parallel and perpendicular electron transport generates a large and abrupt dip of the electron density and a strong peak of the electron temperature in the radial direction. Simultaneously the cross-field electric field is produced partially by the cross-field electron transport and partially by the perpendicular pressure gradient, that is, it is given by \(E_\perp = B_0 \times v_{e\perp} / c - (\nabla \times p_e) / e n_e \left( \frac{p_e}{n_e k_B T_e} \right)\). The small cross-field electron transport is predicted to create the strong cross-field electric field. On the other hand, the parallel electric field \(E_\parallel = - (\nabla \times p_e) / e n_e + m_e v_{e\parallel} v_{e\parallel} / e\) is created by the parallel electron transport and the parallel ambipolar diffusion. Here, \(B_0 = (0,0,B_0)\) is the external magnetic field in the \(z\) direction, \(v_{e\perp} = (v_{e\perp},v_{e\perp},0)\) is the cross-field drift velocity of plasma electrons, \(v_{e\parallel} = (0,0,v_{e\parallel})\) is the parallel drift velocity of plasma electrons, and \(v_{en}\) is the electron-neutral collision frequency. In the second stage the dip of the electron density, the peak of the electron temperature and the electric field vanish almost completely via the turbulent diffusion due to the large amplitude electrostatic waves. The radial and axial profiles of the electron density and temperature and the electric field were obtained numerically and they can explain well the experimental observation in an electron beam-plasma system qualitatively and quantitatively\(^5\)\(^-\)\(^7\).

2. Transport Equations

We analyze the electron transport arising from due to spatially localized unstable electrostatic waves. The transport equations and magnetohydrodynamic equations for
plasma electrons are expressed as
\[
\frac{\partial U_e}{\partial t} = -2\gamma^{(e)}_k U_k , \tag{1}
\]
\[
\frac{\partial \mathbf{P}_e}{\partial t} = -2\gamma^{(e)}_k k U_k - en_e E_\parallel - \nabla_\parallel \mathbf{P}_e - m_e n_e v_e v_{e\parallel} , \tag{2}
\]
\[
n_e v_{e\parallel} - \mu_e n_e E_\parallel - \nabla_\parallel (D_e n_e) = 0 , \tag{3}
\]
\[
- en_e E_\perp - (e/c)n_e v_{e\perp} \times B_0 - \nabla_\perp p_e = 0 , \tag{4}
\]
\[
\frac{\partial n_e}{\partial t} + \nabla \cdot \mathbf{J}_e = 0 , \tag{5}
\]
\[
\mathbf{J}_e = n_e v_e - \mu_e n_e E_\parallel - \nabla_\parallel (D_e n_e + n_e v_{e\parallel}^2/v_{e\perp}) - \nabla_\perp (D_e n_e) , \tag{6}
\]
\[
\mathbf{V} \cdot \mathbf{E} = 0 , \tag{7}
\]
where \( U_k = (1/8\pi) \left( \epsilon (\omega_k / \partial \omega_k) \right) |E_k|^2 \) is the wave energy density, \( \mathbf{k} U_k / \omega_k \) is the wave momentum density, \( U_e = m_e n_e v_e^2 / 2 + 3n_k T_e / 2 \) and \( \mathbf{P}_e = m_e n_e \mathbf{v}_e \) are the energy and momentum densities of plasma electrons, \( \gamma^{(e)}_k \) is the linear Landau and cyclotron damping rate ascribed to plasma electrons, \( \mu_e = e / m_e v_e \) is the electron mobility, \( D_e = k_B T_e / m_e v_{e\perp} \) is the electron diffusion coefficient, \( \epsilon_k = \epsilon_k^0 + i\epsilon_k^\ast \) is the dielectric constant, and \( \mathbf{k} = (k_\parallel, 0, k_\perp) \). \( D_a \) shows the anomalous turbulent diffusion coefficient and it is assumed that \( D_a = D_{a0} U_k / U_k(0) \) and \( D_{a0} = \beta_a k_B T_e / m_e \omega_k \) (\( \beta_a = 1 \)). Here, \( U_k(0) \) and \( T_{e0} \) are \( U_k \) and \( T_e \) at \( t = 0, \ x, y, z = 0 \), respectively. In Eq. (6), the mobility and the diffusion of plasma electrons across the magnetic field is neglected, namely \( \mu_{e\perp} \) and \( \nabla_\perp \left( D_{e\perp} n_e + (v_{e\perp} / \omega_{e\perp}) n_e v_{e\perp}^2 \right) \) are neglected because of \( v_{e\perp} / \omega_{e\perp} \ll 1 \) (\( \omega_{e\perp} \) is the electron cyclotron frequency). On the other hand, Eq. (3) means that the diffusion along the magnetic field is the ambipolar diffusion. According to Eq. (3), the electron flux given by Eq. (6) is reduced to
\[
\Gamma_e = n_e v_{e\perp} - \nabla_\perp (D_{a\perp} n_e) , \tag{8}
\]
under the condition of \( D_{a\perp} n_e \gg n_e v_{e\parallel}^2 / v_{e\perp} \), and the transport equation (2) becomes
\[
\frac{\partial \mathbf{P}_e}{\partial t} = -2\gamma^{(e)}_k k U_k - 2m_e n_e v_{e\parallel} v_{e\parallel} , \tag{9}
\]
Equations (1) and (2) predict that the electrostatic waves create strong electron transport or acceleration along and across the magnetic field. The general Ohm’s law of Eq. (4) shows that the cross-field electric field \( E_\perp \) is produced by the dynamo effect of the cross-field electron drift and the radial pressure gradient induced by the electron transport.

3. Numerical Analysis

The temporal and spatial development of the system has been investigated numerically assuming that the electrostatic waves excited in an electron-beam plasma system are localized radially such that \( U_k \propto \exp \left[ -\left( x^2 + y^2 \right) / a^2 \right] \), and the background plasma is spatially uniform initially. Here \( a \) means the beam radius. The kinetic wave equations
for electrostatic waves are assumed as follows:

\[
\frac{\partial U_k}{\partial t} = \left(2\gamma_k^{(e)} + 2\gamma_N e^{-2\nu_0^2}\right) U_k, \quad \frac{\partial U_k}{\partial z} = \left(2\gamma_i + 2\gamma_N e^{-2\nu_0^2}\right) U_k.
\]

The numerical analysis of Eqs. (1)-(10) was performed under the parameters of \(\omega_k / \omega_{ce} = 0.3\), \(k_z n_{e0} / \omega_{ce} = 1\), \(k_\| / k_z = 0.2\), \(\gamma_k^{(e)} / \omega_{ce} = 0.2\) (\(\gamma_k^{(e)} < 0\)), \(\gamma_N / \gamma_k^{(e)} = 1.2\), \(\gamma_N v_{e0} / \gamma_k^{(e)} = 0.7\), \(\gamma_0 v_{e0} / \gamma_k^{(e)} = 0.2\), \(\gamma_0 v_{e0} / \gamma_k^{(e)} = 0.1\), \(\gamma_1 v_{e0} / \gamma_k^{(e)} = 0.02\), \(\gamma_2 v_{e0} / \gamma_k^{(e)} = 0.1\) (\(\gamma_1, \gamma_2 < 0\)), \(v_{ce} / \gamma_k^{(e)} = 0.8\), \(\alpha \omega_{ce} / v_{e0} = 2\), \(U_k(0) / n_{e0} k_B T_{e0} = 1.1 \times 10^{-4}\), \(D_{e0} / \gamma_N a^2 = 0, 10^{-6}\), \(\beta_a = 1\), and \(v_{e0} = \sqrt{k_B T_{e0} / m_e}\). In Eqs. (1) and (2), \(\gamma_i = -\gamma_2\), and it is assumed that \(\gamma_i = -\gamma_2\) in \(D_a\) in Eq. (6). Thus the temporal evolution of the three-dimensional profiles of the system was obtained. We show the only temporal evolutions of the transverse profiles in the \(x\) and \(y\) directions for the fixed value of \(z\) at \(t = 10\). Figure 1 exhibits the temporal evolutions of the transverse profiles without the anomalous turbulent diffusion due to the electrostatic waves, where \(D_{e0} / \gamma_N a^2 = 0\), that is, \(n_e / n_{e0}\) and \(T_e / T_{e0}\) versus \(x / a\) and \(y / a\) are shown as a parameter of \(\gamma_k^{(e)}\) at \(t = 1, 2\) and 3 for the \(x-y\) profiles of (a), (b) and (c), respectively, where \(n_{e0} = n_{e0}(0)\) and \(T_{e0} = T_{e0}(0)\). It is found that the hollow profiles of the electron density and the peak profiles of the electron temperature develop temporally and axially \((z\) direction). At the same time the cross-field electric field (not shown) is created. This means that the cross-field drift of plasma electrons (not shown) induced by the electrostatic waves generate the abrupt inhomogeneity of radial profile and the strong cross-field electric field. As was predicted in the previous work\(^5\), the transport equations (1) and (2) govern \(v_{ex}\) and \(v_{ez}\), and the generalized Ohm’s law (4) shows that \(E_y\) is determined by \(v_{ex}\) and \(\nabla_y P_e\). The axial electric field \(E_z\) is given by Eq. (3) and \(E_y\) and \(E_z\) produce \(E_x\) so as to satisfy the Poisson’s equation (7). Thus \(E_x\) and \(\nabla_x P_e\) create \(v_{ey}\) via \(E \times B\) drift and the diamagnetic drift. Figure 2 is the temporal evolutions of the transverse profiles of \(n_e / n_{e0}\) and \(T_e / T_{e0}\) with the anomalous turbulent diffusion, where \(D_{e0} / \gamma_N a^2 = 10^{-6}\) and \(\gamma_k^{(e)}\) at \(t = 1, 2\) and 3 for (a), (b) and (c). It can be seen that the hollow profiles of \(n_e\) and the peak profiles of \(T_e\) are suppressed significantly by the anomalous turbulent diffusion.

The typical parameters of the experiment reported by the author et al.\(^5\) are \(T_{e0} = 5\) eV, \(B_0 = 7 \times 10^{-3}\) T and \(a = 1.5 \times 10^{-3}\) m, and hence \(\alpha \omega_{ce} / v_{e0} = 2\) is satisfied. From the present numerical analysis (Fig. 2), we find that \(v_{er} / v_{e0} = (0.02, 0.02, 0.01)\) and \(eEa / k_B T_{e0} = (0.2, 0.2, 0.001)\) at \(\gamma_k^{(e)}\) at \(t = 2\). Thus, we obtain \(Ea = (1.1, 0.005)\) V, \(E = (6.7 \times 10^{2}, 6.7 \times 10^{2}, 3.3)\) V/m.

This value is close comparatively to the experimental value of \(V_{exp} \approx 10\) V. Also, the rather small values of \(v_{er} = v_{ey} = 0.19 \times 10^5\) m/s \((10^{-3}\text{ eV})\), \(v_{ex} = 0.9 \times 10^4\) m/s \((2.3 \times 10^{-4}\text{ eV})\) are obtained. It can be concluded that the obtained results can well explain the experimental observation.
Fig. 1 Temporal evolutions of the transverse profiles of $n_e/n_{e0}$ and $T_e/T_{e0}$. $D_{e0}/\gamma_x a^2 = 0$ and $|\gamma^{(e)}_k| t = 1$ for (a), 2 for (b) and 3 for (c).

Fig. 2 Temporal evolutions of the transverse profiles of $n_e/n_{e0}$ and $T_e/T_{e0}$. $D_{e0}/\gamma_x a^2 = 10^{-5}$ and $|\gamma^{(e)}_k| t = 1$ for (a), 2 for (b) and 3 for (c).

References